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NICHOLSON'S MATHEMATICAL SERIES

NEW
COMPLETE ARITHMETIC

ON THE INDUCTIVE METHOD, WITH PARALLEL
MENTAL AND WRITTEN EXERCISES

BY

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Agricultural and Mechanical College

UNIVERSITY PUBLISHING COMPANY
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PREFACE

In this revision of his complete arithmetic, the aim of the author has been to preserve the distinctive features of former editions, and to bring the work fully up-to-date in matter and method.

The best way to awaken interest, stimulate a spirit of self-reliance, and open the mind to a thorough knowledge of the subject is to lead a pupil by easy and graded steps to discover principles and methods for himself. Therefore a practical union of induction and deduction is one of the strongest possible features of a good arithmetic. A special effort has been made to construct the present work on this plan. Not only are new topics introduced by carefully prepared inductive exercises, but, under the heading of parallel problems, each exercise intended for written work is preceded by an inductive question. The teacher should encourage the pupils to look to these oral questions for such hints as they may need in solving the more difficult problems, and thus train them in induction and deduction, as well as in mental and written work.

The notation of numbers, operations, and relations forms the language of mathematics. It is not only the vehicle of thought, but very largely the means by which thought is directed and energized. In this, as in other things, it is a mistake to pass hastily from the concrete to the abstract.

Unless the student associates the notation with the concrete entities which it represents he will drop into the habit of studying *symbols* rather than *numbers,* and thus "catch the shadow rather than the substance" of a practical and fruitful knowledge of arithmetic. In this book a special effort is made to prevent the student from falling into this error, especially in the important subjects of fractions and percentage.

Definitions and rules serve many good purposes. They should not be formally committed to memory, but should be learned by associating them with the concepts which they define, or the principles on which they are based. They should be used just as a convalescent uses his crutches; that is, until the strength which he acquires by their support renders him independent of that support.

Professors T. H. Harris and R. L. Himes, of the Louisiana State University, and Misses D. L. Bostick, F. A. Stringer, and E. Aitkens, of the New Orleans Teachers Association, have rendered valuable assistance in the preparation of this book. Mr. D. M. Raymond, President of the First National Bank of Baton Rouge, has also given much valuable information in reference to banks. To these and others the author acknowledges his indebtedness, and gives assurance that any suggestions for the further improvement of the book will be thankfully received.

JAMES W. NICHOLSON.

BATON ROUGE, LA., *July*, 1902.

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COMPLETE ARITHMETIC.

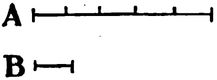
INTEGERS.

NOTATION AND NUMERATION.

1. Numbers arise in counting and measuring.

Thus, in counting these dots • • • • • we get the number five.

In measuring the line A by the line B we get the number five.



Some numbers (integers) are most simply interpreted as *counts,* and others (fractions) as *measures.*

2. A **unit** is the standard by which we count or measure.

Thus:

In counting eggs by ones, one egg is the unit.
In counting pens by the dozen, a dozen pens is the unit.
In measuring a line by the yard, a linear yard is the unit.
In measuring wine by the pint, a pint of wine is the unit.

Units of the Decimal System.

3. In counting a large number of objects it is convenient to arrange them in equal groups, and when these become large to arrange them again in equal groups, and so on. The **decimal system** of counting, which is in common use, consists in grouping by tens, and employing each group as a unit, as follows:

4. One is the name of the *primary* or *basic unit,* and is called a unit of the *first order.*

5. All numbers which may be counted by ones are called **integers** or **whole numbers.** The names of the first ten integers are

one, two, three, four, five, six, seven, eight, nine, ten

6. Ten is the name of a group of ten ones, and is called a unit of the *second order.*

We count by tens thus:

one ten, or ten,
two tens, or twenty,
three tens, or thirty,
four tens, or forty,
five tens, or fifty,
six tens, or sixty,
seven tens, or seventy,
eight tens, or eighty,
nine tens, or ninety,
ten tens, or hundred.

7. Hundred is the name of a group of ten tens, and is called a unit of the *third order.*

We count by hundreds as by tens, thus:

one hundred,
two hundreds,
three hundreds,
four hundreds,
five hundreds,
six hundreds,
seven hundreds,
eight hundreds,
nine hundreds,

ten hundreds, or thousand.

8. Thousand is the name of a group of ten hundreds, and is called a unit of the *fourth order.*

We count by thousands as by tens and hundreds.

9. From ten units of the fourth order is formed **a ten thousand**—a unit of the *fifth order;* and from ten units of the fifth order is formed **a hundred thousand**—a unit of the *sixth order;* and so on. That is, *ten units of any order form one unit of the next higher order.*

10. Names and Orders of the Units:

One	is a unit of the	first	order	Units.
Ten	"	second	"	
Hundred (Hund.)	"	third	"	

Thousand (Thous.)	is a unit of the	fourth	order	Thousands.
Ten thousand	“	fifth	“	
Hund. thousand	“	sixth	“	
Million (Mill.)	“	seventh	“	Millions.
Ten million	“	eighth	“	
Hund. million	“	ninth	“	
Billion (Bill.)	“	tenth	“	Billions.
Ten billion	“	eleventh	“	
Hund. billion	“	twelfth	“	
Trillion	“	thirteenth	order,	Trillions.
. . . .		. . . .		

Formation, Numeration, and Notation of Integers.

11. Integers are designated by *names* and *symbols.*

12. Numeration is the art of naming numbers, and **notation** is the art of expressing numbers by symbols.

13. In the **Arabic Notation** numbers are represented by symbols, called **figures.** The figures

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

stand for, respectively,

none, one, two, three, four, five, six, seven, eight, nine

The figure 0 is called **naught, cipher,** or **zero**; the other nine figures are called **digits** or **significant figures.**

14. All integers greater than nine are regarded as formed of units of different orders, and are named and expressed by figures accordingly. Thus:

An integer formed of 4 thousands, 6 hundreds, 3 tens, 5 ones, is named *four thousand six hundred thirty-five,* and is denoted by 4,635.

15. PRINCIPLE.—*When figures are written side by side, the position or place of each figure, counting from the right, indicates the order of units that the figure represents.*

EXERCISES (Written).

16. Give the name and notation of the number formed of:

1. 2 tens 0 ones. Ans. Name, *twenty;* notation, 20.

2. 3 tens 7 ones. Ans. Name, *thirty-seven;* notation, 37.

3. 2 tens 1 one; 1 ten 3 ones; 1 ten 8 ones; 2 tens 4 ones.

4. 3 tens 0 ones; 3 tens 9 ones; 3 tens 2 ones; 3 tens 7 ones.

5. 4 tens 2 ones; 4 tens 8 ones; 4 tens 9 ones; 5 tens 0 ones.

6. 5 tens 3 ones; 6 tens 1 one; 6 tens 3 ones; 7 tens 5 ones.

7. 8 tens 4 ones; 8 tens 8 ones; 9 tens 0 ones; 9 tens 9 ones.

8. 3 hunds. 5 tens 2 ones. Ans. *three hundred fifty-two,* 352.

9. 5 hunds. 4 tens 7 ones; 8 hunds. 0 tens 5 ones.

10. 6 hunds. 1 ten 6 ones; 4 hunds. 3 tens 0 ones.

11. 7 hunds. 5 tens 8 ones; 3 hunds. 3 tens 3 ones.

12. 5 hunds. 4 tens 3 ones; 8 hunds. 0 tens 0 ones.

13. 2 hunds. 1 ten 9 ones; 9 hunds. 9 tens 9 ones.

Give the formation and notation of:

14. Seventeen.

Ans. Formation, 1 ten 7 ones; notation, 17.

15. Fifteen; twenty-three; twenty-nine; thirty.

16. Sixteen; thirty-seven; forty-eight; fifty-one.

17. Seventy; ninety-four; sixty-seven; thirty-two.

18. Two hundred forty-one; four hundred fifty-five.

19. Six hundred; seven hundred seventy-seven.

20. Three hundred five; four hundred one.

Give the formation and name of :

21. 75. Ans. 7 tens 5 ones, seventy-five.

22. 83, 21, 64, 42, 47, 58.

23. 13, 12, 46, 24, 74, 85.

24. 30, 90, 71, 97, 18, 60.

25. 51, 67, 88, 33, 77, 99.

26. 384. Ans. 3 hunds. 8 tens 4 ones, three hundred eighty-four.

27. 437, 901, 200, 418, 111, 101.

28. 100, 600, 732, 410, 222, 888.

29. 529, 582, 327, 408, 777, 999.

Large Integers.

17. In writing and reading integers greater than a thousand, the figures are separated into groups of three figures each, called **periods.** These periods, beginning at the right, contain the *hundreds, tens,* and *ones* of the **units, thousands, millions, billions, trillions, quadrillions,** and so on.

Thus, the formation of the integer represented by

360504725913876

is shown by the following numeration table :

	Hundreds.	Tens.	Ones.	Hundreds.	Tens.	Ones.	Hundreds.	Tens.	Ones.	Hundreds.	Tens.	Ones.	Hundreds.	Tens.	Ones.
Places of units	15th.	14th.	13th.	12th.	11th.	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.
Figures	3	6	0,	5	0	4,	7	2	5,	9	1	3,	8	7	6.
Periods: Numbers . . .	Fifth.			Fourth.			Third.			Second.			First.		
Periods: Names	Trillions.			Billions.			Millions.			Thousands.			Units.		

Therefore the formation of the integer by periods is

360 trills. 504 bills. 725 mills. 913 thous. 876 units,

and its name is, three hundred sixty trillion five hundred four billion seven hundred twenty-five million nine hundred thirteen thousand eight hundred seventy-six.

The name and notation of an integer formed of 16 billions, 205 thousands, 32 units are sixteen billion two hundred five thousand thirty-two, and 16,000,205,032.

The formation and notation of the integer two trillion forty-five million two hundred nine are 2 trills. 0 bills. 45 mills. 0 thous. 209 units, and 2,000,045,000,209.

18. ***To give the formation and name of a number expressed by figures.***

1. Give the formation and name of 9030321007000.

Separated into periods, 9,030,321,007,000.

Formation, 9 trills. 30 bills. 321 mills. 7 thous. 0 units.

Name, nine trillion thirty billion three hundred twenty-one million seven thousand.

RULE FOR NUMERATION.—*Beginning at the right, separate the number into periods of three figures each.*

Beginning at the left, read each period as if it stood alone, adding its name.

EXERCISES (Written).

Give the formation and name of :

2. 23005 ; 463754 ; 75326424300.
3. 45321 ; 504207 ; 90250472507.
4. 73508 ; 620500 ; 42103021111.
5. 91000 ; 600005 ; 80000160003.
6. 71040 ; 630063 ; 52100125555.
7. 62437521 ; 783042625 ; 5463003702.
8. 53241603 ; 351200300 ; 4900600628.
9. 74630041 ; 210043288 ; 3741005007.
10. 43005006 ; 900037042 ; 6005025125.
11. 80000031 ; 282828282 ; 3745374537.
12. 8321 ; 63900037248.
13. 50430 ; 705271536073.
14. 700721 ; 48000200321657.
15. 66663754 ; 3841324100436.
16. 9501486 ; 250011006548703.

19. ***To give the formation and notation of a number expressed by words.***

1. Give the formation and notation of nine billion six hundred twenty-five million forty-three thousand five.

Formation, 9 bills. 625 mills. 43 thous. 5 units.
Notation, 9,625,043,005.

RULE FOR NOTATION.—*Begin at the left and write the hundreds, tens, and units of each period in their proper order, putting ciphers in all vacant places and periods.*

While writing, separate each period by a comma from the one that follows it.

EXERCISES (Written).

Give the formation and notation of:

2. Six thousand forty-six.
3. Nine thousand four hundred nine.
4. Thirty thousand thirty.
5. Sixty thousand seventy-nine.
6. Twenty thousand five hundred thirty-six.
7. Forty-three thousand eight.
8. Sixty-nine thousand two hundred.
9. Four hundred forty thousand five hundred one.
10. Six million five thousand forty-seven.
11. Nine million forty-nine thousand thirty-one.
12. Twenty-nine million six hundred eleven thousand fifteen.
13. One hundred seven million eleven thousand one hundred sixty-seven.
14. Seven hundred thirty million six hundred nine thousand three hundred ninety-two.
15. Thirteen billion thirteen million thirteen thousand thirteen.
16. Two hundred forty-five billion one hundred seven million fifty-nine thousand eight hundred seventy.

17. Twenty-five trillion two hundred seven million thirty-seven thousand three hundred one.

18. Four trillion seventeen million five hundred one.

19. One hundred billion thirty-five thousand nine hundred.

20. Six quadrillion ten trillion one million twenty-three thousand five hundred sixty-three.

Notation and Numeration of United States Money.

20. The **dollar sign** is $. It is written before the number.

Thus, $45 is read forty-five dollars.

21. A period, called the **decimal point,** is used in connection with $ in expressing cents and mills.

The *first two places* at the right of the decimal point express cents, and the *third* expresses mills.

Thus, $.124 is read twelve cents and four mills.

Dollars and cents are written as one number by placing the decimal point between them, and writing $ before the number of dollars.

Thus, $3.25 is three dollars and twenty-five cents.

When the number of cents is less than ten, a cipher must be written in the first place at the right of the decimal point.

Thus, four dollars three cents five mills is written $4.035.

EXERCISES (Written).

22. Read the following :

1. $125. *2.* $30.25. *3.* $421.08. *4.* $7862.20.
5. $6.40. *6.* $4.213. *7.* $64.005. *8.* $300.054.
9. $.152. *10.* $0.028. *11.* $82375. *12.* $4006.05.
13. $.005. *14.* $.2000. *15.* $637.65. *16.* $125.483.

EXERCISES (Written).

23. Express by signs and figures :

1. Sixteen dollars; five dollars, twenty-two cents, one mill; twelve dollars, nine cents.

2. Sixty-five cents; forty-seven dollars, five cents, two mills; one hundred dollars, two cents.

3. One cent, one mill; three hundred twenty-one dollars.

4. Seventy-four cents; ninety-seven cents, four mills.

5. Thirty-one dollars; two hundred dollars, seven mills.

6. Six thousand seven hundred four dollars, five cents.

7. Forty-three thousand seventy-one dollars, nine mills.

8. Two hundred sixty-two dollars, six cents, one mill.

9. 85 dollars, 70 cents, 8 mills.

10. 721 dollars, 4 cents, 3 mills.

Definitions.

24. A **scale** in Arithmetic is the relation between the successive orders of units.

In the decimal system the scale is ten, because the value of the unit of any order is ten times as great as the unit of the next lower order.

25. In the decimal system of notation each figure has a **simple** and a **local value.** Its local value is the value which its place gives it.

Thus, in 527 the simple value of 5 is *five*, and its local value is *five hundred;* the simple value of 2 is *two*, and its local value is *two tens.*

26. When numbers are used without reference to any particular units, they are called **abstract** or **pure numbers.**

Three, seven, 13, 25, are abstract numbers.

27. When numbers are associated with particular units, they are called **concrete numbers.**

Three bushels, 5 days, 10 pints, are concrete numbers.

28. Like numbers are those which have the same unit.

Five feet and 9 feet are like numbers; 5 feet and 9 days are not like numbers.

ADDITION.

Definitions and Principles.

29. How many beads are 5 beads, 4 beads, and 6 beads ?

Solution: (1) I may make as many marks as there are beads, count them, and thus find that there are 15 beads in all. This is uniting integers by *counting*.

(2) I may say 6 beads and 4 beads are 10 beads, and 10 beads and 5 beads are 15 beads. This is uniting integers by *addition*.

|||||, ||||, ||||||, 15

$$\begin{array}{r} 5 \\ 4 \\ 6 \\ \hline 15 \end{array}$$

30. Addition is a short method of counting the units in two or more numbers.

31. The numbers to be added are called **parts,** and the result is called the **sum of the parts.**

32. The **sign of addition** is +, which is called *plus,* meaning *more.* When + stands between two numbers, it indicates that they are to be added.

33. The **sign of equality** is =, which is read *equals,* or *is equal to.*

Thus, $7 + 4 = 11$ is read : 7 plus 4 equals 11. It may also be read : 7 and 4 are 11.

34. An **equation** is an expression containing the sign of equality (=).

35. PRINCIPLES.—1. *Only like numbers can be added.*

2. *The sum is the same in whatever order the parts may be added.*

Fundamental Combinations.

36. To perform addition one must know the sums of the following forty-five combinations :

2 = 1 + 1, 2 + 9, 3 + 8, 4 + 7, 5 + 6 = 11
3 = 1 + 2, 3 + 9, 4 + 8, 5 + 7, 6 + 6 = 12
4 = 1 + 3, 2 + 2, 4 + 9, 5 + 8, 6 + 7 = 13
5 = 1 + 4, 2 + 3, 5 + 9, 6 + 8, 7 + 7 = 14
6 = 1 + 5, 2 + 4, 3 + 3, 6 + 9, 7 + 8 = 15
7 = 1 + 6, 2 + 5, 3 + 4, 7 + 9, 8 + 8 = 16
8 = 1 + 7, 2 + 6, 3 + 5, 4 + 4, 8 + 9 = 17
9 = 1 + 8, 2 + 7, 3 + 6, 4 + 5, 9 + 9 = 18
10 = 1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5 = 10

The sums of these combinations should be studied until they can be given instantly. They are embraced in the following:

Name the sums of:

5 + 7
6 + 3
2 + 7
7 + 1
2 + 8
6 + 5
1 + 4
4 + 4
5 + 5

2 + 2
2 + 6
3 + 3
8 + 5
7 + 4
7 + 7
9 + 6
2 + 5
8 + 9

6 + 1
4 + 5
3 + 8
6 + 7
6 + 6
1 + 9
4 + 3
3 + 9
4 + 8

2 + 9
3 + 7
1 + 2
3 + 5
9 + 5
7 + 8
9 + 9
6 + 8
4 + 2

8 + 1
6 + 4
4 + 9
1 + 1
8 + 8
7 + 9
3 + 2
5 + 1
1 + 3

Each combination should be read forwards and backwards. Thus, 7 and 4 are 11, 4 and 7 are 11.

EXERCISES (Drill).

37. *1.* Add 2 to 29, to 59, 19, 89, 69, 49, 79, 39, 99.
2. Add 3 to 57, to 27, 67, 17, 87, 37, 77, 97, 47.
3. Add 4 to 68, to 88, 18, 58, 28, 78, 38, 48, 98.
4. Add 5 to 96, to 36, 76, 46, 66, 86, 16, 56, 26.
5. Add 6 to 27, to 57, 17, 87, 67, 47, 77, 37, 97.

6. Add 7 to 49, to 99, 79, 39, 89, 19, 69, 29, 59.
7. Add 8 to 97, to 47, 37, 77, 27, 57, 17, 87, 67.
8. Add 9 to 25, to 55, 15, 85, 65, 45, 75, 35, 95.

---

9. Add by 2's from 0 to 20; from 1 to 21.
10. Add by 3's from 0 to 30; from 2 to 32.
11. Add by 4's from 0 to 40; from 3 to 43.
12. Add by 5's from 0 to 50; from 4 to 54.
13. Add by 6's from 0 to 60; from 2 to 62.
14. Add by 7's from 0 to 70; from 3 to 73.
15. Add by 8's from 0 to 80; from 5 to 85.
16. Add by 9's from 0 to 90; from 7 to 97.

How much is:

17. 8 + 9? 17 + 4? 25 + 8? 38 + 3? 46 + 4?
18. 59 + 5? 64 + 8? 78 + 6? 82 + 8? 95 + 9?
19. 6 + 7? 29 + 8? 67 + 5? 36 + 9? 18 + 4?
20. 43 + 8? 81 + 9? 74 + 5? 55 + 8? 97 + 8?
21. 5 + 7? 99 + 8? 28 + 3? 74 + 8? 47 + 9?
22. 35 + 4? 19 + 6? 36 + 5? 62 + 9? 88 + 7?
23. 24 + 9? 72 + 8? 58 + 7? 16 + 8? 95 + 8?
24. 47 + 5? 66 + 9? 83 + 9? 38 + 7? 99 + 9?

How much is:

25. 37 + 20? 54 + 30? 91 + 70? 62 + 40? 26 + 50?
26. 85 + 60? 13 + 80? 47 + 70? 73 + 50? 57 + 10?
27. 94 + 20? 35 + 40? 66 + 70? 17 + 50? 42 + 30?
28. 78 + 90? 52 + 60? 29 + 10? 84 + 80? 93 + 90?

29. How many are 37 and 28?

37 + 20 = 57, 57 + 8 = 65. Hence, answer thus: 57, 65.

How much is:

30. 43 + 16? 54 + 27? 65 + 46? 46 + 39?
31. 83 + 28? 56 + 79? 48 + 73? 91 + 68?
32. 39 + 85? 73 + 62? 83 + 97? 65 + 99?

33. Add 16 to 24, to 31, 65, 98, 43, 76, 59, 82, 17.
34. Add 23 to 16, to 81, 58, 75, 42, 97, 64, 23, 39.
35. Add 37 to 76, to 98, 43, 65, 31, 24, 17, 59, 82.
36. Add 48 to 15, to 57, 74, 41, 96, 63, 22, 38, 89.
37. Add 51 to 24, to 31, 17, 82, 59, 76, 43, 98, 65.
38. Add 69 to 64, to 98, 43, 59, 76, 82, 17, 31, 25.
39. Add 74 to 39, to 23, 64, 97, 42, 75, 58, 81, 16.
40. Add 82 to 82, to 59, 17, 24, 31, 65, 43, 98, 76.
41. Add 95 to 24, to 31, 65, 98, 43, 76, 59, 82, 17.

EXERCISES (Oral).

38. *1.* If a boy has 15 cents and then earns 10 cents, how many cents will he then have ?

2. What is the cost of a suit of clothes if the material cost $19 and the making $12 ?

3. The head of a fish is 5 inches long, the tail 7 inches, and the body 11 inches. What is the length of the fish ?

4. A boy paid 45 cents for a melon, 8 cents for an orange, and had 9 cents left. How much had he at first ?

5. James gave 37 cents for a slate and 42 cents for a book. What did both cost ?

6. A pole is 64 feet in the air, 9 feet in the earth, and 18 feet in the water. How long is the pole ?

7. A lad, having spent 55 cents, finds he has 45 cents left. How much had he at first ?

8. Mary has 24 apples, and Susan 28 more than Mary. How many has Susan ?

9. Charles read 64 pages one day, 23 pages the next day, and 45 pages the next. How many pages did he read in all ?

10. A drover bought 57 hogs from one man, 74 from another, and 38 from another. How many hogs did he buy of all ?

11. A dealer sold 35 bushels of coal to one man, 45 to another, and 27 to another. How many bushels did he sell to all ?

12. Ada has 42 pins and Ida has 37, and Jane has 22 more than Ada and Ida together. How many pins has Jane ?

13. Henry rode 34 miles one day, and 12 miles farther the next day than he did the first day. How far did he ride in both days ?

14. One side of a square field is 18 rods. What is the distance around the field ?

15. Wilbur has 22 marbles, Charles 12 more than Wilbur, and Luke 15 more than Charles. How many marbles has each, and how many have they all together ?

16. At 6 A.M. the temperature was 42°, at 11 A.M. it was 28° higher, and at 2 P.M. it was 16° higher still. What was the temperature at 2 P.M. ?

17. The distance from A to B is 26 miles, from B to C 38 miles, and from C to D 43 miles. How far is it from A to D ?

Find the number whose parts are :

18. 19 feet, 13 feet, 6 feet, 18 feet, 7 feet, 24 feet.

19. 8 days, 21 days, 5 days, 43 days, 4 days, 56 days.

20. 7 rods, 35 rods, 9 rods, 52 rods, 6 rods, 72 rods.

21. 25 bushels, 46 bushels, 37 bushels, 54 bushels.

22. 51 seconds, 29 seconds, 48 seconds, 71 seconds.

39. ***To add two or more integers.***

1. Find the sum of 5492, 764, and 86.

OPERATION.

$$\begin{array}{r}5492\\764\\86\\\hline 6342\end{array}$$

EXPLANATION.—Write the numbers or *parts* so that units of the same order stand in the same column, *ones* under *ones*, *tens* under *tens*, etc.; and draw a line beneath them.

Adding from the bottom the column on the right, we get 12 (= 1 *ten* and 2 *ones*); write the 2 below the line for the ones of the required sum. Adding the 1 ten with the tens of the given parts, which is the next column, we get 24 (= 2 *hundreds* and 4 *tens*); write the 4 below the line for the *tens* of the required sum. Adding the 2 hundreds to the hundreds of the given parts, which are the numbers of the third column, we get 13 (= 1 *thousand* and 3 *hundreds*); write the 3 in place of hundreds in the sum, and carry the 1 to the next column of thousands, which, added to 5, gives 6 thousands. Hence the sum is 6342. To prove the work, we begin at the top and add down.

2. 45364 + 8965 + 786 + 9374 + 47 = ?

OPERATION.

45364
8965
786
9374
47
64536

In adding it is best to use only the following wording: 7, 11, 17, 22, **26** (emphasize 6, and write it down while pronouncing it), carry 2; 6, 13, 21, 27, **33**, carry 3; **6**, 13, 22, **25**, carry 2; 11, 19, **24**, carry 2; **6**.

RULE.—*Write the parts so that like orders of units shall stand in the same column.*

Begin at the right, add each column separately, write the units' figure of the sum under the column added, and carry the tens, if any, to the next column.

PROOF.—*Perform the addition in the reverse direction, from top to bottom, and if the results agree the work is probably correct.*

EXERCISES (Written).

Copy, add, and prove:

(*3*)	(*4*)	(*5*)	(*6*)	(*7*)	(*8*)
87	49	64	37	65	48
96	70	89	98	34	84
95	21	52	78	80	67
36	54	76	89	63	27
48	79	81	67	94	57

(*9*)	(*10*)	(*11*)	(*12*)	(*13*)
939	818	729	608590	486
827	706	695	587405	3705
705	694	581	446967	46628
963	582	434	993879	37530
47	7316	69	68304	964488
4382	38	7	5275	7291

Copy, add, and prove:

(*14*)	(*15*)	(*16*)	(*17*)	(*18*)	(*19*)
36	74	487	508	671	564
28	49	963	970	582	578
94	68	792	69	908	407
75	91	528	347	763	692
63	35	377	521	857	787
21	43	255	184	946	321
87	98	918	713	437	876

(*20*)	(*21*)	(*22*)	(*23*)	(*24*)
8475	1895	8139	5518	4537
4860	3417	6437	1364	9291
6291	2386	3215	3475	8557
3982	4819	2284	6982	6334
4333	5555	4329	7256	7777
9381	4989	1764	9978	6841
8263	3267	4582	4729	8888
1594	5278	9874	8965	9112
2789	3211	6436	2578	5434
4577	5826	3188	9040	1015

25. A grocer bought three hogsheads of sugar, containing 1,467 pounds, 1,324 pounds, and 1,296 pounds; also, two barrels containing 254 pounds and 237 pounds. How much sugar did he buy?

26. A man bought a farm, paying $2,375 down. After making three other payments of $1,148, $1,096, and $1,260, the amount unpaid was $5,896. What was the cost of the farm?

27. How many days in the first six months of the year, January, March, and May each having 31 days, April and June each 30 days, and February 28 days?

Find the sum of:

28. 25, 364, 2365, 874, 96, 408269, 936, 8.

29. 637, 84375, 9674, 87, 583, 25, 369765.

30. $431, $837, $98, $3475, $806, $25, $679.

31. 37 days, 674 days, 391 days, 6275 days.

32. I bought four loads of hay, which weighed respectively 1340 pounds, 964 pounds, 1273 pounds, 1189 pounds. What was the weight of all?

33. A farmer has 682 sheep in one field, 597 in another, and 875 in another. How many sheep has he in all?

34. A nurseryman sold during the year 3,729 apple trees, 1,415 pear trees, 974 peach trees, 567 plum trees, 918 cherry trees, and 1,584 shade-trees. How many trees did he sell?

35. One year a farmer raised 649 bushels of oats, 422 bushels of corn, 178 bushels of wheat, and 96 bushels of barley. How much grain did he raise?

36. A merchant pays his book-keeper $1,250 a year, two clerks $825 each, and a boy $175. How much do their salaries amount to?

When numbers of dollars and cents are to be added, they must be written so that the points stand in a line, and added as in simple numbers, since the scale is 10.

37. $36.25 + $44.09 + $187. + $6.83 + $.75 = ?

38. $50.04 + $961.20 + $8.75 + $1465.83 + $.09 = ?

39. What is the sum of $87.20, $971.65, $84.07, $3281.64, $5984.30?

40. A farmer paid $45.60 for flour, $86.75 for bacon, $135 for a mule, and $12.75 for a saddle. What did he pay for all?

41. A, B, and C enter into partnership. A puts in $6375; B, $4264.75; and C puts in as much as A and B together. What did C put in, and what sum did all put in?

42. Napoleon's army at Waterloo was composed of: infantry, 48,950; cavalry, 15,765; and artillery, 7,732. What was the number of his whole army?

43. In 1880 the population of New York was 1,206,590:

Philadelphia, 846,984; Brooklyn, 566,689; Chicago, 503,304; Boston, 362,535; St. Louis, 350,522; Baltimore, 332,190; Cincinnati, 255,708; San Francisco, 233,956; New Orleans, 216,140. What was the population of all?

44. In 1900 the population of New York was 3,437,202; Philadelphia, 1,293,697; Chicago, 1,698,575; Boston, 560,892; St. Louis, 575,238; Baltimore, 508,957; Cincinnati, 325,902; San Francisco, 342,782; New Orleans, 287,104. What was the population of all?

45. The total enrollments of Louisiana troops in the Civil War were as follows: infantry, 36,243; cavalry, 10,056; artillery, 4,024; sappers and miners, 276; engineers, 212; signal corps, 76; New Orleans State guards, 4,933. What was the total enrollment?

Parallel Problems.

40. In these and similar exercises the problems whose numbers are in light italic figures are intended for oral work, and as an induction to the examples following them.

1. A lad gave his mother 13 apples, his sister 12, and had 15 left. How many had he at first?

2. A man invested $475.20 in hogs, $764 in sheep, $2387 in horses, and had $328.95 left. How much had he at first?

3. The parts of a number are 12, 9, 16, and 8. What is the number?

4. A farmer has 673 hogs, 978 sheep, 478 cattle, and 47 horses. How many head of stock has he in all?

5. A man came to America when he was 12 years old; 15 years afterward he married; 7 years later his daughter was born, and she is now 11 years old. How old is the man?

6. Traveling on the Mississippi River, it is 130 miles from New Orleans to Baton Rouge; 133 miles from Baton Rouge to Natchez; 101 miles from Natchez to Vicksburg; 339

miles from Vicksburg to Memphis. How far is it from New Orleans to Memphis ?

7. Henry paid $17 for a gun, $12 for a watch, $5 for a wagon, and sold them so as to make $6. What did he receive for all ?

8. A speculator bought a drove of horses for $4728, a drove of sheep for $846.75, a drove of cattle for $3289, and sold them at a profit of $599.65. What did he receive for all ?

9. William has 12 apples, John 5 more than William, and Mary 3 more than John. How many have all ?

10. It is 684 miles from A to B, 237 miles farther from B to C than from A to B, and 1286 miles farther from C to D than from B to C. How far is it from A to D ?

11. Eleven years ago Walter was 8 years old. How old will he be 13 years from now.

12. A number is composed of the two parts 49875 and 8638. How much will it be when increased by 37649 ?

13. Frank is 13 feet ahead of Simon; Simon is 19 feet ahead of Susan, and Susan is 25 feet ahead of Moses. How far is Frank ahead of Moses ?

14. The pyramids of Egypt were built 337 years before the founding of Carthage ; Carthage was founded 49 years before the destruction of Troy ; Troy was destroyed 431 years before Rome was founded ; Carthage was destroyed 607 years after the founding of Rome, and 146 years before the Christian era. How many years before Christ were the pyramids built ?

15. A boy has 11 lines, of which 6 are each 4 yards long and 5 are each 5 yards long. How long are all the lines together ?

16. I have a farm 176 rods long and 115 rods wide. A fence extends around it ; 3 inside fences extend from end to end ; and 4 other fences from side to side. How many rods of fence on the farm ?

17. James has his marbles in 5 sacks. In the first sack there are 6, in the second 8, in the third 12, in the fourth

15, and in the fifth there are 7 more than there are in the other four together. How many marbles has he in all?

18. A man sold five lots. For the first he received $32640, for the second $46875.65, for the third $9837, for the fourth $53981.74, and for the fifth $8369.70 more than for the other four lots. What did he receive for all?

19. A book-case has six shelves. The first two contain 10 books each, the second two 12 each, and the third two 16 each. How many books are there on the six shelves?

20. A merchant's average daily sales for Monday and Tuesday were $4876.45; for Wednesday and Thursday, $6725.36; and for Friday and Saturday, $7994. What was the total value of his sales for the week?

21. A boy paid 18 cts. for each of two spellers, 8 cts. each for three pencils, and 5 cts. for a rubber. What did he pay for all?

22. A merchant pays two book-keepers $1576 each per year, three salesmen $1248 each, five salesmen $946 each, two porters $225 each, and four boys $124 each. What amount of salaries does he pay?

SUBTRACTION.

Definitions and Principles.

41. What number must be added to 5 to make 11? How much more is 11 than 5?

What number must be added to 4 to make 9? What is the difference between 9 and 4?

6 plus *what* = 13 ?
8 plus *what* = 11 ?
14 less 5 = *what?*
12 less 4 = *what?*

13 less 6 = *what?*
11 less 8 = *what?*
5 plus *what* = 14 ?
4 plus *what* = 12 ?

What is the difference between 9 and 4? Why?

What is the difference between 12 and 8? Why ?

42. The **difference** between two numbers is the number which, when added to the less number, will give the greater.

43. **Subtraction** is the process of finding the difference between two numbers.

44. The larger of the two numbers is the **minuend,** and the smaller the **subtrahend.**

45. The result obtained by subtracting is the **remainder** or **difference.**

46. The **sign of subtraction** is $-$, which is called *minus,* meaning *less.* When $-$ stands between two numbers, it indicates that the one *after* it is to be subtracted from the one *before* it.

Thus, $7 - 3 = 4$ is read: 7 minus 3 equals 4; it may also be read: 7 less 3 is 4.

47. PRINCIPLES.—1. *Subtraction is the reverse of addition;* that is, *the sum of the subtrahend and remainder is the minuend.* 2. *Only like numbers can be subtracted.*

Fundamental Combinations.

48. The forty-five fundamental combinations of addition include all those of subtraction.

Thus, there are three pairs of integers whose sum is 7, viz.: $3 + 4 = 7$, $2 + 5 = 7$, $1 + 6 = 7$, from which we get the six differences:

$$7-4=3,\ 7-3=4,\ 7-2=5,\ 7-5=2,\ 7-6=1,\ 7-1=6.$$

How many pairs of integers are there whose sum is 6? What are they? What five differences do we get from them?

How many pairs of integers are there whose sum is 11? What are they? What differences do we get from them?

Let the same questions be asked in succession in reference to 18; 5; 13; 9; 4; 16; 10; 15; 3; 17; 12; 8; 2; 14.

Name the difference of:

15 – 9   13 – 4   9 – 2   5 – 3   7 – 6   12 – 9
11 – 9   10 – 7   14 – 8   15 – 7   10 – 4   9 – 6
8 – 2   3 – 1   5 – 2   4 – 2   2 – 1   3 – 2
4 – 1   6 – 2   17 – 9   6 – 1   16 – 9   5 – 1
5 – 4   10 – 8   13 – 8   4 – 3   13 – 5   6 – 3
6 – 5   13 – 6   7 – 1   6 – 4   7 – 4   15 – 6
16 – 8   7 – 5   14 – 6   13 – 9   14 – 7   7 – 3
13 – 7   8 – 3   12 – 7   8 – 6   12 – 8   8 – 5
7 – 2   12 – 3   9 – 8   14 – 5   9 – 7   12 – 4
14 – 9   9 – 5   11 – 8   8 – 4   9 – 1   8 – 1
8 – 7   10 – 9   8 – 2   10 – 5   11 – 7   12 – 5
12 – 6   11 – 5   9 – 4   16 – 7   10 – 3   9 – 3
11 – 6   17 – 8   10 – 6   11 – 4   11 – 2   10 – 1
10 – 2   18 – 9   11 – 3

EXERCISES (Drill).

49. *1.* Take 2 from 31, 51, 91, 21, 71, 41, 61, 81, 11, 101.
2. Take 3 from 10, 60, 30, 70, 20, 90, 40, 80, 50, 100.
3. Take 4 from 12, 72, 92, 22, 62, 32, 82, 42, 52, 102.
4. Take 5 from 11, 41, 81, 51, 71, 91, 21, 61, 31, 101.
5. Take 6 from 13, 33, 63, 23, 93, 73, 53, 83, 43, 103.
6. Take 7 from 16, 56, 86, 46, 96, 26, 76, 36, 66, 106.
7. Take 8 from 15, 55, 45, 85, 35, 65, 25, 95, 75, 105.
8. Take 9 from 14, 34, 64, 24, 94, 74, 54, 84, 44, 104.

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9. Subtract by 2's from 20 to 0; from 21 to 1.
10. Subtract by 3's from 30 to 0; from 31 to 1.
11. Subtract by 4's from 40 to 0; from 42 to 2.
12. Subtract by 5's from 50 to 0; from 53 to 3.
13. Subtract by 6's from 60 to 0; from 64 to 4.
14. Subtract by 7's from 70 to 0; from 75 to 5.
15. Subtract by 8's from 80 to 0; from 83 to 3.
16. Subtract by 9's from 90 to 0; from 96 to 6.

How much is:

17. 17 − 9? 21 − 4? 33 − 8? 41 − 3? 50 − 4?
18. 106 − 9? 90 − 8? 84 − 6? 72 − 8? 64 − 5?
19. 51 − 8? 13 − 7? 22 − 4? 37 − 8? 90 − 9?
20. 63 − 8? 82 − 5? 79 − 5? 45 − 9? 105 − 8?
21. 12 − 7? 56 − 9? 31 − 3? 39 − 4? 25 − 6?
22. 61 − 5? 82 − 8? 71 − 9? 95 − 7? 107 − 8?
23. 33 − 9? 80 − 8? 65 − 7? 24 − 8? 103 − 8?
24. 52 − 5? 75 − 9? 92 − 9? 45 − 7? 108 − 9?

How much is:

25. 57 − 20? 84 − 30? 161 − 70? 102 − 40? 76 − 50?
26. 75 − 40? 93 − 80? 117 − 70? 123 − 50? 67 − 10?
27. 67 − 50? 72 − 30? 114 − 20? 145 − 60? 112 − 60?
28. 83 − 60? 97 − 80? 136 − 70? 168 − 90? 164 − 80?

29. How much is 65 − 28?

65 − 20 = 35, 35 − 8 = 27; hence, answer thus: **35, 27.**

How much is:

30. 59 − 16? 81 − 27? 111 − 46? 85 − 39?
31. 111 − 28? 135 − 79? 121 − 73? 159 − 68?
32. 124 − 35? 135 − 62? 170 − 97? 164 − 65?

Subtract:

33. 16 from 30, 47, 81, 114, 59, 92, 65, 98, 88.
34. 23 from 41, 92, 73, 104, 35, 56, 80, 62, 81.
35. 37 from 113, 135, 80, 102, 68, 61, 54, 96, 119.
36. 48 from 63, 105, 122, 89, 144, 111, 70, 86, 78.
37. 54 from 92, 73, 85, 61, 106, 123, 118, 140.
38. 69 from 91, 84, 77, 103, 116, 128, 132, 148.
39. 75 from 83, 94, 101, 152, 170, 113, 122, 130.
40. 86 from 97; 121, 153, 117, 135, 172, 160, 181.
41. 97 from 99, 116, 181, 155, 122, 184, 178, 190.

EXERCISES (Oral).

50. *1.* Mary bought 20 apples, and gave Thera 6 of them. How many did Mary then have?

2. How many birds will escape if I shoot 8 birds out of a flock of 41 birds?

3. A man borrowed $32, and afterward returned $17. How much does he still owe?

4. Henry had 53 cents, but he paid 26 cents for some paper. How many cents had he left?

5. 47 gallons of syrup have been drawn from a tank that contained 93 gallons. How many gallons are left?

6. James has caught 65 fishes and Moses 48. How many more fishes must Moses catch to have as many as James?

7. Emma has 81 beads and Susan has 26 less than Emma. How many beads has Susan?

8. A man owing $77 gave his note for $18, and paid the balance in cash. How much cash did he pay?

9. I gave a horse worth $80 for a cow and $27 in money. How much did the cow cost me?

10. William had 42 oranges, and gave 19 to Mary and 17 to James. How many oranges had he left?

11. Edward had $65, and after spending $25 lost all the balance except $16. How much did he lose?

12. A lady bought a shawl for $37 and gave in payment a fifty-dollar bill. How much change should she receive?

13. I gave $61 for a cow and calf. If the cow cost $47, what did the calf cost.

14. A merchant sold a gun for $45, which was $18 more than it cost. Find the cost.

15. An agent bought a bale of cotton for $37 and sold it for $51. How much did he gain?

16. A clerk earns $65 a month and spends $19. How much does he save monthly?

17. The age of a man is 72 years and that of his wife is 57 years. How much older is the man than his wife?

Find the value of :

18. $9 + 4 - 3 + 7 + 8 - 5.$

Combine the numbers in order, calling only results. Thus : 9, 13, 10, 17, 25, 20.

19. $8 + 12 - 6 - 5 + 9 + 7.$

20. $12 - 5 + 7 + 6 - 4 - 8.$

21. $7 + 9 + 8 - 5 - 4 - 3.$

22. $16 + 14 - 9 - 7 + 8 - 5.$

23. $12 - 3 - 6 + 10 + 11 - 7.$

24. $13 - 5 + 20 - 6 - 5 + 12.$

25. $7 + 16 + 9 + 10 - 15 - 13.$

26. $24 - 6 - 7 - 4 - 3 - 2.$

27. $25 + 14 - 7 - 8 - 9 - 12.$

28. $15 + 8 + 9 + 6 + 7 + 5.$

When several numbers are included in a parenthesis ( ), they are to be treated as a single number.

Thus, $(12 - 5) - (2 + 3)$ means that the sum of 2 and 3 is to be subtracted from the difference between 12 and 5. Hence, the solution is $7 - 5 = 2$.

Find the value of :

29. $10 + 5 - (8 + 4) + 6.$

30. $12 - (10 - 7) - (5 + 2).$

31. $16 - (12 - 9) - (10 - 5) + 7.$

32. $5 + (16 - 4) - (15 - 8).$

33. $5 + (13 - 4) - (9 - 6 + 3).$

34. $8 - (13 - 6) - (6 + 2 - 7).$

35. $(13 + 9) - (6 + 7 - 4 - 5).$

36. $(17 - (8 - 3 + 7 - 4 - 2).$

37. James had \$18, earned \$9 more, and spent \$7. How much had he then ?

38. A boy had 48 marbles, of which he sold 15, lost 12, and then bought 25. How many had he then ?

39. If I start from this schoolhouse and go east 21 miles, then west 17 miles, then east 28 miles, then east 25 miles, then west 12 miles, then west 14 miles, how far will I then be from the schoolhouse ?

51. ***To subtract any integer from a greater one.***

1. From 986 subtract 714.

		OPERATION.
EXPLANATION.—	986 = 9 hunds., 8 tens, 6 ones,	986
	714 = 7 hunds., 1 ten, 4 ones,	714
Subtract,	2 hunds., 7 tens, 2 ones =	272

After writing the subtrahend under the minuend, units under units, tens under tens, etc., we say 4 from 6 leaves 2, 1 from 8 leaves 7, 7 from 9 leaves 2, which we write below.

2. From 925 take 673.

		OPERATION.
EXPLANATION.—925 =	8 hundreds, 12 tens, 5 ones,	925
673 =	6 hundreds, 7 tens, 3 ones,	673
Subtracting,	2 hundreds, 5 tens, 2 ones =	252

Proceeding as before, I say 3 from 5 leaves 2, which I write below. Since 7 tens cannot be subtracted from 2 tens, I take 1 hundred, which is equal to 10 tens, from the 9 hundreds, add it to the 2 tens, making 12 tens. Then I say 7 from 12 leaves 5; and since I took 1 hundred from 9 hundred, I say 6 from 8 leaves 2.

Proof: 673, the subtrahend, plus 252, the remainder, equals 925, the minuend.

RULE.—*Write the subtrahend under the minuend, units under units, tens under tens, etc.*

Begin at the right and subtract each figure of the subtrahend from the corresponding figure of the minuend, writing the result beneath.

If a figure in the minuend has a less value than the corresponding figure in the subtrahend, increase the former by ten, and subtract; then diminish by one the units of the next higher order in the minuend, and subtract as before.

PROOF.—*Add together the remainder and subtrahend. If the result is equal to the minuend, the work is correct.*

EXERCISES (Written).

52. Copy, subtract, and prove:

(*1*)	(*2*)	(*3*)	(*4*)	(*5*)
35	42	83	325	431
17	28	67	274	284

(*6*)	(*7*)	(*8*)	(*9*)
608	4200	5034	84375
572	2728	4929	6768

Copy, subtract, and prove:

(*10*)	(*11*)	(*12*)	(*13*)
$640.	$9.70	$16.00	5361 cents
381.	6.35	7.54	809 cents

14. 821 — 375.

15. 100000 — 9374.

16. 792 — 487.

17. 112210 — 8076.

18. 691 — 503.

19. 25624 — 12108.

20. 921 — 799.

21. 20000 — 10001.

22. 643 — 497.

23. 489034 — 6039.

24. 500 — 168.

25. 5002080 — 975.

26. 7113 — 87.

27. 43210000 — 93.

Find the value of:

28. 83576 — 9685.

29. 956873 — 874296.

30. 124308 — 86109.

31. 384221 — 295864.

32. 521000 — 99999.

33. 100090001 — 763019.

34. 252531 — 89899.

35. 200000000 — 10110101.

36. 64201 + 36371 — 4687 + 986 — 74.

37. 75069 — 8384 + 627 — 98 + 5.

38. (374621 — 28375) — (486030 — 379286).

39. 3625800 — (2345629 — 1869387) + (675500 — 48673).

40. A man had $3115 and spent $2725.75. How many dollars did he have left?

Explanation.—We write the amounts so that one *point* stands under the other, annex two 0's to the minuend to supply the vacant *places* of cents, and subtract as in simple numbers.

Operation.
3115.00
2725.75
389.25

41. A man bought a horse for $192, and a buggy for $118.65. How much more did the horse cost than the buggy?

42. A farmer bought a wagon for $137, and gave in exchange his note for $74.35, and the balance in cash. How much was the balance?

43. In an election for Member of Congress, A received

12,031 votes, and B 10,032 votes. Which candidate was elected, and by what majority?

44. A yoke of oxen, before being fattened, weighed 2,586 pounds, and after being fattened 3,174 pounds. How much had they gained?

45. An ox weighed 1,326 pounds on foot, and 997 pounds when slaughtered. What was the difference between the gross and the dressed weights?

46. A man was born in the year 1839. How old was he in 1885?

47. Gunpowder was invented in the year 1330. How long was that before the invention of steamboats, which was in 1807?

48. In 1880 there were 5,083,810 inhabitants in the State of New York, and 4,282,786 inhabitants in the State of Pennsylvania. How many more inhabitants were there in New York than in Pennsylvania?

49. In 1880 the number of white persons in Louisiana was 455,007, and the number of colored 483,794. How many more colored than white were there?

50. Three of the parts of 1250 are 231, 365, and 189. What is the fourth, or remaining part?

EXPLANATON.—Since the required part added to the sum of the other parts makes the *whole* 1250, by subtracting the sum of the given parts from the whole we get the required part. Hence, to find one part when the whole and the other parts are known, *add the given parts and subtract their sum from the whole.*

OPERATION.

$$\begin{array}{r} 231 \\ 365 \\ 189 \\ \hline 785 \end{array} \qquad \begin{array}{r} 1250 \\ 785 \\ \hline 465 \end{array}$$

The result may be obtained by one operation, thus: 9 + 5 + 1 = 15; subtract the 5 from 10 and carry the 1; 1 + 8 + 6 + 3 = 18; subtract the 8 from 14 and carry the 1; 1 + 1 + 3 + 2 = 7, which is to be subtracted from 11.

$$\begin{array}{r} 1250 \\ \hline 231 \\ 365 \\ 189 \\ \hline 465 \end{array}$$

51. Two of the parts of 354 are 128 and 157. What is the other part?

52. Three of the parts of $980 are $19.25, $435.60 and $273.86. What is the other part?

53. A owed B $1525; at one time he paid him $384, at another $365.75, at another $407.85. How much did he still owe him?

54. The United States contain 2,983,153 square miles, of which the Atlantic slope includes 967,576, the Pacific slope 778,266, and the Mississippi Valley the remainder. How many square miles does the Mississippi Valley contain?

55. In 1900 the population of Texas was 3,048,710; Louisiana, 1,381,625; Mississippi, 1,551,270; Alabama, 1,828,697. How many more people were there in Texas than in Alabama? In Alabama than in Mississippi? In Mississippi than in Louisiana?

56. How many more people were there in Texas than in Louisiana and Mississippi together?

Parallel Problems.

53. *1.* A part of 28 is 15. What is the remaining part?

2. A man had $3250, and lost $876.75. How much had he left?

3. Two parts of 40 are 18 and 13. What is the remaining part?

4. Charles and Harry are 3026 feet apart. If Charles goes towards Harry 987 feet, and Harry goes towards Charles 1248 feet, how far apart will they then be?

5. One boy caught 25 fishes, and another boy 35; they sold to one man 12, to another 11, and to another 20. How many fishes had they left?

6. A farmer owed $177.80 for a horse, and $94.20 for a cow. At one time he paid $44, at another $17.90, at another $86.75, and at another $33. How much did he still owe?

7. A boy gathered 40 nuts and ate 18; then gathered 30 and ate 13. How many did he then have?

8. A speculator at one time gained $5374, and then lost $465.85 ; at another time he gained $3249.70, and then lost $4563. How much was his net gain ?

9. A boy paid 55 cents for apples, and 25 cents for having them carried to market. If he sells them for $1, how much will he make ?

10. A dealer paid $18,274 for horses ; his expenses in taking them to market were $6274.55, and he sold them for $25,000. What were his profits ?

11. Two baskets containing fruit weigh 24 and 37 pounds. If the baskets themselves weigh 3 and 5 pounds, what does the fruit weigh ?

12. A farmer received $2134 for his sugar, and $1741 for his cotton. The expense of raising the sugar was $1183, and of the cotton $749.75. What were his profits ?

13. How much more is 52 — 13 than 24 — 9 ?

14. A farmer paid the difference between $1875 and $1619.30 for a horse, and the difference between $913.55 and $879 for a cow. How much more did the horse cost than the cow ?

15. John is 8 — 1 years older than James, James is 13 — 4 years older than William, William is 11 — 5 years older than Samuel. How old will John be when Samuel is 13 years old ?

16. A was 67 years old when B was 55 ; B was 39 when C was 26 ; C was 25 when D was 18 ; D was 47 when E was 34. How old was A when E was 5 ?

17. $14 + 6 - 7 + 8 - 3 + 6 - 5 + 4 - 9 =$ what number ?

18. A man had $164, then won $67, then lost $18.60, then won $374, then lost $218, then won $199.86, then lost $78. How much had he then ?

19. Ben caught 17 fishes, Henry 16 more than Ben, and Robert 51. How many more did Robert catch than Ben and Henry together ?

20. New York contains 47,156 square miles, California 121,825 square miles more than New York, and Texas contains 274,356 square miles. How much larger is Texas than New York and California together?

21. The sum of two numbers is 12, and their sum is 4 more than the greater. What are the numbers?

22. The sum of two numbers is 1024, which is 638 more than the less; what are the numbers?

MULTIPLICATION.

Definitions and Principles.

54. One horse has 4 feet; how many feet have 3 horses?

SOLUTION.

(1) I write 4 feet as many times as there are horses, and add them together. This is *adding equal numbers*.

$$\begin{array}{r} 4 \text{ feet} \\ 4 \text{ feet} \\ 4 \text{ feet} \\ \hline 12 \text{ feet} \end{array}$$

(2) Instead of writing 4 feet *three* times, I write it *once;* then under it place 3, the *number* of *times* it is to be taken, and say, 3 *times* 4 feet are 12 feet. This is *multiplication*.

$$\begin{array}{l} 4 \text{ feet} \\ 3 \\ \hline 12 \text{ feet} \end{array}$$

55. Multiplication is a short method of adding equal numbers; or the process of taking one number as many times as there are units in another.

56. The number to be taken is called the **multiplicand**; the number that shows how many times it is to be taken, the **multiplier**; and the result, the **product**.

The *multiplicand* and *multiplier* are called the **factors** of the product.

57. The **sign of mutiplication** is ×, which is read: *times*, or *multiplied by*. When × stands between two numbers, it indicates that one of them is to be multiplied by the other.

Thus, $4 \times 3 = 12$ is read: 4 times 3 are 12; or, 4 multiplied by 3 equals 12. 4 and 3 are the factors of 12.

58. Principles.—*1. The multiplier must be considered an abstract number.*

2. The multiplicand and product are like numbers.

3. The product is the same in whatever order the factors are multiplied.

Thus, 3 × 4 × 5 = 4 × 3 × 5 = 4 × 5 × 3.

Fundamental Combinations.

59. To perform multiplication one must know the products of the following thirty-six combinations:

4 × 6 = 24
3 × 9 = 27
6 × 6 = 36
8 × 2 = 16
2 × 5 = 10
9 × 4 = 36
6 × 5 = 30
8 × 6 = 48
9 × 8 = 72
7 × 2 = 14
6 × 3 = 18
7 × 5 = 35
9 × 6 = 54
9 × 9 = 81
5 × 4 = 20
3 × 5 = 15
2 × 6 = 12
8 × 8 = 64
3 × 4 = 12
6 × 7 = 42
4 × 4 = 16
5 × 5 = 25
7 × 9 = 63
4 × 2 = 8
3 × 3 = 9
8 × 7 = 56
5 × 9 = 45
2 × 3 = 6
9 × 2 = 18
3 × 8 = 24
4 × 7 = 28
8 × 4 = 32
2 × 2 = 4
3 × 7 = 21
5 × 8 = 40
7 × 7 = 49

In addition to these we have, *any number* × 1 = *that number*, and *any number* × 0 = 0.

Each combination should be read forwards and backwards. Thus, 4 × 3 = 12 is read: 4 times 3 = 12 and 3 times 4 = 12, or 4 threes are 12 and 3 fours are 12.

Find the value of:

1. 7 × 7, 5 × 8, 4 × 7, 3 × 6, 2 × 8.
2. 3 × 3, 3 × 7, 2 × 4, 5 × 9, 6 × 6.
3. 7 × 9, 8 × 8, 4 × 5, 2 × 3, 2 × 4.
4. 2 fives, 6 sevens, 8 sevens, 9 fours, 4 eights.
5. 8 threes, 2 nines, 3 fours, 2 sixes, 4 fours.
6. 3 nines, 6 fives, 5 sevens, 6 fours, 5 fives.

What is the number whose factors are:

7. 8 and 8? 9 and 9? 3 and 5? 6 and 9? 2 and 7?

8. 8 and 9? 6 and 8? 7 and 6? 5 and 8? 6 and 6?

EXERCISES (Induction and Drill).

60. *1.* How much is 6 × 1 ten? Is 6 tens the same as 60? How much, then, is 6 × 10? 7 × 10? 3 × 10? 9 × 10? 2 × 10? 5 × 10? 8 × 10? 4 × 10?

2. How much is 12 × 10? 35 × 10? 241 × 10? 10 × 96?

Any integer is multiplied by 10 *by annexing to the integer one cipher.*

3. How much is 10 × 18? 10 × 346? 10 × 5271?

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4. How much is 6 × 1 hundred? Is 6 hundreds the same as 600? How much, then, is 6 × 100? 8 × 100? 3 × 100? 7 × 100? 12 × 100? 100 × 64? 100 × 472?

5. Give an easy rule for multiplying an integer by 100.

6. How much is 100 × 7? 100 × 11? 100 × 81?

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7. How much is 6 × 1 thousand? Therefore 6 × 1000 are how many ones?

8. How much is 7 × 1000? 13 × 1000? 1000 × 26?

9. Give an easy rule for multiplying an integer by 1000.

10. Multiply 8 by 1000; 16 by 1000; 125 by 1000.

---

11. How much is 6 × 4 tens? Is 24 tens the same as 240? How much, then, is 6 × 40? 8 × 70? 50 × 9? 70 × 4?

12. Is 90 multiplied by 6 by annexing one cipher to 6 × 9? How is 40 multiplied by 8? 60 multiplied by 7?

13. How much is 30 × 9? 70 × 4? 8 × 50?

---

14. How much is 6 × 8 hundred? 48 hundreds is the same as what? How much, then, is 6 × 800?

15. Give an easy rule for multiplying 900 by 5.

16. How much is 700×6? 9×300? 800×4?

17. How much is 4000×9? 7×6000? 5×9000?

18. Give the rule for multiplying any integer with ciphers annexed to it by another integer.

How much is:

19. 40×6? 700×8? 5000×3? 80000×9?
20. 7×50? 6×900? 8×4000? 5×60000?
21. 8×70? 900×4? 6×8000? 70000×7?

---

22. Is 16 equal to $10 + 6$?
Is $3 \times 16 = 3 \times 10 + 3 \times 6$?
How much is $3 \times 10 + 3 \times 6$?
How much, then, is 3×16?

$$\begin{array}{r}16 = 10 + 6\\16 = 10 + 6\\\underline{16} = \underline{10 + 6}\\3 \times 16 = 30 + 18 = 48\end{array}$$

23. How much is 4×15? 4×10 plus $4 \times 5 =$?

24. How much is 17×6? $60 + 42 =$?

How much is:

25. 3×12? 4×18? 5×19? 6×13?
26. 7×14? 8×15? 9×14? 2×19?
27. 5×16? 7×18? 2×16? 8×13?
28. 17×3? 16×9? 17×4? 18×6?
29. 23×4? 21×6? 25×8? 24×7?
30. 2×29? 3×27? 5×26? 6×22?

EXERCISES (Oral).

61. *1.* What is the cost of 9 barrels of flour at $7 a barrel?

Solution. Since 1 barrel cost $7, 9 barrels will cost 9 times $7, or $63.

2. What is the cost of 7 plows at $6 each?

3. If one hat cost $6, what will 8 hats cost?

4. What will 7 pairs of shoes cost at $7 a pair?

5. If a horse walks 5 miles an hour, how far will he walk in 6 hours ?

6. Seven days in a week and 4 weeks in a month. How many days in a month ?

7. At $8 each, what will be the cost of 5 sheep ? Of 13 hogs ?

8. Nine boys can do a piece of work in 3 days. How long will it take 1 boy to do it ?

9. A father gave his son 9 five-cent pieces. How many cents did he give him ? 9 × 5 = ?

10. If a boy sleeps 8 hours per night, how many hours does he sleep in a week ? 8 × 7 = ?

11. There are 6 table knives in a set. How many knives are there in 9 sets ? 6 × 9 = ?

12. How many marbles have 8 boys if each boy has 13 marbles ?

13. If a boy writes 9 words in a minute, how many words will he write in 14 minutes ?

14. 5 girls are in a class and each girl answered 15 questions. How many questions did all answer ?

15. How much will 10 apples cost at 5 cents apiece ? 12 cents apiece ? 15 cents apiece ? 25 cents apiece ?

16. How much will 26 lbs. cotton cost at 10 cents a pound ?

17. There are 10 dimes in $1. How many dimes in $5 ? In $9 ? $27 ? $54 ? $120 ? $255 ?

18. There are 10 square chains in 1 acre. How many square chains in 8 acres ? 23 acres ? 72 acres ?

19. What will 7 cows cost at $40 a head ?

20. What will 8 mules cost at $90 a head ?

21. What will 17 desks cost at $4 per desk ?

22. What will 6 guns cost at $18 per gun ?

23. What will 100 cigars cost at 4 cents each ?

24. At 60 cents a pound, what will 7 pounds of tea cost ?

25. How far will a train go in 8 hours at the rate of 40 miles per hour ?

26. A ton is 2,000 pounds. How many pounds in 9 tons?

27. How much is $6 \times 4 + 5$?

SOLUTION.—$6 \times 4 = 24$; $24 + 5 = 29$.

In expressions like this, the operation indicated by × must be performed first, unless otherwise indicated by a parenthesis. Thus, $12 - 3 \times 2 = 12 - 6 = 6$; $(3 + 4) \times (8 - 2) = 7 \times 6 = 42$.

Find the value of:

28. $6 \times 5 - 8$. *29.* $20 - 6 \times 2 + 3 \times 5$.
30. $12 + 3 \times 5$. *31.* $(18 - 10) \times (20 - 13)$.
32. $4 \times 5 + 6 \times 3$. *33.* $4 \times 3 + 3 \times 2 - 4 \times 2$.
34. $(6 - 2) \times 7 + 8$. *35.* $(6 - 4 + 1) \times 8 - 5 \times 2$.

62. ***To multiply by an integer less than 10.***

Multiply 435 by 7.

OPERATION.

$$\begin{array}{r} 435 \\ \underline{7} \\ 3045 \end{array}$$

EXPLANATION.—For convenience, the multiplier is written under the unit's figure of the multiplicand.

Beginning at the right, we say:

7 times 5 ones are 35 ones, or 3 tens and 5 ones. Write the 5 below in the ones' place, and the three tens are reserved to be added to the tens.

7 times 3 tens are 21 tens, plus 3 tens reserved, are 24 tens, or 2 hundreds and 4 tens. Write the 4 below in the tens' place, and the 2 hundreds are reserved to be added to the hundreds.

7 times 4 hundreds are 28 hundreds, plus 2 hundreds reserved, are 30 hundreds, or 3 thousands and 0 hundreds, which are written below in the thousands' and hundreds' places.

EXERCISES (Written).

Find the product of:

1. 325×3. *2.* 647×6. *3.* 605×5. *4.* 487×6.
5. 792×8. *6.* 986×7. *7.* 739×4. *8.* 876×9.
9. 578×6. *10.* 608×5. *11.* 467×7. *12.* 794×8.

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13. 4576×4. *14.* 3869×9. *15.* 7586×8.
16. 7048×7. *17.* 8679×5. *18.* 9788×6.

19. If one hogshead of sugar weighs 1324 pounds, what will 7 hogsheads weigh?

20. If one bale of cotton weighs 456 pounds, what will 9 bales weigh?

21. There are 640 acres in a square mile. How many acres in 7 sq. miles?

22. There are 5 rows in an orchard, and 123 trees in each row. How many trees in the orchard?

23. A man travels 8 miles. How many feet does he travel, there being 5280 feet in a mile?

24. Sold to one man 3246 acres of land at 6 dollars per acre; to another 4328 acres at 8 dollars per acre; to a third 9546 acres at 5 dollars per acre, and to a fourth 3261 acres at 9 dollars per acre. What was the amount of sales?

63. ***To multiply by the factors of a number.***

25. Multiply 325 by 24.

EXPLANATION.—The factors of 24 are 4 and 6. Hence we multiply 325 by 6, and the product by 4.

ANALYSIS.—Since 24 is 4 times 6, when we take 325, 6 times, the product is 4 times too small; hence the true product is 4 times 1950.

Any other factors of 24 will produce the same result.

OPERATION.

$$24 = 4 \times 6$$

$$\begin{array}{r} 325 \\ 6 \\ \hline 1950 \\ 4 \\ \hline 7800 \end{array}$$

In like manner multiply:

26. 347 by 18, or by 6 × 3.

27. 518 by 36.

28. 582 by 49, or by 7 × 7.

29. 675 by 42.

30. 759 by 63, or by 9 × 7.

31. 852 by 108.

64. ***When there are ciphers at the right of one or both factors.***

32. Multiply 423 by 500.

ANALYSIS.—$500 = 5 \times 100$. Hence we multiply 423 by 5, and the product, 2115, by 100, which is done by annexing two ciphers.

OPERATION.

$$\begin{array}{r} 423 \\ 5 \\ \hline 211500 \end{array}$$

33. Multiply 420 by 600.

ANALYSIS.—$420 = 42 \times 10$, $600 = 6 \times 100$. Hence, $420 \times 600 = 42 \times 10 \times 6 \times 100 = 42 \times 6 \times 10 \times 100 = 252 \times 1000 = 252000$.

OPERATION.

$$\begin{array}{r} 42 \\ 6 \\ \hline 252000 \end{array}$$

Rule.—*Omit the 0's on the right of the factors, multiply the remaining parts of the factors together, and annex the 0's omitted to the result.*

Multiply:

34. 125 by 30.

35. 48950 by 3600.

36. 600 by 80.

37. 654200 by 2500.

38. 437 by 600.

39. 865000 by 72000.

40. What cost 40 horses at $80 each ?

41. A stock train consists of 20 cars, each car containing 180 sheep. How much do all the sheep weigh, their average weight being 60 pounds ?

65. ***To multiply any integer by another.***

Multiply 746 by 523.

Explanation.	Operation.
	746
523 = 500 + 20 + 3 =	523
3 times 746 =	2238
20 times 746 =	14920
500 times 746 =	373000
523 times 746 =	390158

In practice the work is done thus : Multiply 746 by 3, 2, 5, in succession, and write the products and their sum as in the margin.

Operation.

$$\begin{array}{r} 746 \\ 523 \\ \hline 2238 \\ 1492 \\ 3730 \\ \hline 390158 \end{array}$$

The products resulting from multiplying the multiplicand by the *ones*, *tens*, *hundreds*, etc., of the multiplier are called **partial products.**

Rule.—*Write the multiplier under the multiplicand. Begin at the right, multiply the multiplicand by the ones, tens, hundreds, etc., of the multiplier, placing the right hand figure of each partial product under the figure of the multiplier used to obtain it, and add the partial products.*

Proof.—*Multiply the multiplier by the multiplicand, and if the product is the same as before, the work is probably correct.*

EXERCISES (Written).

66. Find the product of :

1. 96 × 16. *2.* 125 × 76. *3.* 542 × 84. *4.* 486 × 64.
5. 85 × 34. *6.* 785 × 72. *7.* 478 × 37. *8.* 960 × 70.
9. 78 × 69. *10.* 609 × 58. *11.* 863 × 68. *12.* 975 × 96.

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13. 1283 × 146. *14.* 6709 × 708. *15.* 8064 × 973.
16. 93245 × 87. *17.* 2346 × 834. *18.* 7895 × 652.
19. 9487 × 946. *20.* 6087 × 307. *21.* 9875 × 480.

---

22. Multiply 68349 by 3254 ; 75080 by 2084.
23. Multiply 44813 by 3591 ; 75603 by 84006.
24. Multiply 809045 by 60703 ; 6570 by 95000.

Find the value of :

25. 34567 × 325. *26.* 346857 × 6807.
27. 46854 × 406. *28.* 654324 × 3097.
29. 58163 × 248. *30.* 683725 × 5618.
31. 58194 × 397. *32.* 398647 × 3009.
33. 28637 × 341. *34.* 468350 × 3915.
35. 32685 × 507. *36.* 304250 × 5039.
37. 56736 × 693. *38.* 416320 × 4069.

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39. A hogshead holds 63 gallons. How many gallons do 82 hogsheads hold ?

40. If a regiment of soldiers contains 1128 men, how many men are there in an army of 84 regiments ?

41. What is the weight of 36 bales of cotton, each bale weighing 463 pounds ?

42. What is the weight of 34 hogsheads of sugar, each hogshead weighing 1395 pounds ?

43. If a carriage-wheel revolves 398 times in going a mile, how many times will it revolve in going 2 miles ? 5 miles ? 100 miles ? 256 miles ?

44. What will 96 horses cost at $127 apiece ?

45. There are 365 days in the year. How many days are there in 74 years?

46. How many trees are there in an orchard in which there are 117 rows, and 83 trees in each row?

47. What is the weight of 236 hogs, if the average weight of a hog is 154 pounds?

48. How many poles will be required for a telegraph 654 miles long, if 24 poles are required for one mile?

67. There are:

60 pounds in 1 bushel of wheat.
56 pounds in 1 bushel of corn.
32 pounds in 1 bushel of oats.
100 pounds in 1 keg of nails.
196 pounds in 1 barrel of flour.
200 lbs. in 1 bbl. of pork or beef.
280 pounds in 1 barrel of salt.
180 pounds in 1 barrel of coal.

What is the weight of:

49. 56 barrels of salt?
50. 97 barrels of flour?
51. 534 barrels of coal?
52. 862 bushels of corn?
53. 3866 kegs of nails?
54. 216 barrels of beef?
55. 175 bushels of oats?
56. 365 bushels of wheat?

Parallel Problems.

68. *1.* What will 9 primers cost at 15 cents each?

2. If 1 watch cost $67, what will 48 watches cost?

3. How long will it take 1 man to do a work which 7 men can do in 18 days?

4. If 63 men can build a wall in 185 days, how long will it take 1 man to build it?

5. Mary had 15 apples, and gathered 7 more on each of the 4 following days. How many had she then?

6. If a boy has $634 when he is 19 years old, and saves $187 each year until he is 43 years old, how much money will he then have?

7. How many units in 12? If the value of each is 25 cents, what will all of them amount to?

8. If the value of each unit of 1760 is 36 inches, how many inches are there in all?

9. If a horse walks 6 miles an hour, how far will he walk in 15 hours?

10. If a locomotive runs 37 miles an hour, how far will it run in 13 hours?

11. A lad had 7 baskets, and 6 oranges in each basket, and sold each orange for 4 cents. What did he receive for all?

12. A farmer sold 17 hogsheads of sugar, averaging 1154 pounds to the hogshead, at 8 cents a pound. What did he receive for all?

13. What is the number whose parts are $15 and 7 times $20?

14. A drover bought 64 hogs at $5.50 a head, and sold them at a profit of $16.75. What did he receive for them?

15. Jacob sold 4 apples at 3 cents each, 7 peaches at 2 cents each, and 6 oranges at 5 cents each. How much did he receive for all?

16. A farmer sold 13 cows at $13.20 each, 24 sheep at $3.25 each, and 4 horses at $137 each. What did he receive for all?

17. A boy sold 9 pigs at $3 each, and took in pay 75 oranges at 4 cents each, 2 pairs of pants at $6 each, and the balance in cash. How much cash did he receive?

18. A farmer sold 16 beef cattle at $24.50 each, and took in exchange 6 barrels of flour, at $7.35 a barrel; 225 pounds of bacon, at 9 cents a pound; and the balance in cash. How much cash did he receive?

19. How much more is 11×6 than $38 + 9$?

20. What is the *difference* between the *product* of 481 and 125, and the *sum* of 37,648 and 16,297?

21. $(18 + 7) \times (36 - 25) =$ what number?

22. If the *multiplicand* is the *sum* of 391 and 264, and the *multiplier* the *difference* between 583 and 196, what is the *product?*

23. There are 7 boys in a class. If to 3 of them the teacher propounds 5 questions each, and to each of the others 6 questions, how many questions will be asked in all?

24. A man has 47 books; 29 of them contain each 147 pages, and each of the others contains 250 pages. How many pages in all the books?

25. A boy sold a pig, a calf, and a pony. For the pig he receved $3, for the calf twice as much as for the pig, and for the pony 3 times as much as for the pig and calf. What did he receive for all?

26. A farmer sold 3 farms. For the first he received $867, for the second 3 times as much as for the first, and for the third 7 times as much as for the first and second. What did he receive for all?

27. In a garden there are 7 rows; each row has 10 vines, and each vine 6 melons. How many melons are in the garden?

28. How many yards of sheeting are there in 91 bales, each bale containing 23 bolts of 45 yards each?

29. Is $4 \times 5 + 6 \times 5 + 8 \times 5$ equal to $(4 + 6 + 8) \times 5$? Prove it. Which is the shorter method of reduction?

30. Find the value of $13 \times 18 + 36 \times 18 + 71 \times 18$ by the shorter method.

31. If a clerk deposits $17 in a bank every month, how much will he have deposited in 9 months?

32. It is estimated that the Mississippi River deposits 137,139,200 cubic yards of solid matter in the Gulf of Mexico every year. How many cubic yards has it deposited in 512 years?

DIVISION.

Definitions and Principles.

69. How many are 4 times 5 yards? How many times, then, does 20 yards contain 5 yards? How much is one fourth of 20 yards?

How many are 7 times $3? How many times, then, does $21 contain $3? How much is one seventh of $21?

The quotient of 40 days by 8 days is 5, because 8 days × 5 = 40 days.

The quotient of 40 days by 5 is 8 days, because 5 × 8 days = 40 days.

What is the quotient of 36 books by 9 books? Why?

What is the quotient of 36 books by 4? Why?

What is the quotient of 28 feet by 7 feet? Why?

What is the quotient of 32 hours by 8? Why?

70. The **quotient** of one number by another is the number which, when multiplied by the second number, will give the first number.

71. **Division** is the process of finding the quotient of one number by another.

That is, to *find the quotient* of 18 by 6 is to *divide* 18 by 6. In multiplication two factors, as 6 and 3, are given to find the product 18; in division a product and one of its factors, as 18 and 6, are given, to find the other factor, 3.

The quotient of 18 inches by 3 inches is 6, because 3 inches × 6 = 18 inches. Here the quotient 6 shows the number of times that 18 inches *contains* 3 inches. In examples like this the quotient is the *measure* of the greater number by the less.

The quotient of 18 inches by 3 is 6 inches, because 3 × 6 inches = 18 inches. Here the quotient, 6 inches, is one of the three *equal parts* of 18 inches, called a *third*. In examples like this the quotient is a *part* of the greater number.

72. Hence, we may say, division is the process of finding how many times one number is contained in another of the same kind, or of finding one of the equal parts of a number.

73. The number to be divided is the **dividend**; the number by which it is divided, the **divisor**; and the result, the **quotient.**

74. When a part of the dividend is left after the division is performed, it is the **remainder,** and must always be less than the divisor. When there is no remainder, the division is said to be exact.

75. The **sign of division** is ÷, which is read *divided by*, or *contains*, and indicates that the number *before* it is to be divided by the number *after* it.

Thus, 36 ÷ 9 = 4, is read: 36 divided by 9 equals 4, or 36 contains 9, 4 times. Here, 36 is the *dividend*, 9 the *divisor*, and 4 the *quotient*. The expressions 9)36 and $\frac{36}{9}$ are each equivalent to 36 ÷ 9, and are read alike.

76. PRINCIPLES.—1. *Division is the converse of multiplication;* that is, *the product of the divisor and quotient, plus the remainder, is the dividend.*

2. *When the dividend and divisor are like numbers, the quotient is an abstract number.*

Fundamental Combinations.

77. The fundamental combinations of division are included in those of multiplication.

Thus, from 3 × 8 = 24 we get 24 ÷ 8 = 3, and 24 ÷ 3 = 8.

Tell the quotients of the following instantly:

24 ÷ 6.
27 ÷ 9.
36 ÷ 6.
16 ÷ 2.
10 ÷ 5.
36 ÷ 4.
30 ÷ 5.
48 ÷ 6.
72 ÷ 8.
24 ÷ 4.
27 ÷ 3.
16 ÷ 8.
72 ÷ 9.

14 ÷ 2.
18 ÷ 3.
35 ÷ 5.
54 ÷ 6.
81 ÷ 9.
20 ÷ 4.
15 ÷ 5.
12 ÷ 6.
64 ÷ 8.
14 ÷ 7.
18 ÷ 6.
35 ÷ 7.
12 ÷ 2.

10 ÷ 2.
12 ÷ 4.
42 ÷ 7.
16 ÷ 4.
25 ÷ 5.
63 ÷ 9.
8 ÷ 2.
9 ÷ 3.
56 ÷ 7.
45 ÷ 9.
12 ÷ 3.
42 ÷ 6.
63 ÷ 7.

6 ÷ 3.
18 ÷ 2.
24 ÷ 8.
28 ÷ 7.
32 ÷ 4.
4 ÷ 2.
21 ÷ 7.
40 ÷ 8.
49 ÷ 7.
6 ÷ 2.
18 ÷ 9.
24 ÷ 3.
40 ÷ 5.

48 ÷ 8.
30 ÷ 6.
36 ÷ 9.
15 ÷ 3.
20 ÷ 5.
54 ÷ 9.
8 ÷ 4.
56 ÷ 8.
45 ÷ 5.
21 ÷ 3.
32 ÷ 8.
28 ÷ 4.
9 ÷ 9.

Name quotient and remainder:

2)7. 4)19. 5)48. 3)7. 7)65. 5)42.
5)31. 6)40. 7)25. 9)50. 8)46. 3)14.
8)75. 9)65. 6)58. 8)59. 9)31. 2)9.
7)19. 3)16. 5)23. 7)32. 6)29. 4)23.
9)77. 2)13. 4)30. 6)33. 5)37. 6)11.
3)11. 8)13. 3)26. 5)19. 4)19. 8)31.

Equal Parts of Numbers.

78. When a number is divided:

Into *two* equal parts, one of the parts is 1 *half* of the number;

Into *three* equal parts, one of the parts is 1 *third* of the number;

Into *four* equal parts, one of the parts is 1 *fourth* of the number;

Into *five* equal parts, one of the parts is 1 *fifth* of the number, and so on.

The parts 1 half, 1 third, 1 fourth, 1 fifth, etc., are written: ½, ⅓, ¼, ⅕, etc., respectively.

To find 1 *half* of a number: *divide the number by* 2.
To find 1 *third* of a number: *divide the number by* 3.
To find 1 *fourth* of a number: *divide the number by* 4.
To find 1 *fifth* of a number: *divide the number by* 5.
And so on.

EXERCISES (Oral).

79. How much is:

1. One half of 12? *2.* One third of 27? *3.* One fourth of 24?
4. One ninth of 45? *5.* One sixth of 48? *6.* One eighth of 56?
7. One fifth of 35? *8.* One tenth of 60? *9.* One seventh of 63?

What is the value of :

10. $\frac{1}{2}$ of 6 ? *11.* $\frac{1}{4}$ of 20 ? *12.* $\frac{1}{9}$ of 72 ? *13.* $\frac{1}{7}$ of 21 ?
14. $\frac{1}{6}$ of 42 ? *15.* $\frac{1}{8}$ of 32 ? *16.* $\frac{1}{5}$ of 50 ? *17.* $\frac{1}{3}$ of 18 ?

18. 2 yards are 6 feet. How many feet are in 1 yard ?

ANALYSIS.—Since 2 yards are 6 feet, 1 yard is half of 6 feet, which is 3 feet.

19. 5 hands in 20 inches. How many inches in 1 hand ?

20. 5 weeks in 35 days. How many days in 1 week ?

21. 6 suits in 30 yards. How many yards in 1 suit ?

22. 7 sets in 42 chairs. How many chairs in 1 set ?

23. If 8 hats cost $32, what is the cost of 1 hat ?

24. If 9 guns cost $63, what is the cost of 1 gun ?

25. If 5 peaches cost 15 cents, what is the cost of 1 peach?

26. If oranges are worth 5 cents each, how many oranges can be bought for 35 cents ?

Since each orange is worth 5 cents, 35 cents will buy as many oranges as 5 cents is contained times in 35 cents, which is 7.

27. How many times 3 days make 24 days ?

28. How many times is 6 cents contained in 42 cents ?

29. How many times is 8 bushels contained in 48 bushels ?

30. I bought 45 cents worth of oranges and paid 5 cents for each orange. How many did I buy ?

31. How many hours will it take a man to walk 27 miles if he walks 3 miles per hour ?

32. There are 40 boys in school, and 5 boys in each class. How many classes are there ?

33. The dividend is 54, the divisor 9. What is the quotient ?

34. The dividend is 63, the divisor 7. What is the quotient ?

35. $56 was divided equally among some boys. If each boy got $8, how many boys were there ?

36. How many weeks in 42 days ?

37. How many rows will 36 trees make if I put 6 trees in each row?

38. One of the factors of 28 is 4. What is the other factor?

39. One of the factors of 64 is 8. What is the other factor?

40. 5 minutes × ? = 40 minutes. 40 ÷ 5 = ?

41. 6 days × ? = 54 days. ? × 5 = 35 pens.

42. 48 cups ÷ ? = 8. $\frac{1}{8}$ of 48 cups = ?

43. 63 boards ÷ ? = 7 boards. 63 boards contains 7 boards how many times?

44. Find the value of 8 + 24 ÷ 6.

SOLUTION.—24 ÷ 6 = 4; 8 + 4 = 12.

The operation denoted by ÷ or × must be performed before those denoted by + and –, unless otherwise indicated by a parenthesis.

45. Find the value of 28 ÷ (5 + 2).

SOLUTION.—5 + 2 = 7; 28 ÷ 7 = 4.

Find the value of:

46. 8 – 10 ÷ 2.

47. 12 ÷ 3 + 4.

48. 24 ÷ 8 + 9.

49. 12 – 40 ÷ 5.

50. $\frac{1}{5}$ of (23 – 18 ÷ 6).

51. $\frac{1}{5}$ of (7 × 8 – 6).

52. 7 – $\frac{1}{6}$ of (8 × 3).

53. $\frac{1}{12}$ of (9 × 8) – 2.

54. (9 × 6 + 2) ÷ 7.

55. (7 × 8 – 4) ÷ 13.

56. (4 × 7) ÷ (8 – 6).

57. (8 × 9) ÷ (5 + 7).

58. (9 × 6 + 2) ÷ (10 – 2 × 3).

59. (11 × 5 –6) ÷ (15 ÷ 3 + 2).

Short Division.

80. When division is performed without writing the work out in full, the process is called **short division.**

1. Divide 16854 by 6.

EXPLANATION.—For convenience the divisor is written at the left of the dividend, with a line between them.

6 is not contained in 1 ten-thousand; hence we say 1 ten-thousand and 6 thousands are 16 thousands, and 6 is contained in 16 thousands 2 thousands times, with a re-

OPERATION.

$$\begin{array}{r} 6\overline{)16854} \\ \hline 2809 \end{array}$$

mainder of 4 thousands, or 40 hundreds. We write the 2 below in the thousands' place.

Uniting the 40 hundreds with the 8 hundreds, we have 48 hundreds. 6 is contained in 48 hundreds 8 hundreds times, with no remainder. We write 8 below in the hundreds' place.

6 is not contained in 5 tens; hence we say 5 tens contains 6 no tens times with a remainder of 5 tens or 50 ones. We write 0 below in the tens' place.

Uniting the 50 ones with the 4 ones, we have 54 ones, which contains 6, 9 ones times, and this we write below in the ones' place.

Proof: The quotient, 2809, multiplied by the divisor, 6, is the dividend, 16854.

EXERCISES (Written).

81. Short division is usually employed when the divisor is less than 11.

1. Divide 4836 by 2; 6978 by 3; 8536 by 4.
2. Divide 5754 by 2; 4872 by 3; 9508 by 4.
3. Divide 8730 by 5; 9102 by 6; 3402 by 7.
4. Divide 50024 by 8; 92187 by 3; 12678 by 6.
5. Divide 75618 by 9; 95328 by 8; 18907 by 7.

Find the quotient and remainder:

6. 2643 ÷ 3.
7. 6235 ÷ 5.
8. 85740 ÷ 9.
9. 6876 ÷ 4.
10. 4571 ÷ 7.
11. 73128 ÷ 9.
12. 25003 ÷ 6.
13. 47904 ÷ 8.
14. 50024 ÷ 7.
15. 510432 ÷ 8.
16. 617386 ÷ 9.
17. 43206 ÷ 6.

18. $\frac{41315}{6}$.
19. $\frac{41853}{7}$.
20. $\frac{21020}{8}$.
21. $\frac{60507}{9}$.
22. $\frac{63018}{4}$.
23. $\frac{370113}{5}$.
24. 3)64732.
25. 5)71250.
26. 2)136158.
27. 8)21604.
28. 7)19000.
29. 9)104875.

30. How many yards of cloth will it take, at 6 cents a yard, to amount to 37296 cents?

31. A merchant spent $33222 for hats, paying on an average $7 apiece. How many hats did he buy?

32. A farmer received $1952 for a lot of land, which he sold at the rate of $8 per acre. How many acres did he sell?

33. 4608 chestnuts were divided equally among 9 boys. How many chestnuts did each boy receive?

34. At $6 a barrel, how many barrels of flour can be purchased with $3756?

35. A wheel is 8 feet in circumference. How many times will it revolve in rolling a distance of 23256 feet?

36. There are 9 square feet in a square yard. How many square yards are there in 45783 square feet?

Long Division.

82. The process of **long division** is the same as that of **short division,** except that the work is written more or less in full.

83. ***When the quotient is not greater than 9.****

1. Divide 91 by 21.

EXPLANATION.—The first figure of the dividend is 9, and that of the divisor is 2; 2 in 9, 4 times. Place the 4 on the right, multiply it by 21, subtract the product 84 from 91, which gives 7 remainder. Hence the answer is: Quo. 4, rem. 7.

OPERATION.

21)91(4
84
7

---

* Most students experience much difficulty in learning long division, which is largely due to an attempt to learn all the steps of a long operation at one time. The object of these exercises is to teach the student the first step only, namely: how to tell when a partial quotient figure is too large or too small.

2. Divide 442 by 75.

The first part of the dividend that contains the first figure (7) of the divisor is 44; 7 in 44, 6 times. Place the 6 on the right, multiply it by 75; and since the result, 450, is *larger* than 442, 6 is *too large*. Hence we take the next less integer (5), put it in place of 6, multiply it by 75; and since the result, 375, is *less* than 442, subtract 375 from 442; and since the remainder, 67, is less than the divisor, 5 is the correct quotient; and the answer is: Quo. 5, rem. 67.

OPERATION.

$$\begin{array}{r} 75)442(6 \\ 450 \\ \hline \end{array}$$

$$\begin{array}{r} 75)442(5 \\ 375 \\ \hline 67 \end{array}$$

3. Divide 337 by 35.

Since 33 is less than 35, we say 3 in 33, 11 times. But instead of 11 we try 9 on the right, multiply it by 35, and since the result, 315, is less than 337, subtract 315 from 337; and since the remainder, 22, is less than the divisor, 9 is the correct quotient.

OPERATION.

$$\begin{array}{r} 35)337(9 \\ 315 \\ \hline 22 \end{array}$$

EXERCISES (Written).

4. Divide 117 by 43; 600 by 76; 511 by 88.
5. Divide 828 by 93; 643 by 75; 360 by 49.
6. Divide 300 by 48; 767 by 98; 573 by 75.
7. Divide 805 by 137; 933 by 475; 108 by 13.
8. Divide 436575 by 77054; 346372 by 97353.
9. Divide 517694 by 68325; 257651 by 63854.

84. ***To divide any integer by any smaller one.***

1. Divide 9156 by 21.

EXPLANATION.—The fewest figures on the left hand of the dividend that will contain 21 is 91. By Art. 83, 21 is contained in 91 hundreds 4 hundreds times with a remainder of 7 hundreds, or 70 tens. We write the 4 on the right in the hundreds' place.

Uniting the 70 tens with the 5 tens of the dividend, we have 75 tens. By Art. 83, 21 is contained in 75 tens 3 tens times with a remainder of 12 tens, or 120 ones. We write the three on the right in the tens' place.

Uniting the 120 ones with the 6 ones of the dividend, we have 126 ones, which contains 21, 6 ones times. Hence the quotient is 436.

OPERATION.

$$\begin{array}{l} 21)9156(436 \\ \quad\;\; 84 \\ \quad\;\; \overline{\;\;75} \\ \quad\;\;\;\; 63 \\ \quad\;\;\;\; \overline{126} \\ \quad\;\;\;\; 126 \\ \quad\;\;\;\; \overline{} \end{array}$$

RULE.—*Write the divisor on the left of the dividend, with a line between them, and draw a line on the right.*

Find how many times the divisor is contained in the least number of the left hand figures of the dividend that will contain it, and place the quotient on the right.

Multiply the divisor by this quotient figure, subtract the product from the figures of the dividend used, and to the remainder annex the next figure of the dividend.

Divide as before, and continue the operation until all the figures of the dividend have been brought down.

When one of the partial dividends is less than the divisor, write 0 *for the next figure of the quotient, and bring down the next figure of the dividend.*

PROOF.—*The same as in short division.*

After all the figures of the dividend have been brought down and divided, the remainder, if there be one, may be set off by itself, or written over the divisor and annexed to the quotient.

EXERCISES (Written).

85. *1.* Divide 455 by 7 ; 592 by 8 ; 429 by 13.
2. Divide 2574 by 11 ; 3780 by 12 ; 3168 by 24.
3. Divide 4494 by 14 ; 2480 by 20 ; 7875 by 35.
4. Divide 9672 by 26 ; 7946 by 29 ; 9591 by 23.
5. Divide 7250 by 25 ; 1792 by 14 ; 8100 by 18.
6. Divide 9630 by 36 ; 8820 by 42 ; 2169 by 19.
7. Divide 3630 by 28 ; 7540 by 49 ; 3575 by 75.

Find the quotient of :

8. 24192 ÷ 32.
11. 31080 ÷ 35.
14. 41220 ÷ 45.
17. 211044 ÷ 86.
20. 317646 ÷ 126.
23. 711287 ÷ 321.

9. 32004 ÷ 42.
12. 42628 ÷ 74.
15. 63168 ÷ 94.
18. 429436 ÷ 49.
21. 238788 ÷ 134.
24. 811332 ÷ 372.

10. 45292 ÷ 52.
13. 21045 ÷ 61.
16. 51875 ÷ 83.
19. 727748 ÷ 98.
22. 456104 ÷ 146.
25. 105056 ÷ 224.

Find the quotient of:

26. 58650 ÷ 625.
27. 98629 ÷ 587.
28. 184752 ÷ 144.
29. 75863 ÷ 5421.
30. 498614 ÷ 3721.
31. 1902825 ÷ 8457.
32. 2100315 ÷ 581.
33. 7481888 ÷ 1021.
34. 3207594 ÷ 767.
35. 8074528 ÷ 6328.
36. 1019806 ÷ 893.
37. 4223745376 ÷ 23456.

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38. Into how many farms of 173 acres each can 4325 acres of land be divided?

39. How many bales of 435 pounds each can be made from 36,540 pounds of cotton?

40. How many bushels of corn are there in 10416 pounds of corn? See Art. 67.

41. How many barrels in 157,140 pounds of coal?

42. In how many days can a paper-mill make 10,176 reams of paper, if it makes 96 reams a day?

43. A Government agent bought cavalry horses at $143 each, and expended $168,454. How many horses did he buy?

44. The yearly cost of keeping a turnpike road in repair is $1438.20, at the rate of $30.60 per mile. How many miles long is the road?

45. If the distance around a wheel is 64 inches, how many times will the wheel turn over in rolling a distance of 7232 inches?

46. At what price per head must I sell 148 hogs to receive $1628?

47. The salary of the President of the United States is $50,000 a year. How much is that a day, there being 365 days in 1 year?

48. There are 196 pounds in a barrel of flour. How many barrels in 241,276 pounds?

49. A owed B $34,789, and after paying $6844, he paid $135 daily until all was paid. How many daily payments did he make?

50. The circumference of the earth is 25,000 miles. How long would it take a vessel to sail around it, going at the rate of 125 miles per day?

51. The moon is 240,000 miles from the earth. How long would it take a balloon to reach it, moving at the rate of 64 miles per hour?

52. The product of two numbers is 54576, and one of the numbers is 379. What is the other number?

53. A man having $24,000 buys 180 acres of land, at $75 per acre. How much land can he buy with what remains, at $84 per acre?

54. A speculator wishes to exchange 75 horses, valued at $117 each, for cows, valued at $39 each. How many cows should he receive?

86. ***When the divisor has ciphers on the right.***

Divide 347 by 10.

OPERATION.

1|0)34|7

34 quo. and 7 rem.

EXPLANATION.—347 = 34 tens, 7 ones, which evidently contains ten 34 times, with a remainder of 7.

Divide 4129 by 100.

OPERATION.

1|00)41|29

41 quo. and 29 rem.

EXPLANATION.—4129 = 41 hunds. 29 ones, which evidently contains a hundred 41 times, with a remainder of 29.

Hence, to divide by 10, 100, 1000, etc., *cut off one, two, three, etc., figures from the right of the dividend for a remainder, and the remaining figures form the quotient.*

Divide 347 by 60.

OPERATION.

6|0)34|7(5 quo.
30

47 rem.

EXPLANATION.—347 = 34 tens, 7 ones. Now 34 tens contains 6 tens 5 times, with a remainder of 4 tens or 40 ones, which when united with 7 ones gives a total remainder of 47.

Divide 73153 by 2700.

EXPLANATION.—73153 = 731 hunds., 53 ones, and 2700 = 27 hunds. Now 731 hunds. contains 27 hunds. 27 times, with a remainder of 2 hunds., or 200 ones, which united with 53 ones gives a total remainder of 253.

OPERATION.

27|00)731|53(27 quo.

54

191

189

253 rem.

RULE.—*Cut off the ciphers from the divisor, and also cut off the same number of figures from the right of the dividend; divide the remaining figures of the dividend by the remaining figures of the divisor, and to the remainder, if any, annex the figures cut off from the dividend for a true remainder.*

EXERCISES (Oral).

87. Divide (give quotient and remainder):

1. 60 by 10; 64 by 10; 80 by 10; 25 by 10.
2. 36 by 10; 90 by 10; 41 by 10; 39 by 10.
3. 33 by 10; 58 by 10; 72 by 10; 94 by 10.

4. 147 by 10; 673 by 10; 504 by 100.
5. 831 by 10; 496 by 10; 620 by 100.
6. 725 by 10; 500 by 10; 917 by 100.

7. 761 by 100; 615 by 100; 700 by 100.
8. 584 by 100; 300 by 100; 911 by 100.
9. 470 by 100; 990 by 100; 637 by 100.
10. 63470 by 10; by 100; by 1000; by 10000.
11. 52420 by 10; by 100; by 1000; by 10000.

EXERCISES (Written).

88. Divide:

1. 271 by 20; by 30; by 50; by 80.
2. 635 by 40; by 70; by 90; by 60.
3. 483 by 50; by 30; by 60; by 80.

Divide :

4. 3837 by 60 ; by 800 ; by 240.
5. 8070 by 30 ; by 400 ; by 510.
6. 7285 by 80 ; by 300 ; by 560.

7. 98354 by 500 ; by 1600 ; by 7000.
8. 72560 by 800 ; by 4400 ; by 4000.
9. 288000 by 1200 ; by 18000 ; by 6400.

10. If 40 barrels of molasses cost $480, what is the price of 1 barrel ?

11. A farmer sold 600 acres of land for $7800. How much was that per acre ?

12. A merchant sold 800 yards of silk for 184,000 cents. How much was that per yard ?

13. If a turnpike 130 miles long cost $325,000, how much did it cost per mile ?

14. A wire fence is 83,200 rods long. How many miles is that, there being 320 rods in a mile ?

15. If a field of 3500 acres produces 84,000 bushels of corn, what is the average yield per acre ?

16. Texas has an area of 265,780 square miles, and Louisiana 48,720 square miles. Into how many states of the size of Louisiana could Texas be divided, and how many square miles would be left over ?

17. The population of Louisiana in 1900 was 1,381,625. About how many persons, on an average, were there living on a square mile ? See ex. 16.

General Principles of Division.

89. The value of the quotient depends upon the *relative* values of the dividend and divisor.

There are six cases of changing the terms by multiplication and division, which may be illustrated by comparing the equation $\frac{24}{6} = 4$ with the following. Thus:

$\frac{24 \times 2}{6}$ *or* $\frac{24}{6 \div 2} = 8$ — *Multiplying the dividend, or dividing the divisor,* MULTIPLIES *the quotient by the same integer.*

$\frac{24 \div 2}{6}$ *or* $\frac{24}{6 \times 2} = 2$ — *Dividing the dividend, or multiplying the divisor,* DIVIDES *the quotient by the same integer.*

$\frac{24 \times 2}{6 \times 2}$ *or* $\frac{24 \div 2}{6 \div 2} = 4$ — *Multiplying or dividing both the dividend and divisor by the same integer* DOES NOT CHANGE *the quotient.*

Hence the following *General Principle:*

A change in the DIVIDEND *by multiplication or division produces a* LIKE *change in the quotient; but such a change in the* DIVISOR *produces an* OPPOSITE *change in the quotient.*

Exercises Involving Multiplication and Division.

90. This is an important class of questions, and they should be studied until the student thoroughly understands the solutions. Let it be observed that in *analyzing* we reason from *given numbers* to *one,* and then from *one* to the *required number.*

EXERCISES (Oral).

91. *1.* If 4 hats cost \$12, what will 7 hats cost?

OPERATION.

$\frac{1}{4}$ of \$12 = \$3

7 × \$3 = \$21

ANALYSIS.—Since 4 hats cost \$12, 1 hat will cost 1 fourth of \$12, or \$3. Again, since 1 hat cost \$3, 7 hats will cost 7 times \$3, or \$21.

2. If 6 apples cost 18 cents, what will 9 apples cost?

3. If 5 balls cost 60 cents, what will 9 balls cost?

4. If 9 tables cost \$45, what will 7 tables cost?

5. If 8 books cost \$24, what will 11 books cost?

6. If 5 hogs cost \$35, what will 7 hogs cost?

7. If 8 cows cost \$64, what will 5 cows cost?

8. If 5 sheep cost $60, what will 11 sheep cost ?

9. If 4 boys kill 48 squirrels, how many will 5 boys kill at the same rate ?

10. If 15 cats catch 45 rats, how many will 7 cats catch at the same rate ?

11. If 9 boys eat 36 biscuits, how many will 20 boys eat at the same rate ?

12. If 5 sheep cost $35, how many sheep can be bought with $63 ?

ANALYSIS.—Since 5 sheep cost $35, 1 sheep will cost one fifth of $35, or $7. Now since 1 sheep cost $7, $63 will buy as many sheep as $7 is contained times in $63, which is 9—Ans.

13. If 8 barrels of flour are worth $48, how many barrels can be bought with $66 ?

14. How many books can I buy with $39, if 7 books cost $21 ?

15. How many oranges will $1 buy, if 9 oranges cost 45 cents ?

16. How many tons of hay will $91 buy, if $63 will buy 9 tons ?

17. If 12 men can do a piece of work in 5 days, how long will it take 20 men to do it ?

ANALYSIS.—Since 12 men do the work in 5 days, it will take 1 man 12 times 5 days, or 60 days, to do the work. If 1 man can do the work in 60 days, 20 men can do it in $\frac{1}{20}$ of 60 days, which is 3 days—Ans.

18. If 9 men can do a work in 8 days, how long will it take 6 men to do the same work ?

19. If 6 men can do a work in 10 days, how many men will it take to do the same work in 15 days ?

EXERCISES (Written).

92. *1.* If 18 acres cost $1620, what will 23 acres cost ?

2. If 25 tables cost $475, what will 17 tables cost ?

3. If 31 cows cost $496, what will 68 cows cost ?

4. If 43 books cost 688 cents, what will 52 books cost ?

5. If 752 sheep cost $4512, what will 173 sheep cost ?

6. If 20 *bushels* = 1280 *pints*, then 37 *bushels* = ?

7. If 15 *yards* = 540 *inches*, then 43 *yards* = ?

8. A dealer bought 43 cows for $1505, and wishes to buy 45 more at the same rate. What should he pay for them ?

9. How many horses may be bought with $2850, if 6 horses cost $450 ?

10. A man had $1005, of which he paid $360 for 24 acres of land ; how much more land could he buy with the remainder of his money, at the same rate ?

11. If it takes 52 men 70 days to do a piece of work, how long will it take 56 men to do it ?

12. If 8 men can do a piece of work in 145 days, how many men will be required to do the work in 29 days ?

13. If 28 acres of land cost $3640, what will 45 acres cost at $6 more an acre ?

14. If 24 horses cost $3096, what will 35 horses cost at $6 less a head ?

Parallel Problems.

93. *1.* One factor of 135 is 9. What is the remaining factor ?

2. What number multiplied by 17 will make 2125 ?

3. If 8 chairs cost $120, what will 1 chair cost ?

4. A man sold 63 horses for $5355. How much did he receive on an average per head ?

5. What number of tables, at $12 each, will $132 buy ?

6. A man invested $106,318 in land, at $59 per acre. How many acres did he purchase ?

7. If 11 girls gathered 121 grapes, how many did each girl gather on an average ?

8. 5544 bushels of corn were raised on 231 acres of land. What was the average product per acre ?

9. What number multiplied by 8 will lack 7 of making 55? $(55 - 7) \div 8 = ?$

10. A man had 1238 walnuts, and, after giving each of his sons 137, had 5 left. How many sons had he?

11. A boy gathers 15 nuts each day, and eats 6 each night. How many days before he will have 72 nuts?

12. A man's yearly salary is $1200, and his expenses $840.60. In how many years can he lay up $4312.80?

13. What is one half of the SUM of 30 and 16?

14. What is the average height of two trees whose heights are 87 feet and 105 feet?

15. What is the average of the THREE numbers 12, 15, and 9? $(12 + 15 + 9) \div 3 = ?$

16. What is the average weight of three bales of cotton weighing respectively 436, 453, and 464 pounds?

17. A lad sold a merchant 10 chickens at 15 cents each, and with the money bought apples at 3 cents each. How many apples did he buy?

18. A farmer sold 405 acres of land at $32 an acre, and invested the money in cows at $24 each. How many cows did he receive?

19. Emma sold 7 cakes at 4 cents each, and 6 apples at 2 cents each, and took her pay in ribbon at 5 cents a yard. How many yards did she get?

20. How much flour at $8 a barrel will pay for 27 hogs at $5 each, and 47 sheep at $7 each?

21. If 11 hats cost $55, what will 18 hats cost?

22. If 87 books cost $107.01, what will 144 books cost?

23. If 4 boys can whitewash a fence in 6 hours, how long would it take 3 boys to do it?

24. If it takes 42 men 25 days to build a bridge, how long will it take 35 men to do it?

25. If apples are worth 3 cents and oranges 5 cents apiece, how many apples will it take to pay for 18 oranges?

26. A farmer exchanged 64 cows for 34 horses valued at

$96 each. What did the farmer receive per head for his cows?

27. A boy bought 9 squirrels for $1.26, and sold 6 of them for the same amount. How much did he gain apiece on those he sold?

28. Twelve men owning a mine sold it to 15 others for $30,600. How much more did each of the original owners receive than each of the new owners pay?

29. A scantling is 4 inches wide and 2 inches thick; how many times can a string 252 inches long be made to wind around it?

30. I have a garden 320 feet long and half as wide; how many times must I walk around it to travel 100 miles—there being 5280 feet in a mile?

31. If 8 coats cost $120, what will 23 coats cost at $5 less apiece?

32. If 16 horses cost $1952, what will 45 horses cost at $8 more a head?

FACTORS AND MULTIPLES.

94. An exact divisor of a number is called a **factor** of that number.

Thus, 3 is a factor of 6, 5 of 20, and 7 of 21.

95. A number that is exactly divisible by another number is called a **multiple** of that number.

Thus, 6 is a multiple of 3, 20 of 5, and 21 of 7.

96. A number which has no exact divisors except itself and 1 is called a **prime number**; and a number which has other exact divisors besides itself and 1 is called a **composite number.**

Thus, 2, 3, 5, 7, 11, 13, 17, 19, are prime numbers, and 4, 6, 8, 9, 10, 12, 14, 15, are composite numbers.

97. Numbers divisible by 2 are called **even numbers,** and numbers not divisible by 2 are called **odd** numbers.

Thus, 2, 4, 6, 8, 10, 12, 14, are even numbers, and 1, 3, 5, 7, 9, 11, 13, are odd numbers.

98. The following simple conditions of divisibility will be found useful:

A number is divisible—

1. By 2 or 5, if its last figure is divisible by 2 or 5.

2. By 4 or 10, if the number expressed by its last two figures is divisible by 4 or 10.

3. By 8, if the number expressed by its last three figures is divisible by 8.

4. By 3 or 9, if the sum of its digits is divisible by 3 or 9.

5. By 6, if it is even and divisible by 3.

Thus, 5226 is divisible by 2, 3, and 6; 5320 by 2, 4, 5, 8, and 10; and 57348 by 2, 3, 4, 6, and 9.

EXERCISES (Oral).

99. *1.* Name the even numbers between 20 and 60.

2. Name the odd numbers between 30 and 70.

3. Name the prime numbers between 10 and 30. Between 30 and 60. Between 60 and 100.

4. Name the composite numbers between 11 and 31. Between 31 and 61. Between 61 and 101.

5. Name three exact divisors of 12. Of 18; 30; 42; 54.

6. What is the smallest factor, except 1, of 15? 21? 35? 49? 54? 121?

7. Excepting the number itself, what is the greatest factor of 21? 33? 36? 40? 49? 63? 72?

8. Name three multiples of 5. Of 6; 8; 9; 15; 20.

Which of the numbers, 2, 3, 4, 6, 8, 9, 10, can be seen by inspection to be factors of:

9. 1935? *10.* 7326? *11.* 6980? *12.* 34536?
13. 29304? *14.* 48105? *15.* 47142? *16.* 71280?

Two Important Theorems.

100. The following are important general theorems:

I. *Every factor of a number is a factor of any multiple of that number.*

If, for example, 7 is a factor of the number, the number can be arranged in groups of *seven*, and so also can any number of repetitions of the number. Similarly for any other factor.

II. *Every common factor of two numbers is a factor of their sum and difference.*

If, for example, 7 is a factor of both numbers, each of the numbers may be arranged in groups of *seven*, and therefore their sum and difference consist of a whole number of *sevens*.

These principles can be made very clear to the beginner by the use of counters.

Factoring.

101. ***To find the prime factors of a composite number.***

102. Factoring is the process of finding the factors of a number.

103. An **exponent** is a small figure written at the right of a number, and a little above, to show how many times the number is used as a factor.

Thus, $4^3 = 4 \times 4 \times 4 = 64$, and $5^4 = 5 \times 5 \times 5 \times 5 = 625$.

1. What are the prime factors of 1050?

OPERATION.

2)1050
3)525
5)175
5)35
7

EXPLANATION.—Divide the number by any prime divisor of it, as 2. Then divide the quotient by any prime divisor of it, and so continue until the quotient, as 7, is a prime number. The several divisors and the last quotient are the prime factors required. Hence the prime factors of 1050 are 2, 3, 5, 5, and 7. $2 \times 3 \times 5^2 \times 7 = 1050$.

EXERCISES (Written).

104. Find the factors of :

1. 252. *2.* 1260. *3.* 10125. *4.* 21504.
5. 675. *6.* 3906. *7.* 25725. *8.* 70070.
9. 693. *10.* 7800. *11.* 39424. *12.* 64384.
13. 855. *14.* 2310. *15.* 11368. *16.* 50568.

Greatest Common Divisor.

105. **A common divisor** of two or more numbers is any number which exactly divides each of them.

Thus, 4 or 2 is a common divisor of 8, 12, and 20; 2, 3, 4, 6, or 12 is a common divisor of 24, 36, and 60.

106. Numbers which have no common divisor are said to be *prime to each other.*

Name all the common divisors of :

1. 6, 12, 18. *2.* 8, 16, 24, 32.
3. 20, 30, 40. *4.* 9, 18, 36, 54.
5. 36, 48, 60. *6.* 18, 36, 54, 90.

Name the greatest common divisor in each of the preceding examples.

107. The **greatest common divisor** of two or more numbers, denoted by G. C. D., is the greatest number which exactly divides each of them.

108. PRINCIPLE.—*The factors of the G. C. D. of two or more numbers are all the common prime factors.*

109. ***To find the G. C. D. of two or more numbers.***

1. Find the G. C. D. of 210, 315, 420.

EXPLANATION.—Divide the numbers by any common prime divisor, as 3; then divide the quotients similarly, and so continue until quotients are obtained that are prime to each other, as 2, 3, 4. The common prime factors are the divisors 3, 5, 7, whose product is the G. C. D. required. Hence the G. C. D. of 210, 315, 420 is 3 × 5 × 7, or 105.

OPERATION.

$$\begin{array}{l} 3)\underline{210,\ \ 315,\ \ 420} \\ 5)\underline{70,\ \ 105,\ \ 140} \\ 7)\underline{14,\ \ \ 21,\ \ \ 28} \\ \quad\ 2,\ \ \ \ 3,\ \ \ \ 4 \end{array}$$

Rule.—*Write the numbers in a horizontal line, divide by any prime number that will exactly divide all of them, divide the quotients in like manner, and so continue till all the quotients have no common divisor. The product of all the divisors is the G. C. D.*

EXERCISES (Written).

110. What is the G. C. D. of:

1.	50, 75, and 100?	*2.*	192, 160, and 224?
3.	95, 190, and 285?	*4.*	252, 324, and 432?
5.	72, 126, and 216?	*6.*	378, 648, and 918?

111. When the numbers cannot be easily factored, the method of finding the G. C. D. is based on this

Principle.—*Any number that will divide the dividend and divisor will also divide the divisor and remainder.*

Thus, 24 into 132 goes 5 times and 12 over. Hence 132 − 5 × 24 = 12. Now, any number which will divide 132 and 24 will divide 132 and 5 × 24 (**100**, I), and also their difference 12 (**100**, II). That is, any divisor of 132 and 24 is also a divisor of 24 and 12.

7. Find the G. C. D. of 301 and 430.

Operation.

```
301)430(1
    301
    ---
    129)301(2
        258
        ---
         43)129(3
            129
            ---
```

Explanation.—We divide the greater number by the less, then divide the less number by the remainder, and continue to divide the last divisor by the last remainder until nothing remaius; the last divisor, as 43, will be the required G. C. D. For any divisor of 301 and 430 is a divisor of 301 and 129; any divisor of these is a divisor of 129 and 43; and since 43 is the greatest number that will exactly divide these, it is also the G. C. D. of 301 and 430.

Find the G. C. D. of:

8.	371 and 636.	*9.*	372 and 589.
10.	481 and 814.	*11.*	1771 and 6132.
12.	779 and 665.	*13.*	7241 and 10907.

Find the G. C. D. of:

14. 3663 and 5439. *15.* 4880 and 10065.
16. 72491 and 103121. *17.* 347387 and 561851.

NOTE.—To find the G. C. D. of more than two numbers, first find the G. C. D. of any two of them; then find the G. C. D. of the number thus found and a *third* number; and so on through all the numbers.

---

18. Three men have respectively $740, $999, and $1147, with which each purchases an exact number of cattle at the same price per head. Required the price of the cattle and the number purchased by each.

19. A boy has three ropes, whose lengths are 45, 60, and 75 feet, which he wishes to cut into plow-lines of the greatest equal lengths. Find the length and number of the lines.

20. A drayman wishes to haul 378 bushels, then 648 bushels, then 918 bushels, with the smallest number of equal loads. Required the amount and number of loads.

21. A lad rolled a wheel 28 feet, then 35 feet, then 63 feet, and in each distance the wheel turned the greatest possible exact number of times. What was the distance around the wheel, and how often did the wheel revolve in all?

22. A man having a piece of land, the sides of which are 616 feet, 671 feet, 737 feet, and 792 feet, wishes to inclose it with a fence having panels the greatest possible equal length. What will be the length and number of the panels?

23. A boy bought three lots of chickens at the same price per chicken. For the first lot he gave 42 cents, for the second 70 cents, and for the third 98 cents. Find the least possible number of chickens he could have bought.

24. In a certain school there are 266 pupils in the high-school department, 418 in the grammar-school department, and 570 in the primary department, and the classes in all the departments are the same size. Find the least possible number of classes in all.

Least Common Multiple.

112. A number which is exactly divisible by two or more numbers is called a **common multiple** of those numbers.

Thus, 12 or 24 or 36 is a common multiple of 3, 4, and 6.

113. The smallest number which is exactly divisible by two or more numbers is called the **least common multiple** (L. C. M.) of those numbers.

Thus, 12 is the L. C. M. of 3, 4, and 6.

114. PRINCIPLES.—1. *A multiple of a number contains the prime factors of that number.*

2. *A common multiple of two or more numbers contains the prime factors of those numbers.*

3. *The least common multiple of two or more numbers is the least number that contains the prime factors of those numbers.*

115. ***To find the L. C. M. of two or more numbers.***

1. Find the L. C. M. of 5, 6, 7, 9, 14, 30, 105.

OPERATION.

$$\begin{array}{r|rrrrrrr} 5) & \cancel{5}, & \cancel{6}, & \cancel{7}, & 9, & 14, & 30, & 105 \\ \hline 3) & & & & 9, & 14, & 6, & 21 \\ \hline & & & & 3, & 14, & \cancel{2}, & \cancel{7} \end{array}$$

EXPLANATION.—5 and 6, being divisors of 30, will divide any number that 30 will; hence we cancel 5 and 6, also 7, as it is a divisor of 14. Now, divide two or more of the remaining numbers by any common prime factor, as 5, and bring down with the quotients 6 and 21 such numbers (9, 14) as do not contain the divisor. Next, divide by 3, as it is a prime divisor of 9, 6, 21, and bring down 14 with the quotients 3, 2, and 7. As 2 and 7 are divisors of 14, we cancel them, and as 14 and 3 are prime to each other, the division terminates here. Now, the L. C. M. is the continued product of the divisors 5 and 3 and the final quotients 3 and 14, or 5 × 3 × 3 × 14 = 630.

RULE.—*Write the numbers in a horizontal line, cancel such of the smaller as are divisors of the larger, divide the remaining numbers by any common prime divisor of two or more of them, and write the quotients and undivided numbers underneath.*

Proceed with these quotients and undivided numbers as with the given numbers, and so continue until the results are prime to each other. The continued product of all the divisors and undivided numbers is the L. C. M.

EXERCISES (Written).

116. Find the L. C. M. of:

1. 6, 10, 16, 20, 24.
2. 18, 25, 36, 42, 100.
3. 8, 12, 18, 24, 36.
4. 13, 17, 33, 39, 119.
5. 12, 15, 20, 36, 60.
6. 20, 43, 60, 86, 120.
7. 7, 19, 57, 84, 112.
8. 210, 315, 420, 1575.
9. 10, 20, 30, 40, 50.
10. 462, 693, 1617, 198.
11. 7, 22, 55, 77, 110.
12. 33, 99, 36, 108, 135.
13. 8, 240, 180, 200.
14. 24, 40, 96, 100, 120.

15. What is the least number of bushels of wheat that would make an exact number of loads for either of three drays, hauling respectively 30, 35, and 36 bushels at a load?

16. What is the smallest number of marbles that can be put in sacks that will hold 12, 15, or 20 marbles each?

17. What is the smallest distance that can be divided into distances of 45, 75, 80, or 100 yards each?

18. James has $6, John $8, Charles $9, and the amount Henry has contains each of these amounts the least possible exact number of times. How much has Henry?

19. What is the smallest sum of money which can be exactly expended for hogs at $7, or cows at $35, or mules at $140, or horses at $150?

20. Simon can walk around a field in 20 minutes, Frank in 25, Walter in 30, and Gordon in 42 minutes. If they all start together, and continue to go around the field, how long before they will all reach the point of starting at the same time, and how many times will each have gone around the field?

Cancellation.

117. Cancellation is the process of shortening operations in division by rejecting equal factors from both dividend and divisor.

118. The **sign of cancellation** is an oblique mark (/) drawn through the number from which the factor is canceled.

When a number is canceled, 1 is supposed to take its place. Thus,

$$\frac{8 \times \cancel{5} \times \cancel{3}}{\cancel{5} \times \cancel{3}} = \frac{8 \times 1 \times 1}{1 \times 1} = 8$$

119. PRINCIPLES.—1. *Rejecting a factor from a number divides the number by that factor.*

2. *Rejecting equal factors from both dividend and divisor does not change the quotient.*

120. ***To shorten operations involving division by cancellation.***

1. Divide the product of 15 and 12 by the product of 3, 4, and 5.

EXPLANATION.—The division is indicated by writing the factors of the dividend above a horizontal line, and the factors of the divisor below it.

Since 3 and 4 are factors of 15 and 12, respectively, they may be rejected from both dividend and divisor, leaving the factors 5 and 3 in the dividend.

Next reject 5 from dividend and divisor, leaving 3 only in the dividend. Hence the quotient is 3.

OPERATION.

$$\frac{\overset{\cancel{5}}{\cancel{15}} \times \overset{3}{\cancel{12}}}{\cancel{3} \times \cancel{4} \times \cancel{5}} = 3.$$

2. Divide 5 × 12 by 16.

EXPLANATION.—Since 4 is a factor of 12 in the dividend and 16 in the divisor, it may be rejected, leaving 3 in the dividend and 4 in the divisor.

The product of the uncanceled factors in the dividend is 15 and the divisor is 4. Hence the quotient is $\frac{15}{4}$ or $3\frac{3}{4}$.

OPERATION.

$$\frac{5 \times \overset{3}{\cancel{12}}}{\underset{4}{\cancel{16}}} = \frac{15}{4} = 3\frac{3}{4}.$$

RULE.—*Cancel the factors common to the divisor and dividend, and divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

EXERCISES (Written).

121. Find the value of:

1. $\frac{12 \times 6}{9 \times 4}$. *2.* $\frac{25 \times 24}{10 \times 15}$. *3.* $\frac{18 \times 40}{15 \times 16}$.

4. $\frac{30 \times 8}{12 \times 5}$. *5.* $\frac{48 \times 36}{27 \times 16}$. *6.* $\frac{50 \times 90}{125 \times 6}$.

7. $\frac{60 \times 18 \times 10}{48 \times 9 \times 5}$. *8.* $\frac{28 \times 49 \times 90}{15 \times 84 \times 7}$.

9. $\frac{33 \times 45 \times 75}{15 \times 27 \times 55}$. *10.* $\frac{84 \times 72 \times 18}{63 \times 12 \times 12}$.

11. $81 \times 16 \times 60 \times 56 \div (45 \times 12 \times 24)$.
12. $25 \times 27 \times 66 \times 18 \div (84 \times 30 \times 45)$.
13. $64 \times 7 \times 100 \times 14 \div (25 \times 16 \times 49)$.
14. $84 \times 90 \times 98 \div (15 \times 21 \times 12 \times 21)$.

15. If 25 men can perform a work in 688 hours, how long will it take 43 men to do the same?

16. How many barrels of flour, each weighing 196 pounds, at 9 cents a pound, must be given in exchange for 3 loads of cotton, each load containing 5 bales, and each bale weighing 441 pounds, worth 8 cents a pound?

17. How many boxes of tea, each containing 24 pounds, at 90 cents a pound, must be given for 27 firkins of butter of 48 pounds each, at 25 cents a pound?

18. A farmer sold a grocer 9 loads of apples, each load containing 18 bags, and each bag 2 bushels, at 35 cents a bushel; and received in payment 12 boxes of sugar, each containing 135 pounds. What was the sugar worth a pound?

COMMON FRACTIONS.

INTRODUCTION.

122. Half. *1.* If an orange be divided into two equal parts, what is each part called?

2. How many halves of an orange make one orange? Two oranges? Three?

3. Count two oranges by halves.

Ans.—1 half, 2 halves, 3 halves, two.

4. Count three oranges by halves.

5. How many oranges in 2 halves of an orange? How many in 4 halves? In 6 halves?

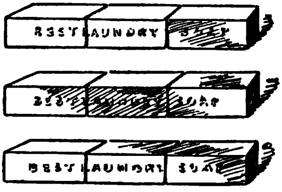
6. If one orange is worth 10 cents, what is half an orange worth?

7. If half an orange weighs 3 ounces, what will an orange weigh?

123. Third. *1.* If a bar of soap be divided into three equal parts, what is each part called?

2. How many thirds of a bar make one bar? Two bars? Three?

3. Count two bars by thirds. How many counts are there? Count three by thirds. How many thirds in four?



4. How many bars in 3 thirds of a bar? In 9 thirds of a bar? How many ones in 6 thirds? How many in 12 thirds? 7 thirds?

5. How many two thirds of a bar make two bars?

6. If one bar weighs 12 ounces, what will a third of a bar weigh? What will 2 thirds of a bar weigh?

7. If 2 thirds of a bar is 6 inches long, how long will 1 third of a bar be? How long will 3 thirds, or one bar, be?

124. Fourth. *1.* If a melon be divided into four equal parts, what is each part called?

2. How many fourths of a melon make one melon? Two melons? How many fourths in one? In two? In four?

3. Count two by fourths. Count three by fourths.

4. How many fourths of a melon make half of a melon? How many fourths in a half? In three halves?

5. How many halves in 4 fourths? In 20 fourths? In 9 fourths?

6. If one melon is worth 36 cts., how much is a fourth of a melon worth? Two fourths of a melon? Three fourths?

7. If three fourths of a melon is worth 18 cts., how much is a whole melon worth?

Answer.—If 3 fourths of a melon is worth 18 cts., then 1 fourth of a melon is worth $\frac{1}{3}$ of 18 cts., or 6 cts.; and if 1 fourth of a melon is worth 6 cts., then 4 fourths of a melon is worth 4 times 6 cts., or 24 cts. That is, a whole melon is worth 24 cents.

8. What is the length of a rod if three fourths of its length is 27 inches?

9. Which is the more, 1 half or 1 fourth of a pie? How much more is 1 half than 1 fourth?

10. Count three halves by fourths. How many fourths, then, in 3 halves? In 5 haves?

11. What is a lot worth if 3 fourths of it is worth $150?

125. Fifth. *1.* If a unit be divided into 5 equal parts, what is each part called ?

2. Count two units by fifths. Count three units by fifths.

3. How many fifths in two units ? In three units ? In six ?

4. Count two units by 2 fifths.

Ans.—2 fifths, 4 fifths, 6 fifths, 8 fifths, two.

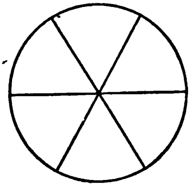
How many counts are there ? How many 2 fifths, then, in two ?

5. Count three units by 3 fifths. How many counts are there ? How many 3 fifths, then, make three ?

6. What is a melon worth if 4 fifths of it is worth 32 cents.

7. What is a pie worth if 3 fifths of it is worth 12 cents ?

8. What is a cake worth if 2 fifths of it is worth 20 cents ?



126. Sixth. *1.* If a circle be divided into six equal parts, what is each part called ?

2. How many circles in 6 sixths of a circle ? In 12 sixths ? In 30 sixths ?

3. Count one by sixths. Count two by sixths.

4. How many sixths make a half ? How many make a third ?

5. What part of a half is a sixth ? What part of a third is a sixth ?

6. What is the third of a cake worth, if a sixth of it is worth 5 cents ?

7. What is the sixth of a pie worth, if half of it is worth 12 cents.

8. What is a cake worth if 5 sixths of it is worth 60 cents ?

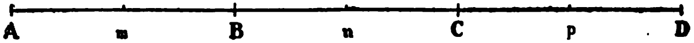
9. What does a stone weigh if 2 sixths of it weighs 40 pounds ?

10. How long is a rope if 4 sixths of it is 36 feet long ?

11. How old is Moses if 3 sixths of his age is 6 years ?

127. Fractional Units.—When anything, regarded as a whole, is divided into any number of equal parts, one of the parts is called a fractional unit.

Thus, $\frac{1}{3}$ of a bar of soap, $\frac{1}{4}$ of a melon, $\frac{1}{5}$ of a gallon, $\frac{1}{6}$ of $1, 1 tenth, etc., are fractional units.



1. Draw a line, as AD. Divide it into 3 equal parts, at the points B and C. What part of the line is one of these parts?

Divide each of the three parts into two equal parts, at *m, n, p*. How many of these smaller parts are there in all? What part is Am of AB? Of AD? What does this prove?

Ans.—$\frac{1}{2}$ of $\frac{1}{3}$ of AD $= \frac{1}{6}$ of AD.

2. How many sixths in one? In a third? In a half?

3. Draw a line; divide it into 4 equal parts, and then divide each of the parts into 3 equal parts. What part of the whole line is one of the smaller parts? What part of a larger part is one of the smaller parts? What does this prove?

4. How many twelfths in one? In a third? In a fourth? Half?

5. Draw a line; divide it into 5 equal parts; divide each of these parts into 3 equal parts. Prove from these that:

$$\frac{1}{3} \text{ of } \frac{1}{5} = \frac{1}{15}, \text{ and } \frac{1}{5} \text{ of } \frac{1}{3} = \frac{1}{15}.$$

6. How many $\frac{1}{15}$ in 1? In $\frac{1}{3}$? In $\frac{1}{5}$?

Prove, by drawing lines and dividing them into equal parts, etc., that:

7. One half of a half = a fourth.

8. $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$.

9. One half of a fifth = a tenth.

10. $\frac{1}{2}$ of $\frac{1}{7} = \frac{1}{14}$.

How much is:

11. $\frac{1}{4}$ of $\frac{1}{5}$ of a pear? $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{3}$ of $\frac{1}{12}$?
12. $\frac{1}{3}$ of $\frac{1}{7}$ of a mile? $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{4}$ of $\frac{1}{15}$?
13. $\frac{1}{5}$ of $\frac{1}{9}$ of a yard? $\frac{1}{7}$ of $\frac{1}{9}$? $\frac{1}{8}$ of $\frac{1}{10}$?
14. $\frac{1}{6}$ of $\frac{1}{8}$ of a week? $\frac{1}{6}$ of $\frac{1}{7}$? $\frac{1}{9}$ of $\frac{1}{11}$?

Prove, by drawing lines and dividing them into equal parts, etc., that:

15. 3 sixths make a half; 2 eighths make a fourth.
16. 2 tenths make a fifth; 4 twelfths make a third.

How many:

17. Fifteenths in a third? In a fifth? In 1?
18. Twentieths in a fifth? In a tenth? In $\frac{1}{4}$?
19. Thirtieths in a sixth? In a fifth? In $\frac{1}{3}$?
20. Fortieths in a fourth? In a half? In $\frac{1}{8}$?

SUGGESTION.—There are just as many *twelfths* in a *fourth* as there are *fours* in a *twelve.*

How many times:

21. Is $\frac{1}{30}$ contained in $\frac{1}{2}$? $\frac{1}{15}$ in $\frac{1}{3}$? $\frac{1}{24}$ in $\frac{1}{8}$?
22. Is $\frac{1}{35}$ contained in $\frac{1}{7}$? $\frac{1}{40}$ in $\frac{1}{8}$? $\frac{1}{42}$ in $\frac{1}{6}$?
23. Is $\frac{1}{49}$ contained in $\frac{1}{7}$? $\frac{1}{28}$ in $\frac{1}{4}$? $\frac{1}{60}$ in $\frac{1}{5}$?
24. Is $\frac{1}{54}$ contained in $\frac{1}{6}$? $\frac{1}{45}$ in $\frac{1}{9}$? $\frac{1}{72}$ in $\frac{1}{8}$?
25. Is $\frac{1}{27}$ contained in $\frac{1}{9}$? $\frac{1}{64}$ in $\frac{1}{4}$? $\frac{1}{63}$ in $\frac{1}{7}$?

Which is the more:

26. $\frac{1}{2}$ of $\frac{1}{9}$ or $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{4}$ of $\frac{1}{10}$ or $\frac{1}{3}$ of $\frac{1}{12}$?
27. $\frac{1}{3}$ of $\frac{1}{8}$ or $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{11}$ or $\frac{1}{4}$ of $\frac{1}{14}$?
28. $\frac{1}{4}$ of $\frac{1}{7}$ or $\frac{1}{3}$ of $\frac{1}{9}$? $\frac{1}{6}$ of $\frac{1}{15}$ or $\frac{1}{5}$ of $\frac{1}{18}$?

29. Are $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{15}$, and $\frac{1}{18}$ exact divisors of $\frac{1}{3}$? How many times is each contained in $\frac{1}{3}$?

30. Name five fractional units that are each exact divisors of $\frac{1}{2}$. Of $\frac{1}{4}$. Of $\frac{1}{5}$. Of $\frac{1}{6}$. Of $\frac{1}{7}$. Of $\frac{1}{8}$. Of $\frac{1}{9}$. Of $\frac{1}{10}$.

DEFINITIONS AND NOTATION.

128. A **fraction** is a fractional unit or a collection of fractional units.

1 half of an orange, 2 thirds of a bar of soap, 3 fourths of a melon, 7 tenths, are fractions.

129. The number which shows into how many parts a thing has been divided is called the **denominator**; and the number which shows how many parts are taken is called the **numerator.**

Thus, the fraction 3 tenths shows that something has been divided into 10 equal parts, and three of these parts are taken. Here 10 is the denominator and 3 is the numerator.

130. The **terms** of a fraction are its numerator and denominator.

131. A **common fraction** is usually expressed by writing the numerator over the denominator with a line between them. Thus:

1 third	is written	$\frac{1}{3}$	5 eighths	is written	$\frac{5}{8}$
1 fourth	"	$\frac{1}{4}$	7 tenths	"	$\frac{7}{10}$
3 fifths	"	$\frac{3}{5}$	12 thirty-thirds	"	$\frac{12}{33}$
4 sevenths	"	$\frac{4}{7}$	17 sixty-ninths	"	$\frac{17}{69}$

Express by figures:

1. 5 ninths.
2. 11 tenths.
3. 25 seventy-fifths.
4. 43 ninety-sevenths.
5. 13 one hundred eighths.
6. 7 one hundred twenty-fifths.
7. 111 one hundred forty-thirds.
8. 143 sixty-fourths.

132. The **value** of a fraction is its relation to one.

The value of 3 thirds is one; of $\frac{10}{5}$, 2; of $\frac{12}{4}$, 3.

133. A **proper fraction** is a fraction whose numerator is less than its denominator; as $\frac{2}{3}$, $\frac{5}{8}$, $\frac{11}{12}$.

The value of a proper fraction is less than a unit.

134. An **improper fraction** is a fraction whose numerator equals or exceeds its denominator; as $\frac{4}{4}$, $\frac{10}{7}$, $\frac{18}{8}$.

The value of an improper fraction is equal to or greater than a unit. It is called *improper* because it is not, properly speaking, a fraction, being equal to or more than 1.

135. A **mixed number** is an integer and a fraction united; as $3\frac{1}{2}$, which means $3+\frac{1}{2}$.

136. An integer is changed to a fractional form by writing 1 under it for a denominator.

Thus, $5 = \frac{5}{1}$, $14 = \frac{14}{1}$.

137. To **invert** a fraction is to interchange the places of its terms.

Thus, $\frac{6}{7}$ inverted is $\frac{7}{6}$; 9 inverted is $\frac{1}{9}$; 6 tenths inverted is 10 sixths.

138. The **reciprocal** of a fraction is the fraction inverted.

Thus, the reciprocal of 7 is $\frac{1}{7}$; of $\frac{3}{8}$, $\frac{8}{3}$; of 3 fifths, 5 thirds.

139. Analyze the fraction $\frac{5}{6}$.

Analysis.—$\frac{5}{6}$ is a proper fraction, 5 and 6 are the terms, 6 is the denominator and 5 the numerator. The fraction means:

(1) 5 of the parts when something has been divided into 6 equal parts.

(2) The fractional unit 1 sixth, or $\frac{1}{6}$ taken 5 times.

In like manner analyze:

1. $\frac{8}{11}$. *2.* $\frac{5}{13}$. *3.* $\frac{7}{7}$. *4.* $\frac{12}{5}$. *5.* $\frac{16}{15}$. *6.* $\frac{23}{3}$

140. A fraction is also a quotient.

This may be shown thus:

$$7 \times 1 \text{ yard} = 7 \text{ yards.}$$

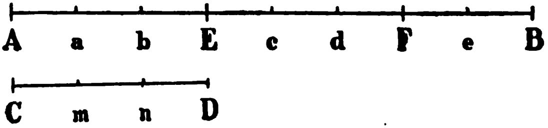
Divide by 10, $\frac{7}{10} \times 1 \text{ yard} = \frac{7 \text{ yards}}{10}$,

or $\frac{7}{10}$ of 1 yard $= \frac{1}{10}$ of 7 yards.

That is, any fraction, as $\frac{7}{10}$, may be regarded as an unexecuted division, the numerator being the dividend and the denominator the divisor.

FRACTIONS AS MEASURES.*

141. To **measure** a quantity is to find how many times it contains another known quantity of the same kind, called the **unit**; and this number of times is called the **measure** of the quantity.



Thus, to measure a length, as AB, a unit of length, as CD, is employed. This is laid on AB so that its first end (C) coincides with the first end (A) of the length to be measured, and the place (E) of its second end (D) is marked; it is then applied again, in the same way, to the remainder (EB), with its first end where the last end previously came; and so on, counting the number of times it is laid down. If there is finally no remainder, the measure will be an integer. But if there remains a length, as FB, less than that of the unit, this is measured by subdividing the length of the unit into a sufficient number of equal parts (as three), and using one of these parts as a secondary unit. In this case the measure will be a mixed number.

Thus, the measure of AF by CD is 2; and the measure of AB by CD is $2\frac{2}{3}$, which means that AB contains CD two whole times with a remainder (FB) which contains a third of CD two whole times.

With CD as a unit, what is the measure of AE? Ad? Ae? Ab? Ac? Aa?

If AB is the unit, what is the measure of CD? Cm? Cn?

If Cn is the unit, what is the measure of CD? Ac? Ae?

142. Considered as the result of a measurement, what does $6\frac{3}{5}$ ft. mean?

Ans.—It means that the thing measured contained the unit (foot) 6 times, with a remainder that contained one fifth of the unit 3 times.

Considered similarly, what do the following mean:

$2\frac{3}{4}$ yards? $5\frac{3}{8}$ rods? $12\frac{3}{7}$ miles?

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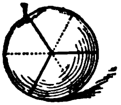
* In this book *unit*, simply, is used instead of "*unit of measure*"; and *measure*, instead of "*numerical measure.*" Hence, measure means a ratio.

Students should be required to measure convenient objects about the schoolroom, such as the length and width of the room, benches, desks, blackboards, books, etc. For this purpose they should be provided with graded yardsticks and foot rules, and should, in each case, write the correct measure.

REDUCTION.

143. Reduction of fractions is the process of changing their terms or forms without changing the values of the fractions.

144. ***To change the terms of a fraction.***



1. Have $\frac{4}{6}$ and $\frac{2}{3}$ the same terms ?

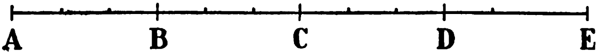
2. Is $\frac{4}{6}$ of an apple the same as $\frac{2}{3}$ of an apple ? Why ?

Ans.—If *one* piece is formed of every *two* of the 6 parts, there will be in all (6 ÷ 2) three equal pieces, of which the 4 parts of the numerator will contain (4 ÷ 2) two. Hence $\frac{4}{6} = \frac{2}{3}$.

3. Is $\frac{3}{6}$ of an apple the same as $\frac{1}{2}$ of an apple ? Why ?

4. Draw a straight line, as AE.

Divide it into four equal parts, at B, C, D. What part of AE is AD ?



5. Now suppose each of the four parts to be divided into three equal parts. How many of these small parts will AE contain ? How many will AD contain ? What part, then, of AE is AD ?

6. What two fractions are thus shown to be equal to each other ?

7. How may the second fraction be obtained from the first ?

8. How may the first fraction be obtained from the second ?

145. Principle.—*Multiplying or dividing both terms of a fraction by the same integer does not change the value of the fraction.*

9. How many fourths in $\frac{1}{2}$? How many tenths?
10. How many thirds in $\frac{6}{9}$? How many eighteenths?
11. How many fifths in $\frac{8}{10}$? How many thirtieths?

Fill the following blanks:

12. $\frac{10}{12} = \frac{(\ )}{6}$. *13.* $\frac{15}{18} = \frac{5}{(\ )}$. *14.* $\frac{21}{35} = \frac{(\ )}{5}$.
15. $\frac{18}{24} = \frac{3}{(\ )}$. *16.* $\frac{25}{75} = \frac{(\ )}{(\ )}$. *17.* $\frac{48}{64} = \frac{3}{(\ )}$.
18. $\frac{3}{5} = \frac{9}{(\ )}$. *19.* $\frac{4}{7} = \frac{(\ )}{28}$. *20.* $\frac{5}{9} = \frac{35}{(\ )}$.

Reduce:

21. $\frac{6}{8}$ to fourths. *22.* $\frac{8}{12}$ to thirds. *23.* $\frac{6}{10}$ to fifths.
24. $\frac{2}{3}$ to sixths. *25.* $\frac{3}{4}$ to twelfths. *26.* $\frac{3}{5}$ to twentieths.
27. $\frac{4}{5}$ to tenths. *28.* $\frac{15}{21}$ to sevenths. *29.* $\frac{5}{6}$ to thirtieths.

146. *To reduce a fraction to its lowest terms.*

A fraction is in its **lowest terms** when its terms have no common divisor.

Reduce $\frac{18}{30}$ to its lowest terms.

Explanation.—Divide both terms of $\frac{18}{30}$ by 2, and then divide both terms of $\frac{9}{15}$ by 3. $\frac{3}{5}$ is in its lowest terms, because 3 and 5 have no common divisor.

Operation.

$$\frac{18}{30} = \frac{9}{15} = \frac{3}{5}$$

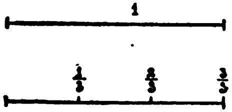
Rule.—*Divide the terms of the fraction by any common divisor, and continue thus to divide until the terms have no common divisor.*

EXERCISES (Written).

Reduce to lowest terms:

1. $\frac{75}{90}$, $\frac{63}{81}$, $\frac{39}{91}$. *2.* $\frac{125}{150}$, $\frac{165}{231}$. *3.* $\frac{315}{693}$, $\frac{336}{504}$, $\frac{308}{506}$.
4. $\frac{84}{231}$, $\frac{156}{195}$, $\frac{84}{168}$. *5.* $\frac{432}{576}$, $\frac{494}{806}$. *6.* $\frac{252}{288}$, $\frac{365}{511}$, $\frac{144}{960}$.
7. $\frac{196}{324}$, $\frac{384}{416}$, $\frac{154}{220}$. *8.* $\frac{225}{450}$, $\frac{315}{745}$. *9.* $\frac{420}{864}$, $\frac{216}{648}$, $\frac{361}{475}$.
10. $\frac{64}{512}$, $\frac{81}{729}$, $\frac{105}{259}$. *11.* $\frac{333}{351}$, $\frac{120}{372}$. *12.* $\frac{275}{875}$, $\frac{345}{552}$, $\frac{289}{323}$.

147. ***To reduce integers and mixed numbers to improper fractions.***



1. Is 1 line the same as 3 thirds of the line? How many fourths are there in an orange? In 2 oranges? In 3? In $2\frac{3}{4}$?

2. Count 2 melons by fifths. How many fifths are there?

3. Count $2\frac{3}{5}$ melons by fifths. How many fifths are there?

4. One pint is the eighth of a gallon; how many pints in 3 gallons? How many eighths in 3? How many pints in 3 gallons and 5 pints? How many eighths in $3\frac{5}{8}$?

Change:

5. 3 to fourths. *6.* 5 to thirds. *7.* $2\frac{1}{3}$ to thirds.
8. 4 to sixths. *9.* $2\frac{1}{2}$ to tenths. *10.* $3\frac{1}{9}$ to ninths.

Fill the following blanks:

11. 6 = ( ) halves. *12.* $3\frac{1}{2}$ = ( ) halves. *13.* $7\frac{1}{2} = \frac{(\ \ )}{2}$.
14. 10 = ( ) thirds. *15.* $5\frac{2}{3}$ = ( ) thirds. *16.* $4\frac{1}{3} = \frac{(\ \ )}{3}$.
17. 13 = ( ) sixths. *18.* $4\frac{5}{6}$ = ( ) sixths. *19.* $3\frac{1}{6} = \frac{(\ \ )}{6}$.
20. 11 = ( ) tenths. *21.* $7\frac{3}{10}$ = ( ) tenths. *22.* $9\frac{7}{10} = \frac{(\ \ )}{10}$.

Reduce $18\frac{3}{5}$ to fifths.

EXPLANATION.—In 1 there are 5 fifths, therefore in 18 there are 18 × 5 fifths, which is 90 fifths. 90 fifths + 3 fifths = 93 fifths, or $\frac{93}{5}$.

OPERATION.

$$\frac{18\frac{3}{5}}{\frac{93}{5}}$$

RULE.—*Multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the result over the denominator.*

EXERCISES (Written).

148. Reduce to improper fractions:

1. $15\frac{2}{3}$.	*2.* $18\frac{4}{7}$.	*3.* $64\frac{8}{13}$.	*4.* $375\frac{3}{8}$.
5. $12\frac{3}{5}$.	*6.* $24\frac{5}{6}$.	*7.* $75\frac{7}{15}$.	*8.* $476\frac{5}{7}$.
9. $18\frac{7}{10}$.	*10.* $70\frac{3}{8}$.	*11.* $86\frac{3}{20}$.	*12.* $238\frac{1}{201}$.
13. $24\frac{5}{9}$.	*14.* $81\frac{5}{11}$.	*15.* $99\frac{7}{51}$.	*16.* $503\frac{201}{300}$.

149. ***To reduce improper fractions to integers or mixed numbers.***

1. How many cakes will 6 thirds of a cake make? Is $\frac{6}{3}$ the same as 2?

2. How many oranges will 11 fourths of an orange make? Is $\frac{11}{4}$ equal to $2\frac{3}{4}$?

3. How many bushels will 5 fifths of a bushel make? How many will 15 fifths make? 26 fifths? $\frac{30}{5}$? $\frac{42}{5}$?

4. Reduce 18 thirds to an integer.

Since 3 thirds is equal to 1 unit, 18 thirds is equal to as many units as 3 thirds is contained times in 18 thirds, or 6. Therefore, $\frac{18}{3} = 6$.

OPERATION.

$18 \div 3 = 6.$

5. Reduce $\frac{18}{5}$ to a mixed number.

Since 5 fifths is equal to 1 unit, 18 fifths is equal to as many units as 5 fifths is contained times in 18 fifths, or $3\frac{3}{5}$. Therefore, $\frac{18}{5} = 3\frac{3}{5}$.

OPERATION.

$18 \div 5 = 3\frac{3}{5}.$

Reduce to integers or mixed numbers:

6. 17 thirds. *7.* 25 eighths. *8.* $\frac{30}{9}$. *9.* $\frac{21}{4}$.
10. 40 sixths. *11.* 45 tenths. *12.* $\frac{75}{12}$. *13.* $\frac{100}{16}$.

Fill the blanks in the following:

14. 12 thirds of a week = ( ) weeks.
15. 15 fifths of a peck = ( ) pecks.
16. 27 sixths of a cake = ( ) cakes.
17. 16 fourths = ( ) ones.
18. 23 fourths = ( ) ones.
19. 48 ninths = ( ) ones.

RULE.—*Divide the numerator by the denominator.*

EXERCISES (Written).

150. Reduce to integers or mixed numbers:

1. $\frac{300}{15}$, $\frac{465}{25}$. *2.* $\frac{399}{28}$, $\frac{471}{45}$. *3.* $\frac{1904}{61}$, $\frac{8173}{76}$.
4. $\frac{341}{62}$, $\frac{600}{40}$. *5.* $\frac{735}{57}$, $\frac{744}{69}$. *6.* $\frac{3864}{98}$, $\frac{4857}{63}$.
7. $\frac{224}{24}$, $\frac{475}{38}$. *8.* $\frac{842}{53}$, $\frac{425}{51}$. *9.* $\frac{7140}{300}$, $\frac{9476}{400}$.
10. $\frac{584}{92}$, $\frac{796}{96}$. *11.* $\frac{847}{107}$, $\frac{693}{420}$. *12.* $\frac{3045}{42}$, $\frac{7140}{112}$.

151. ***To reduce dissimilar fractions to similar fractions.***

152. Similar fractions are those which have the same denominator or fractional unit; and **dissimilar fractions** are those which have not the same denominator or fractional unit.

$\frac{5}{4}$ and $\frac{9}{4}$, 7 tenths and 3 tenths, are similar fractions.
$\frac{5}{4}$ and $\frac{9}{7}$, 7 tenths and 3 fifths, are dissimilar fractions.

1. Are $\frac{1}{2}$ and $\frac{1}{3}$ similar fractions? Is $\frac{1}{2}$ equal to $\frac{3}{6}$? Is $\frac{1}{3}$ equal to $\frac{2}{6}$? Are $\frac{3}{6}$ and $\frac{2}{6}$ similar fractions?

How, then, can $\frac{1}{2}$ and $\frac{1}{3}$ be made similar?

Ans.—By reducing them to sixths, 6 being a multiple of 2 and 3.

2. Are $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ similar fractions? Is $\frac{1}{2}$ equal to $\frac{6}{12}$? Is $\frac{2}{3}$ equal to $\frac{8}{12}$? Is $\frac{3}{4}$ equal to $\frac{9}{12}$? Are $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, similar fractions? How, then, may $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, be reduced to similar fractions?

3. Does multiplying both terms of a fraction by the same integer change the value of the fraction?

4. By what are the terms of $\frac{1}{2}$ multiplied to get $\frac{6}{12}$? The terms of $\frac{2}{3}$ to get $\frac{8}{12}$? Of $\frac{3}{4}$ to get $\frac{9}{12}$?

153. Principle.—*Dissimilar fractions are reduced to similar fractions by multiplying the terms of each fraction by such an integer as will cause all the denominators to be the same.*

1. Reduce $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{5}{12}$ to similar fractions.

All the denominators will exactly divide 24, or 48, or 72, etc.; hence all the denominators may be made 24, or 48, or 72, etc.

To make the denominators 24, the terms of the first fraction must be multiplied by 6, (24 ÷ 4); the terms of the second, by 3, (24 ÷ 8); and the terms of the third, by 2, (24 ÷ 12).

Operation.

$$\frac{3}{4}=\frac{3\times 6}{4\times 6}=\frac{18}{24}$$
$$\frac{7}{8}=\frac{7\times 3}{8\times 3}=\frac{21}{24}$$
$$\frac{5}{12}=\frac{5\times 2}{12\times 2}=\frac{10}{24}$$

154. Fractions may have more than one common denominator. The smallest one they can have is the L. C. M. (least common multiple) of the given denominators, and this is called their **least common denominator.**

2. Change $\frac{3}{4}$, $\frac{4}{9}$, and $\frac{7}{12}$ to similar fractions having their least common denominator.

The L. C. M. of 4, 9, 12, is 36, which is the common denominator.

Now multiply the terms of the fractions by the numbers $36 \div 4$, $36 \div 9$, $36 \div 12$, respectively; that is, by 9, 4, and 3, respectively.

$$\frac{3}{4} = \frac{27}{36}$$
$$\frac{4}{9} = \frac{16}{36}$$
$$\frac{7}{12} = \frac{21}{36}$$

1. If the L. C. M. of the denominators cannot be found by inspection, it should be found as in Art. 115.

2. Integers and mixed numbers, if any, must be reduced to improper fractions.

Hence fractions are reduced to their least common denominator by *multiplying the terms of each fraction by the number of times its denominator is contained in the L. C. M. of the given denominators.*

EXERCISES (Oral).

155. Change to similar fractions having the least common denominator :

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$. *2.* $\frac{1}{3}$, $\frac{1}{4}$. *3.* $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
4. $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$. *5.* $\frac{1}{3}$, $\frac{5}{6}$. *6.* $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{14}$.
7. $\frac{1}{3}$, $\frac{5}{6}$, $\frac{5}{9}$. *8.* $\frac{3}{10}$, $\frac{7}{15}$. *9.* $\frac{3}{8}$, $\frac{1}{6}$, $\frac{7}{12}$.
10. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. *11.* $\frac{5}{9}$, $\frac{7}{12}$. *12.* $\frac{3}{5}$, $\frac{4}{9}$, $\frac{8}{15}$.

EXERCISES (Written).

156. Reduce to similar fractions in their lowest terms :

1. $\frac{5}{6}$, $\frac{13}{36}$, $\frac{17}{48}$. *2.* $\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{10}$, $\frac{7}{12}$, $\frac{4}{15}$.
3. $\frac{5}{21}$, $\frac{9}{28}$, $\frac{4}{7}$. *4.* $\frac{3}{5}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{11}{20}$, $\frac{23}{40}$.
5. $\frac{7}{9}$, $\frac{5}{14}$, $\frac{11}{42}$. *6.* $\frac{2}{3}$, $\frac{3}{5}$, $\frac{19}{25}$, $\frac{8}{15}$, $2\frac{1}{3}$.
7. $\frac{5}{6}$, $\frac{7}{18}$, $\frac{13}{27}$. *8.* $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{7}{12}$, $\frac{11}{18}$.

ADDITION.

157. ***To add two or more fractions.***

1. What is the sum of 2 days and 7 days? 2 tenths and 7 tenths? $\frac{2}{13}$ and $\frac{7}{13}$?

What is the sum of:

2. 4 ninths and 3 ninths? $\frac{3}{10}$ and $\frac{4}{10}$?
3. 3 sixths and 2 sixths? $\frac{5}{12}$ and $\frac{6}{12}$?

158. *Similar fractions are added by adding their numerators, and placing the sum over the common denominator.*

4. Are 3 dimes the same as 30 cents?
Are 5 nickels equal to 25 cents?
What, then, is 3 dimes + 5 nickels?

5. Is $\frac{1}{2}$ of an orange equal to $\frac{3}{6}$ of an orange?
Is $\frac{1}{3}$ of an orange equal to $\frac{2}{6}$ of an orange?
What, then, is $\frac{1}{2}$ of an orange + $\frac{1}{3}$ of an orange?

6. $\frac{1}{3}$ is equal to how many twelfths?
$\frac{1}{4}$ is equal to how many twelfths?
What, then, is $\frac{1}{3} + \frac{1}{4}$?

159. *Dissimilar fractions are added by adding the similar fractions to which the given fractions are respectively equal.*

1. Find the sum of $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{8}$.

OPERATION.

$$\frac{2}{3} + \frac{1}{4} + \frac{3}{8} = \frac{16}{24} + \frac{6}{24} + \frac{9}{24}$$
$$= \frac{31}{24} = 1\frac{7}{24}.$$

EXPLANATION.—Since the fractions are dissimilar, they must be changed to similar fractions before adding. The L. C. M. of 3, 4, and 8 is 24, and $\frac{2}{3} = \frac{16}{24}$, $\frac{1}{4} = \frac{6}{24}$, and $\frac{3}{8} = \frac{9}{24}$. Hence the sum is $\frac{16+6+9}{24} = \frac{31}{24}$, or $1\frac{7}{24}$.

2. What is the sum of $4\frac{2}{3}$, $5\frac{5}{6}$, $7\frac{1}{4}$?

When there are mixed numbers, add the fractions first, and then add this sum to the sum of the integers. Thus, $\frac{3}{12} + \frac{10}{12} + \frac{8}{12} = \frac{21}{12} = 1\frac{3}{4}$. Write the $\frac{3}{4}$ below, and add 1 to the sum of 7, 5, 4, which makes 17. Hence, the entire sum is $17\frac{3}{4}$.

OPERATION.

$$\begin{array}{rcr} 4\frac{2}{3} & = & 4\frac{8}{12} \\ 5\frac{5}{6} & = & 5\frac{10}{12} \\ 7\frac{1}{4} & = & 7\frac{3}{12} \\ \hline & & 17\frac{3}{4} \end{array}$$

RULE.—*Reduce the given fractions to similar fractions, add their numerators, and write the sum over the common denominator.*

When there are mixed numbers, or integers, add the fractions and integers separately, and then add the results.

EXERCISES (Oral).

160. Find the sum of the following:

1. $\frac{1}{3}$, $\frac{1}{3}$.
2. $\frac{3}{4}$, $\frac{2}{4}$.
3. $2\frac{1}{2}$, $3\frac{1}{2}$.
4. $8\frac{7}{8}$, $3\frac{3}{8}$.
5. $\frac{1}{3}$, $\frac{1}{6}$.
6. $\frac{2}{5}$, $\frac{1}{10}$.
7. $3\frac{1}{3}$, $4\frac{2}{3}$.
8. $7\frac{2}{3}$, $5\frac{2}{9}$.
9. $\frac{1}{2}$, $\frac{2}{3}$.
10. $\frac{3}{4}$, $\frac{5}{12}$.
11. $2\frac{1}{4}$, $5\frac{1}{2}$.
12. $6\frac{1}{8}$, $2\frac{5}{6}$.
13. $\frac{3}{8}$, $\frac{5}{6}$.
14. $\frac{1}{9}$, $\frac{5}{6}$.
15. $1\frac{3}{4}$, $2\frac{1}{3}$.
16. $9\frac{9}{10}$, $3\frac{2}{5}$.

EXERCISES (Written).

161. Add the following:

1. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.
2. $\frac{1}{2}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{13}{25}$.
3. $18\frac{1}{2}$, $25\frac{2}{3}$, 17.
4. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$.
5. $\frac{5}{12}$, $\frac{7}{15}$, $\frac{9}{20}$, $\frac{11}{30}$.
6. $15\frac{3}{4}$, $9\frac{5}{6}$, $\frac{7}{8}$.
7. $\frac{4}{5}$, $\frac{1}{6}$, $\frac{7}{10}$.
8. $\frac{5}{7}$, $\frac{7}{10}$, $\frac{13}{14}$, $\frac{9}{35}$.
9. $34\frac{3}{10}$, $41\frac{4}{5}$, $10\frac{5}{6}$.
10. $\frac{4}{9}$, $\frac{7}{12}$, $\frac{11}{18}$.
11. $\frac{2}{5}$, $\frac{7}{8}$, $\frac{9}{16}$, $\frac{13}{20}$.
12. $45\frac{3}{4}$, $9\frac{3}{8}$, $17\frac{5}{6}$.
13. $\frac{5}{8}$, $\frac{7}{10}$, $\frac{13}{20}$.
14. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{7}$.
15. 88, $140\frac{9}{15}$, $7\frac{31}{40}$.
16. $\frac{6}{7}$, $\frac{5}{28}$, $\frac{3}{14}$.
17. $\frac{3}{10}$, $\frac{8}{9}$, $\frac{3}{8}$, $\frac{13}{15}$.
18. $8\frac{3}{4}$, $9\frac{3}{10}$, $4\frac{13}{15}$.

Add:

19.
$$\begin{array}{r} 6\frac{3}{4} \\ \underline{8\frac{2}{3}} \end{array}$$

20.
$$\begin{array}{r} 24\frac{1}{5} \\ \underline{17\frac{3}{8}} \end{array}$$

21.
$$\begin{array}{r} 137\frac{1}{2} \\ \underline{205\frac{5}{6}} \end{array}$$

22.
$$\begin{array}{r} 5\frac{3}{8} \\ 4\frac{5}{6} \\ \underline{7\frac{1}{3}} \end{array}$$

23.
$$\begin{array}{r} 41\frac{1}{5} \\ 25\frac{1}{6} \\ \underline{9\frac{7}{10}} \end{array}$$

24.
$$\begin{array}{r} 307\frac{1}{6} \\ 450\frac{8}{9} \\ \underline{127\frac{2}{3}} \end{array}$$

25.
$$\begin{array}{r} 3\frac{4}{7} \\ 2\frac{3}{5} \\ 1\frac{2}{3} \\ \underline{\frac{1}{2}} \end{array}$$

26.
$$\begin{array}{r} 81\frac{4}{5} \\ 6\frac{7}{20} \\ 43\frac{13}{18} \\ \underline{11\frac{7}{9}} \end{array}$$

27.
$$\begin{array}{r} \$51\frac{13}{120} \\ \$43\frac{37}{150} \\ \$27\frac{43}{200} \\ \underline{\$12\frac{61}{300}} \end{array}$$

28. How much is $\frac{1}{2} + 17\frac{4}{5}$? $121\frac{3}{10} + 25\frac{3}{8}$?

29. How much is $9\frac{1}{5} + 5\frac{2}{3}$? $15\frac{1}{2} + 27\frac{5}{8}$?

30. How much is $\$16\frac{3}{4} + \$13\frac{2}{3}$? $\$25\frac{3}{10} + \$17\frac{5}{8}$?

31. If a knife cost $\$1\frac{1}{4}$ and a hat $\$2\frac{5}{6}$, what will both cost?

32. How much is $94\frac{3}{8}$ pecks, $81\frac{3}{4}$ pecks, and $105\frac{5}{12}$ pecks?

33. I bought three farms containing $120\frac{5}{6}$, $110\frac{1}{2}$, and $84\frac{3}{4}$ acres, respectively. How many acres did I buy in all?

34. A farmer received $\$21\frac{3}{4}$ for oats, $\$32\frac{1}{2}$ for corn, $\$50\frac{3}{5}$ for hay, and $\$61\frac{7}{10}$ for rye. What did he receive for all?

35. Henry is $14\frac{2}{3}$ years old, and Malcolm is $3\frac{5}{8}$ years older than Henry. How old is Malcolm?

36. A merchant sold $2\frac{3}{5}$ yards of cloth for $\$10\frac{3}{8}$, $5\frac{1}{2}$ yards for $\$16\frac{3}{4}$, and $7\frac{5}{8}$ yards for $\$22\frac{1}{5}$. How many yards did he sell, and how much money did he receive?

37. A man traveled $43\frac{3}{8}$ miles on Monday, $25\frac{1}{4}$ miles on Tuesday, and $34\frac{4}{5}$ miles on Wednesday. How far did he travel in all?

38. What is the number whose parts are $3\frac{1}{4}$, $2\frac{5}{6}$, and 7?

39. A man paid $\$13\frac{1}{2}$ for a pair of pants, $\$17\frac{3}{4}$ for a coat, and $\$5\frac{2}{3}$ for a vest, What did he pay for all?

40. Three hogs weigh $160\frac{2}{3}$, $145\frac{1}{5}$, $163\frac{1}{2}$ pounds, respectively. What is the total weight?

41. A father gave his three sons \$5, $\$7\frac{1}{2}$, $\$8\frac{1}{3}$, respectively, and had $\$10\frac{1}{6}$ left. What had he at first?

42. A farmer sold three men, respectively, $140\frac{1}{2}$, $120\frac{3}{4}$, $87\frac{1}{5}$ bushels of wheat, and then had $65\frac{1}{3}$ bushels left. How much wheat had he at first?

SUBTRACTION.

162. *To subtract one fraction from another.*

1. What is the difference between 7 yards and 3 yards? Between 7 tenths and 3 tenths? $\frac{7}{12}$ and $\frac{3}{12}$?

2. What is the difference between 8 ninths and 3 ninths? Between 11 twentieths and 3 twentieths? $\frac{13}{17}$ and $\frac{6}{17}$?

163. *Similar fractions are subtracted by subtracting one numerator from the other, and placing the difference over the common denominator.*

3. Are 4 dimes the same as 40 cents?
Are 3 nickels equal to 15 cents?
What, then, is 4 dimes − 3 nickels?

4. $\frac{1}{2}$ = how many sixths? $\frac{1}{3}$ = how many sixths?
What, then, is $\frac{1}{2} - \frac{1}{3}$?

5. Is the difference between $\frac{1}{3}$ and $\frac{1}{4}$ the same as the difference between $\frac{4}{12}$ and $\frac{3}{12}$? Why? What, then, is $\frac{1}{3} - \frac{1}{4}$?

164. *Dissimilar fractions are subtracted by subtracting the similar fractions to which the given fractions are respectively equal.*

1. From $\frac{3}{4}$ subtract $\frac{2}{5}$.

Explanation.—Since the fractions are dissimilar, they must be made similar before subtracting. The L. C. M. of 4 and 5 is 20, and $\frac{3}{4} = \frac{15}{20}$, and $\frac{2}{5} = \frac{8}{20}$. Therefore the difference is $\frac{15-8}{20}$ or $\frac{7}{20}$.

Operation.

$$\begin{aligned}\frac{3}{4} - \frac{2}{5} &= \frac{15}{20} - \frac{8}{20}\\ &= \frac{7}{20}.\end{aligned}$$

2. From $16\frac{2}{3}$ take $7\frac{1}{5}$.

When there are mixed numbers, subtract the fractions first, and add their difference ($\frac{7}{15}$) to the difference (9) of the integers.

Operation.

$$\begin{array}{rcr} 16\frac{2}{3} &=& 16\frac{10}{15}\\ 7\frac{1}{5} &=& \underline{7\frac{3}{15}}\\ &&9\frac{7}{15}\end{array}$$

3. From $11\frac{1}{4}$ take $9\frac{2}{3}$.

As $\frac{8}{12}$ cannot be taken from $\frac{3}{12}$, take 1, or $\frac{12}{12}$, from 11, leaving 10, and add it to $\frac{3}{12}$, making $\frac{15}{12}$. Then $\frac{8}{12}$ from $\frac{15}{12}$ leaves $\frac{7}{12}$, and 9 from 10 leaves 1.

Operation.

$$\begin{array}{rcr} 11\frac{1}{4} &=& 11\frac{3}{12}\\ 9\frac{2}{3} &=& \underline{9\frac{8}{12}}\\ &&1\frac{7}{12}\end{array}$$

Rule.—*Reduce the fractions to similar fractions, find the difference between the numerators, and write it over the common denominator.*

When there are mixed numbers or integers, subtract the fractions and integers separately.

Mixed numbers may be reduced to improper fractions and subtracted according to the first part of the rule.

EXERCISES (Oral).

165. Find the value of:

1. $6 - \frac{3}{5}$.

SOLUTION.—$6 = 5\frac{5}{5}$; $5\frac{5}{5} - \frac{3}{5} = 5\frac{2}{5}$.

2. $7 - 2\frac{3}{8}$. $7 = 6\frac{8}{8}$; $6\frac{8}{8} - 2\frac{3}{8} = 4\frac{5}{8}$.

3. $\frac{5}{6} - \frac{1}{2}$.

4. $\frac{3}{4} - \frac{1}{6}$.

5. $6 - \frac{3}{7}$.

6. $\frac{3}{5} - \frac{1}{3}$.

7. $\frac{4}{5} - \frac{3}{8}$.

8. $8 - 3\frac{5}{6}$.

9. $\frac{7}{12} - \frac{2}{15}$.

10. $\frac{7}{8} - \frac{5}{6}$.

11. $9 - 4\frac{5}{11}$.

12. $2\frac{1}{2} - 1\frac{1}{3}$.

13. $5\frac{1}{3} - 4\frac{1}{6}$.

14. $6\frac{1}{4} - 2\frac{5}{8}$.

EXERCISES (Written).

166. Find the value of:

1. $\frac{9}{10} - \frac{8}{11}$.

2. $\frac{21}{32} - \frac{17}{56}$.

3. $\frac{37}{45} - \frac{23}{60}$.

4. $\frac{8}{21} - \frac{5}{27}$.

5. $\frac{13}{16} - \frac{11}{24}$.

6. $\frac{17}{30} - \frac{7}{24}$.

7. $\frac{19}{35} - \frac{7}{25}$.

8. $\frac{12}{15} - \frac{9}{16}$.

9. $\frac{17}{33} - \frac{11}{30}$.

10. $5\frac{1}{4} - 3\frac{1}{3}$.

11. $6\frac{7}{8} - 4\frac{5}{12}$.

12. $8\frac{1}{4} - 3\frac{4}{7}$.

13. $9 - 2\frac{11}{63}$.

14. $7\frac{1}{9} - 3\frac{5}{12}$.

15. $17\frac{2}{15} - 9\frac{23}{50}$.

16. $16\frac{3}{7} - 5\frac{13}{14}$.

17. $25\frac{4}{9} - 7\frac{5}{8}$.

18. $41\frac{5}{7} - 2\frac{2}{9}$.

19. $84\frac{5}{16} - 73\frac{4}{21}$.

20. $93\frac{1}{2} - 6\frac{21}{41}$.

21. $8\frac{13}{40} - 6\frac{1}{3}$.

Copy and subtract:

22. $\begin{array}{r} 9\frac{1}{2} \\ 4\frac{1}{5} \\ \hline \end{array}$

23. $\begin{array}{r} 3\frac{1}{3} \\ 2\frac{4}{5} \\ \hline \end{array}$

24. $\begin{array}{r} 6\frac{5}{6} \\ 4\frac{7}{8} \\ \hline \end{array}$

25. $\begin{array}{r} 1\frac{2}{5} \\ \frac{6}{7} \\ \hline \end{array}$

26. $\begin{array}{r} 43 \\ 25\frac{7}{12} \\ \hline \end{array}$

27. $\begin{array}{r} 125\frac{2}{9} \\ 48\frac{7}{12} \\ \hline \end{array}$

28. $\begin{array}{r} 408\frac{17}{20} \\ 327\frac{13}{15} \\ \hline \end{array}$

29. $\begin{array}{r} 71 \\ 60\frac{18}{23} \\ \hline \end{array}$

30. $\begin{array}{r} 300\frac{1}{5} \\ 299\frac{8}{11} \\ \hline \end{array}$

31. $\begin{array}{r} 25\frac{121}{150} \\ 17\frac{201}{250} \\ \hline \end{array}$

Find the value of:

32. $\frac{2}{3}+\frac{3}{4}-\frac{1}{2}-\frac{1}{3}$.   *33.* $\frac{4}{5}-\frac{1}{6}-\frac{7}{10}+\frac{2}{3}$.

34. $\frac{3}{4}+\frac{5}{6}-\frac{2}{3}+\frac{1}{2}$.   *35.* $\frac{1}{2}-\frac{1}{3}+\frac{5}{6}-\frac{1}{4}$.

---

36. From a cheese weighing 48 pounds a grocer cut $23\frac{3}{5}$ pounds. How much was left?

37. If $12\frac{5}{16}$ yards are cut from a piece of cloth containing $34\frac{17}{20}$ yards, how many yards will be left?

38. Henry had \$$25\frac{1}{5}$, and bought a coat for \$$16\frac{2}{3}$. How much had he left?

39. James and John weigh, respectively, $64\frac{1}{2}$ and $57\frac{1}{6}$ pounds. How much more does James weigh than John?

40. Two bales of cotton weigh $425\frac{4}{17}$, $417\frac{4}{15}$ pounds. How much heavier is one than the other?

41. From a bin containing $47\frac{1}{2}$ bushels of wheat, $22\frac{2}{3}$ bushels were sold, and then $5\frac{3}{4}$ bushels were sold. How many bushels remained?

42. Two pieces of cloth, containing $42\frac{2}{3}$ and $38\frac{3}{4}$ yards, shrank in dyeing $2\frac{2}{3}$ and $1\frac{5}{6}$ yards, respectively. How much did both pieces then measure?

43. A lady paid \$$12\frac{1}{2}$ for a bonnet, \$$32\frac{3}{4}$ for a dress, and \$$52\frac{5}{8}$ for a cloak. How much change should she receive if she gave in payment a hundred-dollar bill?

MULTIPLICATION AND DIVISION.

167. *To multiply a fraction by an integer.*

1. How much is 3 times $\frac{2}{7}$?

SOLUTION.—$\frac{2}{7}$ taken 3 times is $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{6}{7}$.

Therefore, $3\times\frac{2}{7}=\frac{3\times 2}{7}$, or $\frac{6}{7}$.

2. How much is $5\times\frac{3}{4}$? $8\times\frac{4}{5}$? $10\times\frac{7}{12}$?

168. ***A fraction is multiplied by an integer by multiplying the numerator of the fraction by the integer.***

A common factor in the integer and denominator may be canceled before multiplying, for it is equivalent to dividing both terms by that factor after multiplication.

Thus, by the rule, $8 \times \frac{5}{6} = \frac{8 \times 5}{6} = \frac{40}{6} = \frac{20}{3} = 6\frac{2}{3}$.
By cancellation, $8 \times \frac{5}{6} = 4 \times \frac{5}{3} = \frac{20}{3} = 6\frac{2}{3}$.

Always use cancellation when possible.

EXERCISES (Oral).

169. How much is :

1. $6 \times \frac{5}{7}$? $6 \times \frac{5}{7} = \frac{30}{7} = 4\frac{2}{7}$.
2. $4 \times \frac{7}{8}$? $4 \times \frac{7}{8} = 1 \times \frac{7}{2} = 3\frac{1}{2}$.
3. $12 \times \frac{5}{6}$? $12 \times \frac{5}{6} = 2 \times \frac{5}{1} = 10$.

4. $4 \times \frac{1}{2}$? *5.* $6 \times \frac{1}{12}$? *6.* $8 \times \frac{1}{3}$? *7.* $5 \times \frac{3}{25}$?
8. $7 \times \frac{4}{21}$? *9.* $9 \times \frac{13}{45}$? *10.* $11 \times \frac{7}{22}$? *11.* $3 \times \frac{4}{5}$?
12. $5 \times \frac{3}{4}$? *13.* $8 \times \frac{5}{16}$? *14.* $12 \times \frac{3}{8}$? *15.* $24 \times \frac{2}{3}$?
16. $27 \times \frac{5}{18}$? *17.* $12 \times \frac{5}{9}$? *18.* $36 \times \frac{7}{27}$? *19.* $45 \times \frac{11}{18}$?

20. How much is 8 times $6\frac{2}{5}$?

Solution.—We may reduce $6\frac{2}{5}$ to an improper fraction, and proceed as before, but it is generally better to multiply thus :

$$8 \times 6 = 48,\ 8 \times \tfrac{2}{5} = 3\tfrac{1}{5},\ 48 + 3\tfrac{1}{5} = 51\tfrac{1}{5}.$$

Multiply :

21. $5\frac{1}{2}$ by 8. *22.* $6\frac{2}{3}$ by 9. *23.* $3\frac{3}{4}$ by 6. *24.* $5\frac{5}{8}$ by 12.
25. $3\frac{1}{3}$ by 6. *26.* $7\frac{1}{2}$ by 5. *27.* $5\frac{2}{9}$ by 12. *28.* $3\frac{3}{16}$ by 8.

---

29. Each of 6 boys has \$$\frac{3}{4}$. How much have they all?

30. What will 16 pounds of tea cost at \$$\frac{5}{8}$ a pound?

31. If the length of a man's step is $\frac{5}{6}$ of a yard, how far will he go in making 50 steps?

32. What will 12 yards of prints cost at $6\frac{2}{3}$ cents a yard? At $5\frac{3}{4}$ cents a yard? At $3\frac{3}{8}$ cents? At $4\frac{5}{9}$ cents?

170. ***To divide a fraction by an integer.***

1. How much is $\frac{1}{3}$ of $\frac{2}{7}$, or $\frac{2}{7} \div 3$?

SOLUTION.—$\frac{1}{3}$ of $\frac{1}{7} = \frac{1}{21}$.
Therefore, $\frac{1}{3}$ of $\frac{2}{7} = 2 \times \frac{1}{21} = \frac{2}{21}$.

2. How much is $\frac{1}{5}$ of $\frac{3}{4}$? $\frac{1}{8}$ of $\frac{4}{5}$? $\frac{1}{10}$ of $\frac{7}{12}$?

171. *A fraction is divided by an integer by multiplying the denominator of the fraction by the integer.*

A common factor in the numerator and integer may be canceled before dividing, for it is equivalent to dividing both terms by that factor after the division.

Thus, by the rule, $\frac{6}{7} \div 8 = \frac{6}{7 \times 8} = \frac{6}{56} = \frac{3}{28}$.
By cancellation, $\frac{6}{7} \div 8 = \frac{3}{7} \div 4 = \frac{3}{7 \times 4} = \frac{3}{28}$.

Always use cancellation when possible.

EXERCISES (Oral).

172. How much is :

1. $\frac{1}{3}$ of $\frac{2}{5}$? *2.* $\frac{1}{3}$ of $\frac{3}{4}$? *3.* $\frac{1}{5}$ of $\frac{2}{3}$? *4.* $\frac{1}{6}$ of $\frac{3}{5}$?
5. $\frac{1}{6}$ of $\frac{2}{7}$? *6.* $\frac{1}{5}$ of $\frac{10}{11}$? *7.* $\frac{1}{8}$ of $\frac{4}{7}$? *8.* $\frac{1}{9}$ of $\frac{12}{13}$?
9. $\frac{1}{6}$ of $\frac{4}{5}$? *10.* $\frac{1}{8}$ of $\frac{5}{7}$? *11.* $\frac{1}{12}$ of $\frac{9}{10}$? *12.* $\frac{1}{15}$ of $\frac{10}{17}$?

---

13. $\frac{12}{13} \div 9$? $\frac{12}{13} \div 9 = \frac{4}{13} \div 3 = \frac{4}{39}$.
14. $\frac{3}{7} \div 4$? *15.* $\frac{5}{8} \div 6$? *16.* $\frac{4}{11} \div 7$? *17.* $\frac{2}{5} \div 8$?
18. $\frac{6}{7} \div 8$? *19.* $\frac{9}{11} \div 12$? *20.* $\frac{16}{17} \div 8$? *21.* $\frac{24}{25} \div 36$?

22. How much is $15\frac{3}{4} \div 6$?

SOLUTION.—We may reduce $15\frac{3}{4}$ to an improper fraction, and proceed as before, but it is generally better to divide thus: 6 is contained in $15\frac{3}{4}$ 2 times, with a remainder of $3\frac{3}{4} = \frac{15}{4}$, and $\frac{15}{4} \div 6 = \frac{5}{4} \div 2 = \frac{5}{8}$. Therefore the quotient is $2\frac{5}{8}$.

Find the value of :

23. $6\frac{3}{4} \div 3$. *24.* $16\frac{4}{5} \div 4$. *25.* $18\frac{6}{7} \div 2$. *26.* $20\frac{15}{17} \div 5$.
27. $8\frac{4}{7} \div 5$. *28.* $11\frac{2}{3} \div 7$. *29.* $16\frac{3}{4} \div 9$. *30.* $14\frac{2}{9} \div 8$.

31. If 6 pounds of sugar cost \$$\frac{3}{10}$, what will 1 pound cost ?

32. If 8 pounds of rice cost \$$\frac{2}{9}$, what will 1 pound cost ?

33. If 3 pounds of cheese cost 85$\frac{1}{2}$ cents, what will 1 pound cost ?

34. I bought 6 turkeys for \$7$\frac{1}{2}$. What was the average price of the turkeys ?

35. I paid \$3$\frac{3}{4}$ for 5 bushels of potatoes. What did I pay per bushel ?

36. A boy walked $\frac{9}{10}$ of a mile in 15 minutes. What rate was that per minute ?

173. ***To multiply an integer or fraction by a fraction.***

Multiply 7 by $\frac{2}{3}$.

SOLUTION.—Since $\frac{1}{3}$ is a *third* of 1, the product of 7 by $\frac{1}{3}$ is a *third* of the product of 7 by 1, which is $\frac{7}{3}$. And since $\frac{2}{3}$ is 2 times $\frac{1}{3}$, the product of 7 by $\frac{2}{3}$ is 2 times the product of 7 by $\frac{1}{3}$, which is $\frac{14}{3}$, or 4$\frac{2}{3}$.

$$
\begin{aligned}
7 \times 1 &= 7; \\
7 \times \tfrac{1}{3} &= \tfrac{1}{3} \text{ of } 7 = \tfrac{7}{3}; \\
7 \times \tfrac{2}{3} &= 2 \times \tfrac{7}{3} = 4\tfrac{2}{3}.
\end{aligned}
$$

Multiply $\frac{7}{5}$ by $\frac{2}{3}$.

EXPLANATION.

$$
\begin{aligned}
\tfrac{7}{5} \times 1 &= \tfrac{7}{5}; \\
\tfrac{7}{5} \times \tfrac{1}{3} &= \tfrac{1}{3} \text{ of } \tfrac{7}{5} = \tfrac{7}{15}; \\
\tfrac{7}{5} \times \tfrac{2}{3} &= 2 \times \tfrac{7}{15} = \tfrac{14}{15}.
\end{aligned}
$$

OPERATION.

$$\frac{7}{5} \times \frac{2}{3} = \frac{7 \times 2}{5 \times 3} = \frac{14}{15}$$

Hence, to multiply an integer by a fraction, the rule is the same as in Art. 168 ; and to multiply a fraction by a fraction, *find the product of the numerators for the required numerator, and of the denominators for the required denominator.*

Reduce mixed numbers, if any, to improper fractions.

When possible, use cancellation.

Thus, $\frac{10}{9} \times \frac{12}{25} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

The word *of* following a fraction is equivalent to ×.

Thus, $\frac{2}{3}$ of 9 = $\frac{2}{3} \times 9$; $\frac{3}{5}$ of $\frac{5}{4} = \frac{3}{5} \times \frac{5}{4}$.

Since integers and mixed numbers may be expressed as improper fractions, any two numbers may be multiplied by the last rule.

EXERCISES (Written).

174. Find the value of:

1. $\frac{1}{2}$ of $\frac{4}{5}$.
2. $\frac{2}{3} \times \frac{6}{7}$.
3. $\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{6}{7}$.
4. $\frac{3}{4}$ of $\frac{8}{9}$.
5. $\frac{7}{8} \times \frac{2}{14}$.
6. $\frac{1}{2}$ of $\frac{4}{7}$ of $3\frac{1}{2}$.
7. $\frac{2}{3}$ of $\frac{9}{11}$.
8. $\frac{36}{49} \times \frac{7}{16}$.
9. $\frac{3}{5}$ of $\frac{2}{7}$ of $5\frac{5}{6}$.
10. $\frac{6}{5}$ of $\frac{10}{3}$.
11. $3\frac{1}{8} \times \frac{16}{5}$.
12. $2\frac{1}{2} \times 3\frac{1}{3} \times \frac{3}{10}$.
13. $\frac{5}{6}$ of $\frac{12}{25}$
14. $3\frac{1}{3} \times 4\frac{2}{5}$.
15. $\frac{3}{5} \times 7\frac{1}{2} \times 6\frac{2}{3}$.
16. $\frac{16}{7}$ of $\frac{21}{4}$.
17. $\frac{18}{35} \times 5\frac{1}{4}$.
18. $16\frac{1}{2} \times \frac{2}{3} \times 17\frac{1}{3}$.

What is the cost:

19. Of $\frac{2}{3}$ of a yard of cloth, at \$$\frac{3}{4}$ a yard?

20. Of $\frac{5}{6}$ of a barrel of cider, at \$$2\frac{2}{5}$ a barrel?

21. Of $1\frac{1}{5}$ pounds of coffee, at $12\frac{1}{2}$ cts. a pound?

22. Of $\frac{6}{7}$ of a barrel of flour, at \$$8\frac{3}{4}$ a barrel?

23. Of $8\frac{3}{4}$ yards of silk, at \$$\frac{5}{10}$ per yard?

24. Of $9\frac{1}{8}$ bushels of corn, at $62\frac{1}{2}$ cts. a bushel?

25. Of $11\frac{1}{9}$ cords of wood, at \$$3\frac{1}{2}$ a cord?

26. Of $3\frac{1}{5}$ pounds of butter, at $22\frac{1}{2}$ cts. a pound?

27. What is the worth of 483 pounds of cotton, at $7\frac{3}{4}$ cts. per pound?

OPERATION.

$$\begin{array}{rl} 483 & \\ 7\frac{3}{4} & \\ \hline 4)1449 & \text{Prod. by 3.} \\ \hline 362\frac{1}{4} & \text{Prod. by } \frac{3}{4}. \\ 3381 & \text{Prod. by 7.} \\ \hline 3743\frac{1}{4} & \text{Prod. by } 7\frac{3}{4}. \end{array}$$

What is the cost of:

28. 460 hats, at \$$4\frac{3}{4}$ each?

29. 183 sheep, at \$$3\frac{2}{3}$ each?

Multiply:

30. 24 by $15\frac{3}{8}$.
31. 36 by $23\frac{7}{10}$.
32. 45 by $18\frac{7}{12}$.
33. 42 by $16\frac{5}{6}$.
34. 86 by $40\frac{1}{4}$.
35. 74 by $19\frac{2}{3}$.
36. 53 by $20\frac{3}{5}$.
37. 94 by $25\frac{1}{2}$.
38. 82 by $12\frac{4}{9}$.

What is the cost of:

39. 76 cows, at \$$25\frac{3}{4}$ each?

40. 134 yards of cloth, at $8\frac{2}{3}$ cts. a yard?

41. 82 barrels of flour, at \$$6\frac{3}{5}$ a barrel?

What is the cost of :

42. 280 pounds of coffee, at $12\frac{3}{4}$ cts. a pound ?

43. 12 hams, each weighing $13\frac{2}{3}$ pounds, at $11\frac{3}{4}$ cts. a pound ?

44. A, B, and C bought a lot for \$575, of which A paid $\frac{7}{12}$ and B $\frac{3}{10}$. What did each pay ?

45. How many dollars did a man spend who had \$3644, and spent $\frac{5}{8}$ of it ?

46. A boy had $\$42\frac{3}{4}$, but gave $\frac{4}{5}$ of it for a gun. What did the gun cost ?

47. A merchant paid $\$51\frac{1}{3}$ for a buggy, and sold it for $\frac{4}{7}$ of what it cost him. How much did he lose ?

48. What will be the cost of $\frac{5}{9}$ of a piece of cassimere containing $23\frac{1}{4}$ yards, at $\$3\frac{3}{5}$ per yard ?

49. How much money will it take to buy 16 tables, at $\$8\frac{5}{6}$ apiece ?

50. A is $46\frac{2}{5}$ years old. B is $\frac{3}{4}$ as old as A, and C is $\frac{3}{7}$ as old as B. How old are B and C ?

51. A wheel in making one revolution goes $16\frac{4}{5}$ feet. How far will it go in making $41\frac{1}{3}$ revolutions ?

52. If the rent of a house is $\$42\frac{3}{4}$ a month, what will be the rent for $3\frac{1}{4}$ years ?

53. How far will a horse travel in $4\frac{4}{5}$ hours at the rate of $8\frac{1}{3}$ miles per hour ?

54. A man owned $\frac{7}{12}$ of a factory, and sold $\frac{8}{11}$ of his share. What part of the factory did he still own ?

55. If a water-pipe discharges $25\frac{2}{3}$ barrels of water in an hour, how many barrels will it discharge in $12\frac{2}{7}$ hours ?

56. One man bought $642\frac{1}{2}$ acres of land, at $\$7\frac{2}{5}$ an acre ; and another $584\frac{3}{5}$ acres, at $\$8\frac{4}{7}$. How much more did one pay than the other ?

57. A speculator bought 840 acres of land for \$9318. He sold $\frac{3}{5}$ of the land at the rate of $\$12\frac{3}{4}$ an acre, and the remainder at the rate of $\$13\frac{2}{3}$ an acre. What did he gain by the transaction ?

175. ***To divide an integer or fraction by a fraction.***

Divide 7 by $\frac{2}{3}$.

EXPLANATION.

Since 1 contains $\frac{1}{3}$ three times,
1 " $\frac{2}{3}$, $\frac{1}{2}$ of 3 times, or $\frac{3}{2}$ times.
Hence 7 " $\frac{2}{3}$, 7 × $\frac{3}{2}$ times, or $10\frac{1}{2}$ times.

OPERATION.

$$7 \div \frac{2}{3} = 7 \times \frac{3}{2} = \frac{21}{2} = 10\frac{1}{2}.$$

Divide $\frac{7}{8}$ by $\frac{3}{5}$.

EXPLANATION.

Since 1 contains $\frac{1}{5}$ five times,
1 " $\frac{3}{5}$, $\frac{1}{3}$ of five times, or $\frac{5}{3}$ times.
Hence $\frac{7}{8}$ " $\frac{3}{5}$, $\frac{7}{8}$ of $\frac{5}{3}$ times, or $1\frac{11}{24}$ times.

OPERATION.

$$\frac{7}{8} \div \frac{3}{5} = \frac{7}{8} \times \frac{5}{3} = \frac{35}{24} = 1\frac{11}{24}.$$

Hence, to divide an integer or fraction by a fraction, *multiply the dividend by the reciprocal of the divisor.*

The preceding form of analysis shows at once why we invert the divisor, namely: *to find how many times it is contained in* 1.

Thus, to divide $\frac{5}{9}$ by $\frac{2}{7}$ we say, 1 contains $\frac{1}{7}$, 7 times; hence 1 contains $\frac{2}{7}$, $\frac{7}{2}$ times, and therefore $\frac{5}{9}$ contains $\frac{2}{7}$, $\frac{5}{9}$ of $\frac{7}{2}$ times.

After inverting the divisor, use cancellation when possible.

Integers and mixed numbers must be expressed as improper fractions.

176. It is often most convenient to divide by simply reversing the rule for multiplication.

Thus, since $\frac{3}{7} \times \frac{4}{5} = \frac{3 \times 4}{7 \times 5} = \frac{12}{35}$,
therefore $\frac{12}{35} \div \frac{4}{5} = \frac{12 \div 4}{35 \div 5} = \frac{3}{7}$.

That is, divide the terms of the dividend by the corresponding terms of the divisor, and the quotients will be, respectively, the terms of the quotient.

Give the values of the following instantly:

$\frac{9}{10} \div \frac{3}{2}$
$\frac{6}{12} \div \frac{3}{4}$
$\frac{35}{45} \div \frac{7}{5}$

$\frac{12}{15} \div \frac{3}{5}$
$\frac{10}{18} \div \frac{2}{3}$
$\frac{10}{40} \div \frac{2}{4}$

$\frac{8}{25} \div \frac{4}{5}$
$\frac{21}{28} \div \frac{3}{7}$
$\frac{9}{80} \div \frac{3}{8}$

$\frac{64}{81} \div \frac{8}{9}$
$\frac{45}{60} \div \frac{9}{5}$
$\frac{34}{90} \div \frac{2}{5}$

When the terms of the dividend are not divisible by those of the divisor, we may multiply the former by such an integer as will make them divisible.

But in cases of this kind it is more convenient to invert the divisor, and multiply.

EXERCISES (Written).

177. Find the value of:

1. $\frac{4}{7} \div \frac{4}{5}$.	*2.* $\frac{4}{21} \div \frac{2}{3}$.	*3.* $\frac{9}{16} \div \frac{3}{8}$.
4. $\frac{2}{3} \div \frac{3}{10}$.	*5.* $3\frac{1}{5} \div \frac{4}{7}$.	*6.* $6\frac{5}{6} \div \frac{21}{2}$.
7. $\frac{4}{5} \div \frac{8}{11}$.	*8.* $9 \div 4\frac{1}{5}$.	*9.* $18 \div 7\frac{1}{3}$.
10. $\frac{17}{33} \div \frac{5}{6}$.	*11.* $3\frac{3}{4} \div 1\frac{2}{3}$.	*12.* $8\frac{3}{4} \div 6\frac{2}{3}$.
13. $\frac{18}{49} \div \frac{3}{7}$.	*14.* $8\frac{4}{9} \div \frac{4}{5}$.	*15.* $16\frac{1}{2} \div 9\frac{5}{8}$.
16. $\frac{15}{16} \div \frac{5}{9}$.	*17.* $11 \div 3\frac{1}{3}$.	*18.* $12\frac{1}{11} \div 7\frac{4}{13}$.
19. $\frac{21}{52} \div \frac{7}{13}$.	*20.* $6\frac{3}{7} \div \frac{9}{5}$.	*21.* $16\frac{1}{5} \div 5\frac{1}{7}$.
22. $\frac{20}{27} \div \frac{14}{9}$.	*23.* $12 \div 5\frac{5}{11}$.	*24.* $25\frac{1}{2} \div 18\frac{7}{9}$.
25. $\frac{17}{19} \div \frac{7}{9}$.	*26.* $28\frac{1}{5} \div 6$.	*27.* $121\frac{1}{3} \div 8$.

Divide:

28. $\frac{3}{4}$ of $\frac{6}{5}$ by $\frac{10}{9} \times \frac{3}{5}$.	*29.* $\frac{4}{9}$ of $\frac{3}{5}$ of $\frac{7}{8}$ by $\frac{14}{15}$.
30. $\frac{3}{8}$ of $\frac{6}{5}$ by $\frac{3}{8}$ of $\frac{1}{4}$.	*31.* $\frac{15}{28} \div \frac{5}{7}$ by $\frac{3}{4}$ of $\frac{5}{2}$.
32. $\frac{19}{22} \div \frac{8}{5}$ by $\frac{17}{21} \div \frac{4}{3}$.	*33.* $\frac{20}{21}$ of $\frac{3}{7}$ by $2\frac{1}{2} \div \frac{5}{4}$.
34. $7\frac{4}{13} \div \frac{81}{65}$ by $5\frac{5}{7} \div \frac{7}{3}$.	*35.* $5\frac{1}{6}$ of $\frac{19}{27} \div \frac{95}{68}$ by $18\frac{3}{4}$.

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36. Divide $11\frac{11}{12}$ of $2\frac{9}{13}$ by $\frac{11}{4}$ of $6\frac{2}{9} \times \frac{1}{3}$.

Change the mixed numbers to improper fractions, invert all factors of the divisor, and use cancellation. Thus:

$$\left(\frac{143}{12} \times \frac{35}{13}\right) \div \left(\frac{11}{4} \times \frac{56}{9} \times \frac{1}{3}\right) = \frac{143}{12} \times \frac{35}{13} \times \frac{4}{11} \times \frac{9}{56} \times \frac{3}{1} = 5\frac{5}{8}.$$

Divide:

37. $3\frac{1}{5}$ of $7\frac{1}{2}$ by $1\frac{1}{3}$.	*38.* $3\frac{3}{4}$ of $2\frac{3}{16}$ by $5\frac{5}{8}$ of $8\frac{8}{9}$.
39. $4\frac{1}{2}$ of $1\frac{1}{9}$ by $\frac{2}{3}$ of $\frac{5}{7}$.	*40.* $3\frac{1}{2} \times \frac{4}{5}$ by $2\frac{1}{10} \times \frac{4}{13}$.
41. $\frac{2}{3}$ of $\frac{7}{11}$ of $\frac{3}{5}$ by $\frac{28}{15}$.	*42.* $7\frac{1}{5} \times 5\frac{5}{6} \times 3\frac{3}{7}$ by $1\frac{1}{8}$ of $2\frac{2}{3}$.

43. Find the value of $\dfrac{17\frac{1}{2}}{\frac{5}{6}}$.

SOLUTION.

$$\frac{17\frac{1}{2}}{\frac{5}{6}} = 17\tfrac{1}{2} \div \tfrac{5}{6} = \frac{\overset{7}{\cancel{35}}}{\cancel{2}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{5}} = 21.$$

EXPLANATION.—The expression means $17\frac{1}{2}$ divided by $\frac{5}{6}$, and is solved like other examples in division of fractions. Expressions of unexecuted division of fractions, when written in the form of a fraction, are often called *complex fractions.*

Reduce to simple fractions:

44. $\dfrac{2\frac{1}{2}}{\frac{3}{4}}$. *45.* $\dfrac{16\frac{2}{3}}{\frac{2}{3} \text{ of } \frac{5}{6}}$. *46.* $\dfrac{\frac{4}{3} - \frac{3}{4}}{\frac{6}{5} + \frac{5}{6}}$. *47.* $\dfrac{6\frac{2}{3} - \frac{5}{6}}{2\frac{1}{2} + \frac{3}{4}}$.

48. $\dfrac{7\frac{1}{5}}{9}$. *49.* $\dfrac{\frac{1}{2} + \frac{1}{3}}{2\frac{1}{2}}$. *50.* $\dfrac{\frac{2}{3} - \frac{2}{5}}{\frac{2}{3} \text{ of } \frac{2}{5}}$. *51.* $\dfrac{5\frac{1}{4} \times 5\frac{1}{7}}{3\frac{1}{6} + 2\frac{3}{4}}$.

52. Simplify $\frac{2}{3} \div \frac{5}{6} \times \frac{6}{35} \div \frac{2}{7} \div \frac{3}{4}$.

Invert all factors preceded by the sign ÷, and use cancellation. Thus:

$$\tfrac{2}{3} \div \tfrac{5}{6} \times \tfrac{6}{35} \div \tfrac{2}{7} \div \tfrac{3}{4} = \tfrac{2}{3} \times \tfrac{6}{5} \times \tfrac{6}{35} \times \tfrac{7}{2} \times \tfrac{4}{3} = \tfrac{16}{25}.$$

Simplify:

53. $\frac{3}{4} \times \frac{5}{6} \div \frac{10}{9}$. *54.* $\frac{3}{5} \times 2\frac{1}{2} \div \frac{12}{19} \div 6\frac{1}{3}$.

55. $\frac{2}{3} \div \frac{3}{4} \times \frac{4}{5}$. *56.* $2\frac{2}{3} \times 1\frac{1}{4} \div 5\frac{1}{3} \div 3\frac{3}{4}$.

57. $\frac{15}{16} \div \frac{7}{12} \div \frac{5}{7}$. *58.* $\frac{8}{15} \div \frac{2}{3} \div \frac{9}{10} \div \frac{20}{27}$.

59. $1\frac{1}{8} \div \frac{1}{2} \div 2\frac{1}{4}$. *60.* $\frac{14}{5} \div \frac{5}{8} \times \frac{10}{19} \div \frac{7}{3}$.

61. Divide $185\frac{1}{3}$ by 8. See solution of Ex. 22, page 91.

Divide:

62. $26\frac{2}{5}$ by 6. *63.* $41\frac{1}{4}$ by 5. *64.* $53\frac{1}{2}$ by 7.

65. $37\frac{1}{3}$ by 12. *66.* $45\frac{3}{5}$ by 9. *67.* $67\frac{1}{5}$ by 8.

68. $169\frac{1}{2}$ by 5. *69.* $236\frac{1}{8}$ by 2. *70.* $437\frac{1}{3}$ by 4.

71. If $\$98\frac{2}{3}$ were divided equally among 4 boys, how much would each receive?

72. If $932\frac{4}{7}$ bushels of wheat were put equally into 6 bins, how many bushels would each bin contain?

73. If 11 barrels of sugar are worth $\$489\frac{1}{13}$, what is one barrel worth?

74. A farmer cut 8 lines of equal length from a rope $101\frac{1}{3}$ feet long. How long was each line?

75. A railroad train was nineteen hours in going $486\frac{2}{5}$ miles. At what average rate per hour did it run?

76. If one barrel of flour cost $5, how many barrels of flour can be bought with $\$434\frac{2}{7}$?

77. Divide 140 by $21\frac{1}{4}$.

EXPLANATION.

$21\frac{1}{4} = \frac{85}{4} = 85$ fourths.
$140 = \frac{560}{4} = 560$ fourths.
560 fourths ÷ 85 fourths
$= 560 \div 85 = 6\frac{10}{17}$.

OPERATION.

$$\begin{array}{c|l} 21\frac{1}{4} & 140 \\ 4 & \ \ 4 \\ \hline 85 & 560(6\frac{10}{17} \\ & 510 \\ \cline{2-2} & \ \frac{50}{85} = \frac{10}{17} \end{array}$$

Divide:

78. 135 by $29\frac{2}{5}$.
79. 125 by $18\frac{3}{4}$.
80. 204 by $12\frac{4}{7}$.
81. 321 by $25\frac{1}{2}$.
82. 240 by $11\frac{1}{9}$.
83. 360 by $17\frac{1}{7}$.

84. Divide $59\frac{1}{2}$ by $30\frac{1}{3}$:

Here we multiply both terms by 6, the L. C. M. of the two denominators 3 and 2, and divide one product by the other. The operation is equivalent to reducing the dividend and divisor to sixths, and dividing the numerator of the first by that of the second.

$$\begin{array}{c|c} 30\frac{1}{3} & 59\frac{1}{2} \\ 6 & 6 \\ \hline 182 & 357(1\frac{25}{26} \end{array}$$

Divide:

85. $42\frac{1}{2}$ by $13\frac{3}{4}$.
86. $57\frac{1}{2}$ by $10\frac{5}{6}$.
87. $65\frac{1}{4}$ by $14\frac{5}{8}$.
88. $46\frac{1}{2}$ by $10\frac{1}{5}$.
89. $71\frac{3}{4}$ by $18\frac{1}{5}$.
90. $143\frac{1}{2}$ by $9\frac{1}{10}$.

91. How many bushels of clover seed can be bought for $\$23\frac{1}{3}$, if each bushel cost $\$4\frac{2}{3}$?

HINT.—As many bushels can be bought as $4\frac{2}{3}$ is contained times in $23\frac{1}{3}$.

92. If $9\frac{1}{3}$ bushels of corn weigh $528\frac{1}{2}$ pounds, what is the average weight of a bushel ?

93. If it takes $1\frac{7}{8}$ bushels of wheat to sow one acre, how many acres can be sown with $156\frac{1}{4}$ bushels ?

What will :

94. 1 yard of cloth cost, if $3\frac{1}{2}$ yards cost \$$7\frac{3}{5}$?

95. 1 acre of land cost, if $7\frac{1}{3}$ acres cost \$$30\frac{1}{4}$?

96. 1 bushel of corn cost, if $2\frac{3}{4}$ bushels cost \$$2\frac{5}{7}$?

97. How many pounds of tea can be bought for \$$40\frac{1}{3}$, at \$$\frac{3}{8}$ a pound ?

98. How many tons of hay can be bought for \$$165\frac{1}{3}$, at \$$12\frac{3}{4}$ a ton ?

99. A mechanic, whose wages are \$$3\frac{1}{3}$ a day, receives \$$16\frac{1}{4}$ for his services. How many days does he work ?

100. A farmer raised $232\frac{1}{2}$ bushels of wheat on $10\frac{5}{7}$ acres. What was the average yield per acre ?

101. How many times will a wheel, $11\frac{1}{4}$ feet in circumference, revolve in rolling $58\frac{1}{2}$ feet ?

102. In how many minutes will a locomotive run $22\frac{1}{2}$ miles, at the rate of $\frac{9}{14}$ of a mile per minute ?

103. If a piano is worth \$$308\frac{1}{40}$, in how many months can a lady pay for it by making monthly payments of \$$22\frac{1}{5}$?

104. What is the price of hay, when $8\frac{1}{4}$ tons are worth \$143 ?

105. How many pounds of cotton, worth $8\frac{1}{3}$ cents a pound, must be given in exchange for $32\frac{1}{7}$ pounds of bacon, worth $12\frac{4}{9}$ cents a pound ?

106. A man rode 44 miles. After riding $4\frac{2}{3}$ hours at the rate of $5\frac{1}{7}$ miles an hour, he rode the rest of the way at the rate of $6\frac{1}{4}$ miles an hour. In how many hours did he complete the entire journey ?

107. The circumference of the fore and hind wheels of a wagon are $13\frac{3}{4}$ and $19\frac{1}{5}$ feet. How many more turns will be made by the former than by the latter in going a distance of 5 miles, of 5280 feet each ?

FRACTIONAL RELATION OF NUMBERS.

178. ***To find what part one number is of another.***

1. What part is 3 of 12 ?

1 is $\frac{1}{12}$ of 12 ; hence 3 is 3 times $\frac{1}{12}$, or $\frac{1}{4}$ of 12.

OPERATION.

$$3 \div 12 = \frac{3}{12} = \frac{1}{4}.$$

2. What part of 15 is $4\frac{1}{2}$?

1 is $\frac{1}{15}$ of 15 ; hence $4\frac{1}{2}$ is $4\frac{1}{2}$ times $\frac{1}{15}$ of 15, or $\frac{9}{30}$ of 15, or $\frac{3}{10}$ of 15.

OPERATION.

$$4\tfrac{1}{2} \div 15 = \frac{9}{2} \times \frac{1}{15} = \frac{3}{10}.$$

3. What part of 3 is 2 ? *4.* What part of 9 is 3 ?
5. What part of 12 is 8 ? *6.* What part of 48 is 36 ?
7. What part of 2 is $\frac{1}{2}$? *8.* What part of $\frac{3}{4}$ is $\frac{3}{5}$?
9. What part of 10 is $3\frac{1}{3}$? *10.* What part of 9 is $2\frac{1}{4}$?

11. A bale of cotton is worth \$36. What part of a bale can be bought for \$12 ? \$8 ? \$20 ? \$24 ?

12. A man sold a watch which cost \$30 for \$36. What part of the cost was the gain ?

RULE.—*Divide the part by the number of which it is a part.*

EXERCISES (Written).

179. What part of :

1. $\frac{3}{4}$ is $\frac{2}{3}$? *2.* $2\frac{7}{8}$ is $1\frac{1}{2}$? *3.* $3\frac{1}{3}$ is $\frac{7}{12}$? *4.* $8\frac{3}{4}$ is $1\frac{1}{4}$?
5. 51 is $3\frac{2}{5}$? *6.* $8\frac{1}{5}$ is $1\frac{3}{7}$? *7.* $8\frac{4}{7}$ is 6 ? *8.* $6\frac{2}{3}$ is 5 ?
9. $2\frac{1}{2}$ is $3\frac{1}{3}$? *10.* 22 is $2\frac{3}{4}$? *11.* 21 is $\frac{3}{7}$? *12.* $\frac{5}{7}$ is $\frac{3}{4}$?
13. $3\frac{6}{7}$ is $2\frac{1}{4}$? *14.* $3\frac{1}{4}$ is $5\frac{7}{9}$? *15.* $6\frac{2}{5}$ is $2\frac{2}{3}$? *16.* $17\frac{1}{2}$ is $1\frac{2}{5}$?

17. If a barrel of flour is worth \$$7\frac{1}{2}$, what part of a barrel can be bought for \$$1\frac{2}{3}$? \$$2\frac{1}{4}$? \$$\frac{5}{8}$? \$$1\frac{3}{7}$?

18. A man had \$$56\frac{1}{4}$ and spent \$$9\frac{3}{8}$. What part of his money did he spend ?

180. ***A number and the part which it is of another number given, to find that other number.***

1. $\frac{3}{8}$ of a certain number is 45. What is the number?

Since $\frac{3}{8}$ of the number is 45, $\frac{1}{8}$ of the number is $\frac{1}{3}$ of 45, or 15; and since $\frac{1}{8}$ of the number is 15, the number must be 8 times 15, or 120. Hence 45 is $\frac{3}{8}$ of 120.

Find the number of which:

2. 20 is $\frac{4}{5}$. *3.* 16 is $\frac{4}{7}$. *4.* 12 is $\frac{2}{3}$. *5.* 30 is $\frac{3}{10}$.
6. 36 is $\frac{9}{11}$. *7.* 63 is $\frac{7}{8}$. *8.* 70 is $\frac{5}{8}$. *9.* 20 is $\frac{10}{3}$.
10. 40 is $\frac{8}{5}$. *11.* 14 is $\frac{4}{5}$. *12.* 50 is $\frac{8}{3}$. *13.* 30 is $\frac{9}{5}$.

14. James has 40 cts., which is $\frac{5}{9}$ of what John has. How much has John?

15. A man sold a gun for $27, which was $\frac{9}{7}$ of the cost. How much did the gun cost?

EXERCISES (Written).

181. *1.* Find the number of which $7\frac{1}{5}$ is $\frac{9}{10}$.

The number is 10 times $\frac{1}{9}$ of $7\frac{1}{5}$, or $\frac{10}{9} \times 7\frac{1}{5}$, or $7\frac{1}{5} \div \frac{9}{10}$.

OPERATION.

$$7\frac{1}{5} \div \frac{9}{10} = \frac{36}{5} \times \frac{10}{9} = 8.$$

RULE.—*Divide the number by the fractional part.*

Find the number of which:

2. $16\frac{1}{2}$ is $\frac{11}{13}$. *3.* $5\frac{1}{4}$ is $\frac{7}{8}$. *4.* 180 is $\frac{5}{6}$. *5.* 324 is $\frac{8}{9}$.
6. $15\frac{1}{3}$ is $\frac{2}{7}$. *7.* $8\frac{1}{3}$ is $\frac{15}{17}$. *8.* 497 is $\frac{7}{12}$. *9.* 609 is $\frac{21}{25}$.

10. A man lost $124, which was $\frac{8}{25}$ of what he had. How much had he?

11. One bale of cotton weighs 520 pounds, which is $\frac{13}{17}$ of what another bale weighs. What does the other weigh?

12. A trader bought a horse for $132, which was $\frac{8}{9}$ of what he received from him. How much did he gain?

13. A lady bought a dress for $32.25, and found that she had spent $\frac{5}{8}$ of her money. How much money had she?

14. A man sold a house for $24000, which was $\frac{15}{11}$ of its cost. How much did the house cost?

15. A farmer raised 960 bushels of wheat, which was $\frac{16}{31}$ of the amount of oats he raised. How much oats did he raise?

16. In buying a farm for $8370, a man spent $\frac{5}{6}$ of his fortune. What was his fortune?

17. An oak is $85\frac{3}{4}$ feet high, which is $\frac{7}{9}$ of the height of a pine. How high is the pine?

18. If cotton to-day is worth $8\frac{5}{100}$ cents a pound, and is $\frac{7}{8}$ of what it was yesterday, what was it worth yesterday?

Problems of Pursuit.

EXERCISES (Written).

182. *1.* A hare has 120 feet the start of a hound. In how many seconds will the hound overtake the hare if the hound runs 30 feet, and the hare 24 feet, per second?

Solution.—To overtake the hare, the hound must gain 120 feet on the hare. In 1 second the hound gains 30 − 24 feet, or 6 feet; therefore to gain 120 feet will require as many seconds as 6 feet is contained times in 120 feet, or 20 seconds.

2. At what time between 3 and 4 o'clock are the hands of a clock together?

Solution.—At 3 o'clock the hour hand is 15 minute spaces ahead of the minute hand; that is, the latter will have to gain 15 minute spaces on the former before they will be together.

Since the minute hand moves 60 spaces while the hour hand moves 5 spaces, the minute hand gains 55 spaces in 60 minutes, or $\frac{55}{60}$, or $\frac{11}{12}$ of a space in 1 minute. Hence, to gain 15 spaces it will take as many minutes as $\frac{11}{12}$ is contained times in 15, or $16\frac{4}{11}$ minutes. That is, the hands will be together at $16\frac{4}{11}$ minutes after 3 o'clock.

3. At what time between 3 and 4 o'clock are the hands of a clock opposite to each other?

Solution.—In this case the minute hand has first to gain 15 minute spaces to overtake the hour hand; and then it has to gain 30 minute spaces to be opposite to the hour hand, for the hands are just opposite each other when they are 30 minute spaces apart. Hence the minute hand has to gain in all 45 spaces, and the number of minutes is therefore $45 \div \frac{11}{12}$, or $49\frac{1}{11}$.

Hence the time is 3:$49\frac{1}{11}$, or $49\frac{1}{11}$ min. after 3 o'clock.

4. At what time between 2 and 3 o'clock will the hands of a watch be together? At what time will they be opposite to each other?

5. At what times between 4 and 5 o'clock are the hands of a watch perpendicular to each other?

The hands are perpendicular to each other when they are 15 minute spaces apart. Hence, in the first instance, the minute hand has to gain 20 − 15, or 5 spaces; and in the second, 20 + 15, or 35 spaces.

6. At what times between 5 and 6 o'clock are the hands of a clock perpendicular to each other?

7. A fox has 100 yards the start of a hound. If the hound runs $12\frac{1}{2}$ yards while the fox runs 10, how far will the hound run before he overtakes the fox?

Solution.—He has to gain 100 yards. He gains $2\frac{1}{2}$ yards in every $12\frac{1}{2}$ yards, or 1 yard in every 5 yards; therefore, to gain 100 yards, he must go 100 × 5 yards, or 500 yards.

8. A sets out and travels at the rate of 18 miles in 5 hours. Twelve hours afterwards B sets out from the same place and travels in the same direction, at the rate of 15 miles in 4 hours. In how many hours will B overtake A?

9. A courier who goes at the rate of $31\frac{1}{2}$ mi. in 5 hr., is followed, after 8 hours, by another who goes at the rate of $22\frac{1}{2}$ mi. in 3 hr. In how many hours will the second overtake the first?

10. At what time are the hands of a clock together between 1 and 2 o'clock? Between 10 and 11 o'clock?

11. At what time are the hands of a watch opposite to each other between 1 and 2 o'clock? Between 4 and 5 o'clock? Between 8 and 9 o'clock?

12. A and B start from the same point, A traveling at the rate of 44 mi. in 6 hr., and B at the rate of 34 mi. in 5 hr. How far apart will they be in $3\frac{3}{4}$ hr. (1) if they go in the same direction? (2) If they go in opposite directions?

13. At what time are the hands of a clock perpendicular to each other between 1 and 2 o'clock? Between 7 and 8 o'clock? Between 11 and 12 o'clock?

14. At what times between 4 and 5 o'clock are the hands of a clock 12 minute spaces apart?

Problems of Time and Work.

EXERCISES (Written).

183. *1.* A can do a piece of work in 5 days, and B can do it in 4 days. In how many days can they do the work together?

SOLUTION.—In 1 day A can do $\frac{1}{5}$ of the work, and B in 1 day can do $\frac{1}{4}$ of it. Therefore both together in 1 day can do $\frac{1}{5} + \frac{1}{4} = \frac{9}{20}$ of it. That is, if the work were divided into 20 equal parts they could do 9 of the parts in 1 day; hence it will take them as many days to do the entire work as 9 is contained times in 20, or $2\frac{2}{9}$ days.

2. A can do a piece of work in 10 days, B in 24 days, and C in 40 days. In how many days can they do the work together?

3. A can do a piece of work in 15 days, and A and B together can do it in 10 days. How many days will it take B alone to do the work?

4. A, B, and C can do a piece of work in 20 days; A and B in 40 days; and A and C in 30 days. In how many days can each alone do it?

5. One pipe can fill a reservoir in 10 days, and another can fill it in 15 days, and these two together with the help of a third can fill it in 5 days. How many days would it take the third alone to fill it?

6. A man went from A to B, and from B back to A in 15 hours. His rate in going from A to B was 6 mi. an hour, and in returning from B to A 4 mi. an hour. How far was it from A to B?

Solution.—If the distance from A to B were 1 mile, it would have required $\frac{1}{6}$ of an hour to go and $\frac{1}{4}$ of an hour to return, or $\frac{1}{6} + \frac{1}{4} = \frac{5}{12}$ of an hour to go and return. Therefore the distance from A to B is as many miles as $\frac{5}{12}$ is contained times in 15, or 36 miles.

7. A crew can row down stream 8 mi. an hour, and up stream 6 mi. an hour. How far down stream can they row and return in 7 hours?

8. A can ride on a bicycle 12 mi. an hour, and return on the cars at the rate of 30 miles an hour. How far should he go to make the round trip in $5\frac{1}{4}$ hours?

9. Two men, A and B, start from different places to meet each other. In how many hours will they meet if A can travel the entire distance from A to B in 10 hours, and B can travel it in 12 hours?

10. A tank can be filled by three pipes in $1\frac{1}{3}$ hr., $3\frac{1}{3}$ hr., and 5 hr. respectively. In what time will the tank be filled when all three pipes are running together?

11. A could have done a certain work in 12 days, but after working on it 3 days he called in B, and with his help he finished it in 8 days. In how many days could B alone do the work?

12. A tank can be filled by two pipes in 24 min. and 30 min. respectively, and emptied by a third in 20 min. In what time will it be filled if all three are running?

13. A does $\frac{6}{11}$ of a piece of work in 12 days, when B comes to help him, and they finish the work in 6 days more. How long would it have taken B alone to do the whole work?

14. A and B together can do a piece of work in 12 days, A and C in 15 days, and B and C in 20 days. In what time can they do it all working together, and in what time can each do it alone?

Parallel Problems.

184. *1.* What is the sum of $5\frac{1}{3}$ and $3\frac{1}{2}$?

2. What is the sum of $17\frac{2}{3}$ and $8\frac{3}{8}$?

3. What is the difference between $5\frac{1}{4}$ and $2\frac{1}{3}$?

4. What is the difference betweeen $18\frac{3}{8}$ and $6\frac{2}{11}$?

5. A father gave one son \$$5\frac{1}{2}$, another \$$6\frac{1}{3}$, another \$$7\frac{1}{6}$. How much did he give them all?

6. Four bales of cotton weigh $450\frac{1}{2}$, $439\frac{3}{8}$, $447\frac{2}{3}$, and $457\frac{1}{4}$ pounds. What is the total weight?

7. The parts of a number are $3\frac{1}{2}$, $6\frac{1}{6}$, and $5\frac{2}{3}$. What is the number?

8. James has \$$24\frac{3}{4}$, John \$$22\frac{5}{6}$ more than James, and Henry \$$17\frac{1}{5}$ more than John. How much has Henry?

9. How much is $3\frac{1}{3}$ times $1\frac{1}{5}$?

10. Find the product of $7\frac{1}{2}$ by $4\frac{3}{5}$.

11. How much is $\frac{5}{6}$ divided by $\frac{2}{3}$?

12. Find the quotient of $17\frac{1}{2}$ by $1\frac{3}{4}$.

13. How far will a boy go in $2\frac{1}{2}$ hours, if he goes $3\frac{1}{5}$ miles per hour?

14. How far will a horse travel in $6\frac{1}{3}$ days at the rate of $32\frac{1}{4}$ miles per day?

15. James steps $2\frac{1}{2}$ feet each step. How many steps does he make in walking $12\frac{1}{2}$ feet?

16. The distance around a wheel is $8\frac{1}{3}$ feet. How many times will the wheel revolve in rolling $69\frac{4}{9}$ feet?

17. Mary had \$11, and gave Emma \$$2\frac{1}{2}$ and Susan \$$5\frac{1}{3}$. How much had she left?

18. Two of the parts of $15\frac{1}{2}$ are $6\frac{2}{3}$ and $5\frac{1}{7}$. What is the third part?

19. What is the sum of $(5 - 2\frac{2}{3})$ and $(6 - 1\frac{1}{2})$?

20. A man sold a horse which cost \$$123\frac{1}{5}$ at a loss of \$$10\frac{3}{4}$, and a cow which cost \$$37\frac{2}{3}$ at a loss of \$$3\frac{5}{8}$. What did he receive for both?

21. What is the value of $\frac{3}{7}$ of $\frac{5}{6}$ of 42?

22. Jane's grandmother is 70 years old, her mother is $\frac{4}{7}$ as old as her grandmother, and Emma is $\frac{5}{8}$ as old as her mother. How old is Emma ?

23. If 1 pound of cheese cost $\$\frac{4}{5}$, how many pounds can be bought for $\$6\frac{2}{3}$?

24. If 3 yards of cloth cost $\$\frac{5}{8}$, how many yards can be bought for $\$17\frac{1}{7}$?

25. How many bushels of corn, at $\$\frac{3}{4}$ a bushel, will pay for 9 yards of cloth, at $\$\frac{2}{3}$ a yard ?

26. How many acres of land, at $\$12\frac{2}{3}$ an acre, will pay for $28\frac{1}{2}$ barrels of flour, at $\$6\frac{3}{4}$ a barrel ?

27. How many pounds of butter, at $18\frac{3}{4}$ cts. a pound, will pay for $15\frac{1}{2}$ pounds of sugar, at $10\frac{1}{2}$ cts. a pound ?

28. How much larger or smaller will $\frac{1}{2}$ be, if both terms are increased by 2 ?

29. How much larger or smaller will $\frac{5}{7}$ be, if both terms are decreased by 2 ?

30. What is the cost of a barrel of flour, if $\frac{3}{5}$ of a barrel cost $4.

31. What is the cost of $\frac{5}{8}$ of a lot of land, if $\frac{7}{9}$ of it cost $\$37\frac{1}{3}$.

32. William lost $\frac{1}{3}$ of his nuts and then had 24 left. How many had he at first ?

33. $\frac{3}{7}$ of a pole was broken off, and the pole was then $16\frac{1}{2}$ feet long. What was its original length ?

34. A owns $\frac{1}{2}$ of a farm, B $\frac{1}{3}$, and C the remainder. If C's part is worth $700, what is the whole farm worth ?

35. A farmer has $\frac{3}{8}$ of his sheep in one field, $\frac{2}{5}$ in another, and the remainder, which is 27 sheep, in another. How many sheep has he in all ?

36. Henry can perform an errand in $\frac{1}{3}$ of a day, and Ben in $\frac{1}{4}$ of a day. How many errands can both perform in 1 day ?

37. A can mow 1 acre in $\frac{2}{3}$ of a day, and B can mow 1 acre in $\frac{3}{4}$ of a day. How many acres can both mow in 1 day ?

38. A father had 30 apples and gave $\frac{1}{5}$ of them to his daughter, $\frac{1}{4}$ of the remainder to his son, and the balance to his wife. How many did each receive?

39. Of a 60-acre field $\frac{2}{5}$ was planted to corn, $\frac{5}{9}$ of the remainder to wheat, and the balance to oats. How many acres were there of each kind of crop?

40. William and Walter bought 960 nuts, for which William paid $1 and Walter $2. What part of the nuts is each entitled to? How many nuts should each receive?

41. Three men, A, B, and C, bought 640 acres of land. A paid $4,000, B paid $5,000, and C $7,000. What part of the land is each entitled to? How many acres should each receive?

42. Frank can hoe the garden in 3 days, John in 8 days, and Ben in 24 days. How long would it take them all together to hoe the garden?

43. One pipe can fill a tank in 5 hours, another in 8 hours, and another in 10 hours. How long would it take all together to fill the tank?

44. If a boy can earn $1.80 in $1\frac{1}{3}$ days, counting 9 hours to the day, how much can he earn in $3\frac{3}{4}$ days, counting 8 hours to the day?

45. If a man can earn $\$37\frac{1}{2}$ in $8\frac{3}{4}$ days, working $7\frac{3}{7}$ hours a day, how much can he earn in $9\frac{1}{3}$ days, working $5\frac{3}{4}$ hours a day?

46. A man found $11, of which he gave each of his sons $\$1\frac{1}{2}$, and had $2 left. How many sons had he?

47. A man having $10\frac{1}{6}$ acres of land retained $2\frac{2}{3}$ acres, and sold the rest in lots of $\frac{5}{6}$ of an acre. How many lots did he sell?

48. An engineer, who received $\$\frac{1}{2}$ per hour, ran a train 255 miles at the rate of 30 miles per hour. What did he receive for the trip?

49. A man bought 280 pounds of oats, at $\$\frac{3}{8}$ per bushel. Allowing 32 pounds to a bushel, what did he pay for them?

50. John is 18 years old, and $\frac{3}{4}$ of Simon's age is equal to $\frac{5}{6}$ of John's age. How old is Simon?

51. How much tea at \$$\frac{5}{8}$ a pound must be given in exchange for 50 pounds of coffee at \$$\frac{4}{15}$ a pound?

52. An empty can weighs $\frac{1}{2}$ of a pound, and when full of milk $5\frac{3}{4}$ pounds. How many cans will be required to hold $31\frac{1}{2}$ pounds of milk?

53. A clerk receives \$$85\frac{1}{2}$ per month, and his average monthly expenses are \$$46\frac{4}{5}$. In how many years can he save enough money to buy a house valued at \$$1171\frac{2}{3}$?

54. Reduce $\frac{\frac{2}{3}}{\frac{3}{4}}$ and $\frac{1\frac{3}{4}}{2\frac{4}{5}} \times \frac{4}{15}$ to simple fractions.

55. Reduce $\frac{7\frac{1}{3}}{9\frac{1}{6}}$ and $\frac{3\frac{3}{4} \div 6\frac{2}{3}}{4\frac{3}{8} \div 23\frac{1}{3}} \times \frac{\frac{2}{3}}{1\frac{2}{5}}$ to simple fractions.

56. John spent $\frac{5}{8}$ of his money for a book that cost 40 cents, and with the remainder bought apples at 3 cents apiece. How many apples did he buy?

57. A part of a quantity of rye being damaged, a grocer sold that which was not damaged for $\frac{4}{7}$ of the value of all, at \$$\frac{3}{5}$ per bushel, and received \$480 for it. How many bushels were damaged?

58. Susan spent $\frac{7}{15}$ of her money for clothes. She then gave $\frac{1}{6}$ of what she had left to pay a debt of \$$6\frac{2}{5}$. How many dollars had she at first?

59. A farmer deposited $\frac{11}{20}$ of his money in a bank, and then spent $\frac{5}{12}$ of what he had left for a carriage. He paid \$420 for the carriage. How much money had he at first?

60. A man went to town and back in 10 hours. In going his rate was 3 miles per hour, and in returning it was 2 miles per hour. How far was it to town?

61. A man sold an equal number of ducks and chickens for \$21. He sold the ducks at the rate of 3 for \$1, and the chickens at the rate of 4 for \$1. How many chickens or ducks did he sell?

DECIMALS.

DEFINITIONS AND NOTATION.

185. If a yardstick is divided by marks into ten equal parts, each part is a *tenth* of a yard; if each tenth is divided by marks into ten equal parts, each part is a *hundredth* of a yard; if each hundredth is divided by marks into ten equal parts, each part is a *thousandth* of a yard; and so on.

The divisions of anything into tenths, hundredths, thousandths, etc., are called *decimal divisions*.

186. A **decimal fraction,** or simply a **decimal,** is one or more of the decimal divisions of a unit.

Thus, 3 tenths or $\frac{3}{10}$, 17 hundredths or $\frac{17}{100}$, are decimals.

187. Since the scale for the decimal units, tenths, hundredths, etc., is the same as that for integers, decimals may be written as integers; that is, *the unit or denominator may be indicated by the position of the figures.*

In the decimal system of notation the local value of a figure is decreased tenfold by each removal one place to the right. Hence, according to this system,

A figure at the *right of units* expresses *tenths;* a figure at the *right of tenths* expresses *hundredths;* a figure at the *right of hundredths* expresses *thousandths;* and so on.

188. To indicate where a decimal begins, a period, called the **decimal point,** is placed before the figure that represents *tenths.*

Thus, 5.273 yards may be interpreted as follows: a distance was measured by the unit yardstick; it contained the yardstick 5 times, with a remainder that contained 1 tenth of the unit (yardstick) 2 times, with a smaller remainder that contained 1 hundredth of the unit 7 times, with a still smaller remainder that contained 1 thousandth of the unit 3 times.

189. Let the student interpret, in a similar manner, the following:

1. 3.4 yards.	*2.* 14.37 feet.	*3.* .625 of an inch.
4. 1.48 pints.	*5.* 6.308 rods.	*6.* 8.031 miles.
7. $2.4.	*8.* $3.75.	*9.* $12.253.

Students should be required to measure convenient objects about the schoolhouse by a yardstick or foot-rule graded in tenths, hundredths, etc., and write the correct measures.

190. The relation of integrals and decimals is clearly shown in the following

Table.

Names.	**Millions.**	Hund.-thousands.	Ten-thousands.	**Thousands.**	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	**Thousandths.**	Ten-thousandths.	Hund.-thousandths.	**Millionths.**
Orders.	**7th.**	**6th.**	**5th.**	**4th.**	**3d.**	**2d.**	**1st.**	**2d.**	**3d.**	**4th.**	**5th.**	**6th.**	**7th.**
Units.	3	3	3	3	3	3	3.	3	3	3	3	3	3
	Integers.							Decimals.					

The number is read, 3 million 333 thousand 333, *and* 333 thousand 333 *millionths.*

All integral orders on the left of the *unit's* place are *multiples* of this unit, all decimal orders on the right are *decimal parts* of this unit, and the values of the units, equally distant from this *fundamental* unit, are *reciprocals* of each other.

Thus, the 2d order on the left is 1 *ten,* and the 2d order on the right is 1 *tenth;* the 3d order on the left is 1 *hundred,* and the 3d order on the right is 1 *hundredth.*

191. ***To read decimals.***

Read the expression 17.205.

Ans. Seventeen *and* two hundred five thousandths.

Read the expression .04063.

Ans. Four thousand sixty-three hundred thousandths.

RULE.—*Read the decimal as an integral number, and give it the denomination of the right hand figure.*

NOTE.—The conjunction *and* is never used in reading an integer or decimal except for connecting the integral and decimal parts of a number. This should be carefully noted to prevent confusion and errors.

Thus, .325 is *three hundred twenty-five thousandths*; and 300.025 is *three hundred* AND *twenty-five thousandths.*

EXERCISES (Oral).

Read:

1. .3; .8; .25; .06; 3.5; 6.12; 9.08.
2. .43; .094; .305; 437.1; 32.05; 4.062.
3. .0321; 43.909; 617.00617; 990.99099.
4. .568; 500.068; 1000.001; 17000.0017.
5. .0007; 6405.014; 69.34875; .0000008.

192. ***To express decimals by figures and the decimal point.***

1. Express decimally forty-three thousandths.

Ans. .043.

2. Express decimally twenty-six *and* two hundred seventy-five hundred thousandths.

Ans. 26.00275.

RULE.—*Write the integral part (if any), after it the decimal point, then write the numerator of the decimal, prefixing ciphers if necessary to indicate the denominator.*

EXERCISES (Written).

Express decimally :

1. Five tenths ; nine tenths ; seven hundredths.

2. Twenty-nine hundredths ; three hundred five thousandths.

3. Two thousandths ; fifteen ten thousandths ; five hundred sixty-eight hundred thousandths.

4. Five *and* twelve hundredths ; fourteen *and* one tenth.

5. Five hundred twenty-one thousandths ; five hundred *and* twenty-one thousandths.

6. 24 millionths ; 3271 ten-millionths ; 13 billionths.

7. 142 *and* 4005 hundred thousandths ; 9043 and 52109 hundred millionths. 16 and 16 ten-billionths.

8. $\frac{7}{10}$; $\frac{25}{100}$; $\frac{8}{100}$; $\frac{43}{1000}$; $\frac{253}{1000}$; $\frac{9}{1000}$.

9. $5\frac{3}{10}$; $18\frac{5}{100}$; $9\frac{73}{100}$; $146\frac{3}{1000}$; $13\frac{13}{10000}$.

Fundamental Principles.

193. I. *The denominator of a decimal, when expressed, is* 1 *with as many ciphers annexed as there are figures in the decimal.*

Thus, $.043 = \frac{43}{1000}$; $.000605 = \frac{605}{1000000}$.

II. *Annexing ciphers to a decimal does not alter its value.*

For, $.40 = 4$ tenths + 0 hundredths = 4 tenths = .4. $.530 = 5$ tenths + 3 hundredths + 0 thousandths = 5 tenths + 3 hundredths = .53.

Or, $.40 = \frac{40}{100} = \frac{4}{10} = .4$; $.530 = \frac{530}{1000} = \frac{53}{100} = .53$.

III. *Each decimal cipher, prefixed to a decimal, diminishes the value of the decimal tenfold.*

For, .5 = 5 tenths, and .05 = 5 hundredths.
.006 = 6 thousandths, and .0006 = 6 ten thousandths.

COROLLARIES.—1. *To multiply a decimal by* 10, 100, 1000, *etc., remove the decimal point as many places to the right as there are ciphers in the multiplier.*

2. *To divide a decimal by* 10, 100, 1000, *etc.*, ***remove the decimal point to the left as many places as there are ciphers in the divisor.***

Thus, $.43 \times 10 = 4.3$; $2.706 \times 100 = 270.6$.
$.43 \div 10 = .043$; $63.75 \div 1000 = .06375$.

IV. *A simple fraction annexed to a decimal is of the same order as the last integral figure of the decimal.*

Thus, $.5\frac{1}{2} = 5$ tenths + $\frac{1}{2}$ tenths.
$.64\frac{2}{3} = 6$ tenths + 4 hundredths + $\frac{2}{3}$ hundredths.

COROLLARY.—Prefixing a decimal point to a simple fraction does not alter its value.

Thus, $\cdot\frac{1}{2} = \frac{1}{2}$; for $\cdot\frac{1}{2} = \frac{1}{2}$ of one $= \frac{1}{2}$.

REDUCTION.

194. ***To reduce dissimilar to similar decimals.***

1. Reduce .8, .43, and 5.028 to similar decimals.

EXPLANATION.—The lowest order of decimals in the given numbers is thousandths ; hence all the other decimals are changed to thousandths by annexing ciphers. (Prin. II., Art. 193.)

$.8 = .800$
$.43 = .430$
$5.028 = 5.028$

RULE.—*Give all the decimals the same number of places by annexing ciphers.*

EXERCISES (Written).

Reduce to similar decimals :

1. .3, .21, .342, .00123.
2. .05, .037, .6041, 5.3.
3. 7, .035, .06, .14, .1.
4. .305, .4029, .001016.
5. 6.1, .007, .41, .0082.
6. 45.2, 24.03, 1.24308.
7. 10, .01, 1.0104, 13.4.
8. 2.1, 16, .374, .00036.

195. ***To reduce decimals to common fractions.***

1. Reduce .375 to a common fraction.

Operation.

$.375 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$.

Explanation.—.375 expressed as a common fraction is $\frac{375}{1000}$, which, when reduced to lowest terms, is $\frac{3}{8}$.

Rule.—*Omit the decimal point, supply the proper denominator, and reduce the fraction to its lowest terms.*

EXERCISES (Written).

Reduce to common fractions :

1. .25. *2.* .75. *3.* .64. *4.* .225.
5. .725. *6.* .625. *7.* .0625. *8.* .3125.
9. .256. *10.* .448. *11.* .5125. *12.* .15625.
13. .045. *14.* .008. *15.* .7024. *16.* .41275.
17. $.26\frac{2}{3}$.

$$.26\frac{2}{3} = \frac{26\frac{2}{3}}{100} = \frac{80}{3} \times \frac{1}{100} = \frac{8}{30} = \frac{4}{15}.$$

18. $.62\frac{1}{2}$. *19.* $.08\frac{1}{3}$. *20.* $.12\frac{1}{2}$. *21.* $.156\frac{1}{4}$.
22. $.000\frac{3}{5}$. *23.* $.00\frac{3}{5}$. *24.* $.0\frac{3}{5}$. *25.* $.\frac{3}{5}$.
26. $.028\frac{1}{8}$. *27.* $.0284\frac{1}{11}$. *28.* $.444\frac{4}{9}$. *29.* $.01339\frac{2}{7}$.

Express by an integer and common fraction :

30. 6.5. *31.* 8.6. *32.* $17.3\frac{1}{3}$. *33.* $11.05\frac{5}{9}$.
34. 9.75. *35.* $3.41\frac{2}{3}$. *36.* $3.87\frac{1}{2}$. *37.* $71.58\frac{1}{3}$.
38. 4.16. *39.* $12.0\frac{3}{8}$. *40.* $15.1\frac{2}{3}$. *41.* $9.012\frac{3}{11}$.

196. ***To reduce a common fraction to a decimal.***

1. Reduce $\frac{5}{8}$ to an equivalent decimal.

Operation.

$$8\overline{)5.000}$$
$$.625$$

Explanation.—5 = 5 ones 0 tenths 0 hundths 0 thous'ths, which, when divided by 8, as in integers, gives 6 tenths 2 hundths 5 thous'ths, or .625.

Rule.—*Annex ciphers to the numerator, divide by the denominator, and point off as many decimal places in the quotient as there are ciphers annexed.*

EXERCISES (Written).

Express decimally :

1. $\frac{4}{5}$. *2.* $\frac{1}{4}$. *3.* $\frac{3}{8}$. *4.* $\frac{7}{8}$. *5.* $\frac{3}{20}$.
6. $\frac{5}{9}$. *7.* $\frac{4}{25}$. *8.* $\frac{5}{16}$. *9.* $\frac{17}{40}$. *10.* $\frac{11}{80}$.
11. $\frac{13}{16}$. *12.* $\frac{5}{32}$. *13.* $\frac{15}{44}$. *14.* $\frac{1}{24}$. *15.* $\frac{3}{125}$.
16. $\frac{9}{60}$. *17.* $\frac{1}{18}$. *18.* $\frac{74}{75}$. *19.* $\frac{1}{600}$. *20.* $\frac{7}{400}$.
21. $18\frac{3}{4}$. *22.* $13\frac{3}{5}$. *23.* $9\frac{1}{8}$. *24.* $10\frac{5}{16}$. *25.* $19\frac{4}{15}$.
26. $3.2\frac{1}{5}$. *27.* $4.7\frac{3}{8}$. *28.* $.0\frac{5}{16}$. *29.* $.03\frac{5}{8}$. *30.* $.002\frac{17}{80}$.

ADDITION.

197. ***To add two or more decimals.***

1. Find the sum of 16.54, 7.389, and 125.3.

EXPLANATION.—The numbers are written so that units of the same order stand in the same column, and they are added precisely as in integers, since 10 units of any order make 1 unit of the next higher order. The sum is expressed decimally by placing the point beneath the other points.

OPERATION.

```
  16.54
   7.389
 125.3
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 149.229
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EXERCISES (Written).

Find the sum of :

1. 4.16, 31.4, .035. *2.* 15, 8.3, 45.075, 4.610.
3. .042, .493, 2.81. *4.* 3.804, 91.64, 18.0365.
5. .375, 80.06, 45.0084, .00755, 84.635.
6. 84.08, 16.075, 2.9, 1.96, 1.003, 5.08.
7. 8.365, 27.4, 543, 16.04, .0357.
8. 63.75, 9.6, .075, 725, .8, .035.
9. 15.19, 7.4, 5364.437, 68.0048.
10. .271, .0185, .000477, 16.0489.
11. 340.4, 580.768, 68.3745, 78.6.
12. 173.625, 6.40625, 84.35, .764.

13. How many pecks of oats in four sacks, there being 9.5 pecks in the first, 11.4 pecks in the second, 8.75 pecks in the third, and 12.675 pecks in the fourth ?

14. A lady bought 16.25 yards of silk, 12.75 yards of alpaca, 6.5 yards of merino, 11.875 yards of delaine, and 23.5 yards of French calico. How many yards of goods did she purchase ?

15. A farmer sold four lots of land containing 79.76, 85.08, 36.125, 140.309 acres, respectively. How many acres did he sell ?

16. In five piles of wood there are 18.79, 147.072, 856.709, 185.8761, 397.05784 cords, respectively. How many cords in all ?

17. A dealer bought six horses, for which he paid $117.5, $124, $129.3175, $104.75, $130.0625, $131.5625, respectively. What did he pay for all ?

18. A merchant sold seven lots of sugar for $137.25, $94.5, $216.375, $56.4, $113.7, $80.625, $296.3, respectively. What did he receive for all ?

19. A lady paid $1.08 for buttons, $.43$\frac{1}{2}$ for ribbon, 1.27\frac{3}{4}$ for muslin, $.37$\frac{1}{4}$ for needles, 2.54\frac{2}{5}$ for a hat, and $10.625 for a dress. How much did her purchases amount to ?

20. A boy traveled 4.2$\frac{3}{4}$ miles, then 3.35 miles, then $\frac{7}{8}$ mile, then 12.08 miles. How far did he travel in all ?

21. John is 12.025 years older than Ben, and Ben is 13$\frac{1}{16}$ years older than Henry. How old will John be when Henry is 24.685 years old ?

SUBTRACTION.

198. ***To subtract one decimal from another.***

1. From 23.7 take 19.385.

Operation.

$$\begin{array}{r} 23.700 \\ 19.385 \\ \hline 4.315 \end{array}$$

Explanation.—The subtrahend is written under the minuend, so that units of the same order stand in the same column, and they are subtracted as in integers.

The decimals may be made similar by annexing ciphers, or the ciphers may be supposed to be there, though not written.

EXERCISES (Written).

Find the value of :

1. 37.16 — 21.643.

2. 7 — 3.464.

3. 29.063 — 7.5374.

4. 8.4 — 5.3635.

5. 83.75 — 64.003275.

6. .125 — .00325.

7. 475.675 — 99.00088.

8. 184000 — .184.

9. 25 — 18.3465.

10. 1.0001 — .0020087.

11. 57.25 — 33.0439.

12. $16.5\frac{1}{2} - 8.24\frac{3}{4}$.

13. $8.5 + 3\frac{3}{4} - 4\frac{7}{8}$.

14. $16 - (5.23 + 7\frac{3}{5})$.

15. $125\frac{7}{16} - 117.38\frac{1}{2}$.

16. $18\frac{2}{5} - (16.63 - 12\frac{3}{4})$.

17. Frank is 5.025 feet high and Walter 4.6347 feet. How much taller is Frank than Walter ?

18. From a safe containing \$34.26, \$$31\frac{1}{40}$ were stolen. How much remained ?

19. A silver dollar weighs 412.5 grains, and contains 41.25 grains of alloy. How much silver does it contain ?

20. A cubic foot of gold weighs $1203\frac{5}{8}$ pounds, and a cubic foot of iron 446.875 pounds. How much more does the gold weigh than the iron ?

21. There are 118.375 barrels of water in a cistern which will hold 130.25 barrels. How much more water will the cistern hold ?

22. The owner of a lot sold .4073 of it. What part of the lot did he still own ?

23. A man starts on a journey of 576.45 miles. After going 309.8295 miles, how much farther will he have to go ?

24. A man sold four lots together for \$1,200 : for the first he received \$276.5 ; for the second, \$318.37 ; and for the third, \$294.2. What did he receive for the fourth lot ?

25. A farmer sold a merchant a bale of cotton for \$34.25, and took in payment a pair of pants for \$8.375, a coat for \$12.50, a barrel of flour for \$5.3775, a hat for \$4, and the rest in money. How much money did he receive ?

MULTIPLICATION.

199. *To multiply one decimal by another.*

1. Multiply 4.23 by .8.

OPERATION.

$$\begin{array}{r} 4.23 \\ .8 \\ \hline 3.384 \end{array}$$

EXPLANATION.—$4.23 = \frac{423}{100}$, $.8 = \frac{8}{10}$.
$\frac{423}{100} \times \frac{8}{10} = \frac{3384}{1000} = 3.384$.

RULE.—*Multiply as if the numbers were integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

If the product does not contain as many figures as there are decimals in both factors, supply the deficiency by prefixing ciphers.

EXERCISES (WRITTEN).

Find the value of:

1. 4.5 × 6.4.
2. 7.05 × 3.2.
3. 16.5 × .065.
4. 122 × .75.
5. .0061 × .05.
6. 125.05 × .25.
7. 4508 × .240.
8. .1866 × 5.4.
9. 2.456 × 3.55.
10. 2.713 × .67.
11. 4.285 × .235.
12. 12.3107 × .152.
13. .432 × .0025.
14. 6.05 × .0075.
15. .054 × .00175.
16. 400 × .0004.
17. 1.44 × .0066.
18. .0321 × 72.5.
19. 32.6 × .0437.
20. 75.3 × .672.
21. 4.625 × 7.14.
22. 928 × 8.302.
23. 20.52 × .0087.
24. .00948 × .029.

25. Multiply 63.542 by 100.

Remove the decimal point two places to the right (Prin. III, Cor. 1), and we have 6354.2—Ans.

Find the product of :

26. 3.463 × 10.
27. 457.6 × 100.
28. .5473 × 100.
29. .07301 × 1000.
30. .0003 × 1000.
31. 4.07 × 100000.

32. Multiply 3.7405 by 600.

OPERATION.

$$\begin{array}{r} 374.05 \\ 6 \\ \hline 2244.30 \end{array}$$

EXPLANATION.—600 = 6 × 100.
First, multiply 3.7405 by 100, which gives 374.05; next multiply 374.05 by 6.

Find the product of :

33. 45.218 × 500.
34. 402.73 × 40.
35. 3.6041 × 600.
36. .06809 × 8000.
37. 8.102 × 9000.
38. .004025 × 160000.
39. 23.14 × 70000.
40. .8888 × 25000.
41. 63.05 × $.8\frac{3}{5}$.
42. 626.4 × $.09\frac{3}{4}$.
43. $.0062\frac{1}{2}$ × $51\frac{2}{5}$.
44. $.0234\frac{1}{2}$ × $60.2\frac{3}{8}$.

45. What cost 137.5 acres of land, at $76.25 per acre ?

46. What is the value in United States money of 15.4 pounds English money, if 1 pound is worth $4.8665 ?

47. A cubic foot of water weighs 62.5 pounds; what does a cubic foot of gold weigh, gold being 19.258 times heavier than water ?

48. What does a cubic foot of salt weigh, salt being 2.13 times heavier than water ?

49. What is the value of 25.64 bushels of corn, at $.625 per bushel ?

50. What is the value of 5.5 hogsheads of sugar, averaging 1164.74 pounds a hogshead, at $.08 per pound ?

51. A man owned .3275 of a mill, and sold .1375 of his share. What part of the mill did he sell ?

52. If the average rate of a freight train, including stops, is 16.87 miles per hour, how far will it go in 48.4 hours ?

53. A farmer bought 35.75 yards of cloth, at $4.50 per yard, and paid in part 13.375 cords of wood, at $4.62 per cord. How much did he still owe ?

54. A man bought 85 pounds of coffee at \$.25 per pound, 56 pounds of soap at \$.0625 per pound, 36 pounds of tea at \$.94 per pound, 75 yards of cassimere at \$.875 per yard, 63 gallons of molasses at \$.375 per gallon, and 125 pounds of rice at \$.08⅓ per pound. What was the cost of the whole ?

DIVISION.

200. ***To divide one decimal by another.***

1. Divide .0884 by 3.4.

ANALYSIS.

$.0884 = \frac{884}{10000},\ 3.4 = \frac{34}{10}.$

$\frac{884}{10000} \div \frac{34}{10} = \frac{884 \div 34}{10000 \div 10} = \frac{26}{1000}$

$= .026.$

OPERATION.

```
3.4).0884(.026
      68
     ---
     204
     204
     ---
```

RULE.—*Divide as if the numbers were integers, and in the quotient point off as many decimal figures from the right as the decimal places of the dividend exceed those of the divisor, prefixing ciphers, if necessary, to supply the deficiency.*

NOTES.—1. When there are more decimal places in the divisor than in the dividend, make them equal by annexing ciphers to the dividend before dividing.

2. If there is a remainder, ciphers may be annexed to it as decimals, and the division continued at pleasure.

3. When there is a remainder at the close of the operation, the sign + should be annexed to the quotient to show that it is not complete.

EXERCISES (Written).

Find the quotient of :

1. .625 ÷ 2.5.

2. 1.776 ÷ .24.

3. 313.9 ÷ .043.

4. .28375 ÷ 2.5.

5. .00639 ÷ 71.

6. 45625 ÷ .125.

Find the quotient of:

7. 234.21 ÷ .0211.
8. 428.1 ÷ .346.
9. 128.2561 ÷ 301.
10. 18.1771 ÷ 27.13.
11. 1.51088 ÷ 79.52.
12. 8.16508 ÷ 16.87.
13. .0012342 ÷ .3025.
14. 45031.25 ÷ .3275.
15. .39645 ÷ .01762.
16. 5.99795 ÷ 17.137.
17. .03 ÷ .0625.
18. 1828.125 ÷ 1.875.
19. 46.103 ÷ 2.14.
20. 6.2512 ÷ .37.
21. 16.025 ÷ .045.
22. 3.1812 ÷ 482.
23. 272.636 ÷ 6.37.
24. .2818585 ÷ 73.21.

25. Divide 637.5 by 100.

Remove the decimal point two places to the left—6.375 Ans.

Find the quotient of:

26. 17.63 ÷ 10.
27. 63.659 ÷ 1000.
28. 324.7 ÷ 100.
29. 3.465 ÷ 10000.
30. .0419 ÷ 100.
31. 43 ÷ 100000.

32. Divide 381.6 by 400.

OPERATION.

$$\begin{array}{r|l} 4 & \underline{3.816} \\ & .954 \end{array}$$

EXPLANATION.—400 = 4 × 100.

First, divide 381.6 by 100, which gives 3.816; next divide 3.816 by 4.

Find the value of:

33. 23.5 ÷ 50.
34. 45.36 ÷ 600.
35. 1236.9 ÷ 300.
36. 35.128 ÷ 8000.
37. 6.5016 ÷ 400.
38. 5254.2 ÷ 9000.
39. 17 ÷ 12000.
40. 4.8165 ÷ 1500.

41. What is the quotient of $8\frac{3}{8}$ by 6.25?

42. The dividend is \$$10\frac{3}{4}$, the divisor 2.125. What is the quotient?

43. A farmer sold $7\frac{1}{2}$ acres for $70.125. How much was that per acre?

44. Among how many persons can $403 be distributed if each person receives $1.625?

45. The circumference of a circle is 3.1416 times the diameter. If the distance *around* a circular garden is 285.6 yards, how far is *through* it ?

46. How many yards of calico, at $.0775 per yard, can be bought for $9.25 ?

47. How many casks, each holding 41.315 gallons, will be required to hold 11278.995 gallons of wine ?

48. How many days will it take a man to earn $35.0625 if he earns $4.125 per day ?

49. At $53.4 per hogshead, how many hogsheads of sugar can be bought with $1321.65 ?

50. A farmer had $1500, of which he spent $200.625 and invested the remainder in land at $31.5 per acre. How many acres did he purchase ?

51. How many barrels of flour, at $5.125 per barrel, are equivalent to 91.25 bushels of wheat, at $1.40 per bushel, and 85 bushels of rye, at $1.25 per bushel ?

BUSINESS TRANSACTIONS.

Definitions.

201. Quantity, in business transactions, is the amount of any commodity bought or sold.

202. Price is the value in money of a given unit of the quantity.

203. Cost is the value in money of the entire quantity.

204. An aliquot part of a number is an exact divisor of that number.

205. ***Aliquot parts of one dollar.***

$50 \text{ cts.} = \frac{1}{2} \text{ of } \$1.$
$25 \text{ cts.} = \frac{1}{4} \text{ of } \$1.$
$20 \text{ cts.} = \frac{1}{5} \text{ of } \$1.$
$10 \text{ cts.} = \frac{1}{10} \text{ of } \$1.$
$5 \text{ cts.} = \frac{1}{20} \text{ of } \$1.$

$33\frac{1}{3} \text{ cts.} = \frac{1}{3} \text{ of } \$1.$
$16\frac{2}{3} \text{ cts.} = \frac{1}{6} \text{ of } \$1.$
$12\frac{1}{2} \text{ cts.} = \frac{1}{8} \text{ of } \$1.$
$8\frac{1}{3} \text{ cts.} = \frac{1}{12} \text{ of } \$1.$
$6\frac{1}{4} \text{ cts.} = \frac{1}{16} \text{ of } \$1.$

206. ***Abbreviations.***

@, At.
℅, Account.
Acc't, Account.
Bal, Balance.
Bbl, Barrel.
Cr, Creditor.
Dr, Debtor.
Doz, Dozen.
Hhd, Hogshead.
Lb, Pound.
Pay't, Payment.
Pd, Paid.
Per, by.
Rec'd, Received.
Yd, Yard.

Generally, in business calculations, if in the final result the mills are less than 5 they are not regarded ; if 5 or more than 5, they are considered a cent.

Thus, we write $12, 23 cts. 6m., $12.24.

The amount written after @ is the price.

Short Methods.

207. ***To find the cost when the quantity is given, and the price is an aliquot part of $1.***

1. What cost 75 yards of cloth, @ 33⅓ cts. ?

Explanation.—If the price were $1 a yard, the cost would be $75 ; hence, since the price is ⅓ of $1, the cost is ⅓ of $75, or $25.

Operation.
⅓ of $75 = $25

Rule.—*Take such a part of the given quantity, regarded as dollars, as the price is of* $1.

EXERCISES (Oral and Written).

Find the cost of :

1. 120 yd. of calico, @ 12½ cts.
2. 136 lb. of sugar, @ 6¼ cts. Also, @ 8⅓ cts.
3. 78 lb. of coffee, @ 16⅔ cts. Also, @ 33⅓ cts.
4. 57 doz. eggs, @ 20 cts. Also, @ 16⅔ cts.
5. 164 bushels of corn, @ 25 cts. Also, @ 12½ cts..
6. 326 gallons of syrup, @ 33⅓ cts. Also, @ 50 cts.
7. 87 yd. of cloth, @ $1.33⅓.

Operation.—$87 × 1⅓ = $87 + $29 = $116.

8. 24 hats, @ $1.25. Also, @ $1.16⅔ ; @ $1.12½.
9. 186 chairs, @ $1.06¼. Also, @ $1.08⅓ ; @ $1.25.

208. ***To find the quantity when the cost is given, and the price is an aliquot part of $1.***

1. How many books, @ 25 cts., can be bought for $7?

Explanation.—If the price were $1 a book, $7 would buy 7 books; hence, since the price is $\frac{1}{4}$ of $1, $7 will buy 4 times 7 books.

Operation.

4×7 books $=$ 28 books.

Rule.—*Multiply the number of dollars in the cost by the number of times the price is contained in one dollar.*

EXERCISES (Oral and Written).

1. How many boxes of grapes can be bought with $4, @ $33\frac{1}{3}$ cts.? With $5, @ 25 cts.? With $11, @ 20 cts.?

2. How many pounds of lard can be bought with $4.50, @ $6\frac{1}{4}$ cts.? Also, @ $8\frac{1}{3}$ cts.? @ $16\frac{2}{3}$ cts.? @ $12\frac{1}{2}$ cts.?

3. How many books can be bought with $120, @ 25 cts.? Also, @ 20 cts.? @ $16\frac{2}{3}$ cts.? @ $12\frac{1}{2}$ cts.?

4. How many pounds of tea can be bought with $7.35, @ 25 cts.? @ $33\frac{1}{3}$ cts.? @ 75 cts.?

5. How many yards of prints, @ $6\frac{1}{4}$ cts., can a lady buy with $14? How many with $$1\frac{3}{4}$?

6. A lad has $7; how many melons can he buy @ $33\frac{1}{3}$ cts.? How many @ 25 cts.?

7. If sugar is worth $6\frac{1}{4}$ cts. per pound, coffee $16\frac{2}{3}$ cts. per pound, and tea 25 cts. per pound, how many pounds of each can be bought with $18.40?

209. ***To find the cost, when the quantity is sold by the 100 or 1000.***

1. What cost 575 rails at 75 cts. per hundred?

Explanation.—575 = 5.75 times 100; hence 575 rails cost 5.75 times as much as 100 rails, or $4.3125.

Operation.

$575 \div 100 = 5.75$

5.75×75 cts. $=$ $4.3125

Rule.—*Multiply the price per* 100 *or* 1000 *by the hundredth or thousandth part of the given quantity.*

The letter *C* is sometimes used for *hundred*, and *M* for *thousand.*

EXERCISES (Written).

1. What is the cost of 1540 boards, at $1.75 per C. ?

2. What will be the freight on 468 pounds of merchandise, at $62½ per C. ? At $1.84 per M. ?

3. What will be the cost of 50,000 laths, at $.26 per C., and 1570 feet of boards, at $16.50 per M. ?

Find the cost of :

4. 746 pounds of pork, at $12½ per hundred. At $10⅜.

5. 1425 feet of square timber, at $28 per M. At $26.50.

6. 3846 cedar rails, at $6¼ per 100. At $56¾ per 1000.

7. 27890 bricks, at $9.70 per 1000. At $9¼ per M.

210. ***To find the cost when the thing or quantity is sold by the ton of 2000 pounds.***

1. At $5.60 a ton, what will 1750 pounds of coal cost ?

Explanation.—Since $5.60 is the price of 2000 pounds, ½ of $5.60, or $2.80, is the price of 1000 lbs. The cost of 1750 lbs. is now determined as in Art. 209.

Operation.

$\$5.60 \div 2 = \$2.80$

$1.750 \times \$2.80 = \$4.90$

Rule.—*Multiply ½ of the cost per ton by the one thousandth part of the given quantity.*

EXERCISES (Written).

1. Find the cost of 2760 pounds of hay, at $16⅔ a ton ?

2. A potter bought 3475 pounds of porcelain clay, at $17.50 a ton. What did it cost him ?

3. Find the freight on 8760 pounds of tea, at $2⅝ a ton.

4. What will a load of plaster weighing 1784 pounds cost, at $2.75 a ton ? At $2.50 ?

5. What is the cost of 120 sacks of manure, each weighing 112½ pounds, at $26 a ton. At $32.40 ?

ACCOUNTS AND BILLS.

211. A debt is the amount which one person owes another. A *debtor* is a person who owes a debt; a *creditor*, one to whom a debt is due.

212. A bill of goods is a written statement given by the seller to the buyer, containing the date of the purchase, the names of the buyer and seller, a list of the goods bought, with their prices, and the total amount.

213. An item is any article in the bill; *extending* an item is finding its cost, and the *footing* is the entire cost of all the items in a bill.

214. An account is a written statement of debits and credits between two parties.

When a bill or account is paid, it should be *receipted;* that is, the creditor should write at the bottom of the same: *Received payment,* and after it his name.

Bills.

EXERCISES (Written).

215. Copy, extend the items, and find the footings of the following bills:

(1)

MONTGOMERY, ALA., *May* 3, 1902.

Mr. J. W. Jones,

Bought of C. G. Bates.

7 yd. Ribbon, @ $.12½		
4 yd. Cambric, @ $.16¼		
5 doz. Buttons, @ $.15		
12 skeins Sewing-silk, @ $.06¼		
13 yd. Prints, @ $.08		
Amount		

Received payment,

C. G. Bates.

(2)

Athens, Ga., *Sep.* 23, 1901.

Mr. C. D. Pate,

Bought of J. D. Young & Co.

64 lb. Tea, @ $.85		
48 lb. Starch, @ $.04		
63 gal. Molasses, @ 37½ct.		
125 lb. Bar-soap, @ $.05½		
120 lb. Coffee, @ $.10⅓		
56 lb. Rice, @ $.06¼		
Amount		

Received payment,

J. D. Young & Co.,

per Johnson.

Make out, extend the items, find the footings, and receipt:

3. Mrs. A. P. Jackson bought of Boyd, Rice & Co., of London, Miss., as follows: 23 yards of muslin, @ 16⅔ cts.; 40 yards sheeting, @ 12½ cts.; 14 yards of silk, @ $1.33⅓; 10 handkerchiefs, @ 25 cts.; 3 pairs kid gloves, @ $1.62½; 9 neckties, @ 66⅔ cts.; 25¾ yards of prints, @ 8 cts.; 12½ yards of alpaca, @ 75 cts.; 3 dozen lemons, @ 62½ cts.

4. Guilbeau & Martin, Breaux Bridge, La., sold Arcade Patin, Dec. 3, 1902, 94¾ lb. bacon, @ 9 cts.; 75 lb. boneless bacon, @ 14 cts.; 256 lb. ham, @ 12½ cts.; 325 lb. shoulder, @ 10 cts.; 68 lb. dried beef, @ 16 cts.; 32¾ lb. Bologna sausage, @ 18 cts.

5. Col. Thos. D. Boyd bought of Jones & Whitaker, Baton Rouge, La., Dec. 2, 1901, 37 lb. sugar, @ $.06¼; 138 lb. coffee, @ $.17¾; 25¼ lb. rice, @ $.06; 6½ lb. starch, @ $.03½; 3½ gal. syrup, @ $.45; 60 lb. flour, @ $.03½; 42 lb. grits, @ $.01¾; 2 lb. tea, @ $.57; 15 lb. dried apples, @ $.07½. The amt. due was paid to Parish.

6. I. A. Newton bought of John A. Doyle Co., Jackson, Miss., May 5, 1902, ⅓ gross knives and forks, @ $7.75; ¾ gross tablespoons, @ $12.25 ; 250 lb. rod iron (1 in.), @ $.02⅝ ; ⅔ doz. axes, @ $6 ; 2 kegs nails (10d.), @ $2.50 ; 1 doz. L. H. shovels, @ $6.35 ; 2 B. F. Avery & Son stubble-diggers, @ $75 ; 55 lb. sad irons, @ $.03¼ ; 2 J. H. Hall cotton plows, @ $4.90. The amt. due was paid to A. K. Smith for the Firm.

7. Fuqua Hardware Co., Baton Rouge, La., sold George Hill, Aug. 10, 1901, 1 coil sisal rope, 64 lb., @ 8¼ ct. ; 2 sides harness leather, 36 lb. @ 32 ct. ; 2 doz. Istrouma axes, @ $6 ; 4 doz. ax handles, @ $1.25 ; 4 doz. Istrouma eye hoes, at $4.50 ; 5 doz. poplar hoe handles, at $1.25 ; 6 Deere Pelican plows, @ $13.29 ; 3 doz. Arcade mill files, @ $1.40 ; 25 coils Waukegan barbed wire, @ $3.55 ; 1 keg galvanized staples, @ $3.55 ; 1 Adriance Buck-eye mower, @ $45. The amount due was paid to F. B. Jones, Aug. 14, 1901.

Parallel Problems.

216. *1.* What cost 6 pounds of butter, at $.25 a pound ?

2. What will 124.45 feet of wire cost, at 6.5 cts. a foot ?

3. If 6 hats cost $12.48, what will 1 hat cost ?

4. If 6.35 bushels of corn cost $5, what will 1 bushel cost ?

5. If 5 caps cost $12.50, what will 7 caps cost ?

6. If 84.37 acres of land cost $696.0525, what will 96.24 acres cost ?

How many hours will it take a man :

7. To travel 49.2 miles, if he travels at the rate of 6 miles per hour ?

8. To travel 34½ miles, if he travels at the rate of 4.6 miles per hour ?

9. A laborer is paid $2 a day. How much does he save in 12 days, if he spends $1.25 per day ?

10. A man's income is $1800 a year, and his expenses are $1172.75. How much does he save in 5.64 years ?

11. A lad bought 12 hogs at $4 each; he sold ⅔ of them at $5, and the remainder at $3, each. How much did he make or lose?

12. A grocer bought 1485 pounds of sugar @ 9 cts.; he sold .64 of it @ 9.5 cts., and the remainder @ 8.4 cts. How much did he make or lose?

13. A boy bought apples at 3 cts. each, sold them at 4½ cts. each, and thereby made 30 cts. on all he had. How many had he?

14. A drover bought cattle @ $18.75, sold them @ $24.1, and thereby gained $663.40. How many cattle did he buy?

15. What cost 4500 shingles, at $8 per M.?

16. What cost 637.5 pounds of hay at $16.50 per ton?

17. A girl bought 15 readers at 33⅓ cts. each, 20 copybooks at 12½ cts. each, and 32 pencils at 6¼ cts. each. What was the cost of all?

18. A farmer bought 256.4 pounds of flour @ $.25, 138.7 pounds of bacon @ $.08⅓, 42.3 pounds of lard @ $.16⅓. What did he pay for all?

Show that any number may:

19. Be multiplied by .5 by taking ½ of it.

20. Be multiplied by .25 by taking ¼ of it.

21. Be multiplied by .125 by taking ⅛ of it.

22. Reduce to a common fraction .5; .25; .125.

23. Reduce to a common fraction .625; .5625.

24. Add as fractions and then as decimals: ½, ⅖, ¾. Show that the results are the same.

25. Add as fractions and then as decimals: $\$\frac{3}{8}$, $\$\frac{4}{25}$, $\$\frac{3}{32}$. Show that the results are the same.

Solve the two following examples, (1) by the rules of fractions, (2) by the rules of decimals; and then show that the results are the same:

$$\textit{26.}\quad (16\tfrac{2}{5} + 11\tfrac{3}{4}) \times (13\tfrac{1}{2} - 6\tfrac{7}{8}) = ?$$

$$\textit{27.}\quad (19\tfrac{2}{5} - 16\tfrac{5}{16}) \div (\tfrac{3}{4} \times 2\tfrac{5}{8}) = ?$$

28. Reduce $\frac{2\frac{1}{2}}{4}$ and $\frac{3\frac{1}{2}}{6\frac{2}{5}}$ to decimals.

29. .6 of a number is 42. What is the number?

30. .37 of a number is 444. What is the number?

31. James gave his mother .7 of his apples and had 12 left. How many apples had he at first?

32. A merchant, after losing .7254 of his money, had $1088.10 left. How much had he at first?

33. James, John, and Henry own a sack of marbles: ⅖ of them belong to James, .3 to John, and Henry has 5 marbles less than James. How many marbles are in the sack? How many has each boy?

34. A, B, and C entered into business. A put in .365 of the capital; B, .4725 of it, and C put in $620 less than B. What was the amount of the capital?

35. A lad, after eating .5 of his apples, and .6 of the remainder, had 12 left. How many had he at first?

36. A man spent .375 of his fortune, then spent .488 of the remainder, and had $32754 left. What was the value of his fortune?

37. A farmer sold a horse, cow, and sheep for $175; he received $125¾ for the horse, and $34.2745 for the cow. What did he receive for the sheep?

38. A laborer received $12 for his week's work. He paid $4.37½ for board, $.65 for a knife, and $3.27¾ for a vest. How much money had he left?

39. A grocer sold a farmer 240 hams @ $1.75; 33 gallons of syrup @ 42.75 cts.; and took in part payment 15 bushels of oats @ 66.50 cts. How many bushels of corn, @ 70.5 cts., will pay the balance?

40. A man has four piles of wood containing, respectively, 13⅞, 15.66⅔, 18¼, and 21⅓ cords. How many cords has he in all, and what is it worth at $4.25 per cord?

41. What is the cost of 4565 feet of joist at $23 per M. feet; 13640 feet of boards at $53.55 per M. feet; and 15250 shingles at $4.50 per M.?

DENOMINATE NUMBERS.

STANDARD UNITS AND REDUCTION.

217. Standard units are units established by law or custom.

Thus, 1 yard, 1 pound, 1 bushel, are standard units.

218. A number whose unit is a standard unit is called **a denominate number.**

219. A denominate number which is formed of units of one denomination only is called a **simple denominate number.**

Thus, 6 yards, 9 pounds, $7, are simple denominate numbers.

220. A denominate number which is formed of related units of different denominations is called **a compound denominate number.**

Thus, 4 yards 2 feet 5 inches is a compound denominate number.

221. Denominate numbers arise in measuring quantities by standard units.

Thus, 4 yards 2 feet 5 inches is the result of measuring something, say the length of a line, by a yardstick. The line contains the yardstick 4 times, with a remainder which contains $\frac{1}{3}$ of a yard, or 1 foot, 2 times, with a small remainder which contains $\frac{1}{12}$ of a foot, or 1 inch, 5 times.

Students can best acquire a clear conception of denominate numbers by measuring convenient objects about the schoolroom by a yardstick or other units.

Units of Extension.

222. Extension is that which has one or more of the dimensions, *length*, *breadth*, and *thickness*.

There are three kinds of extensions; viz., *lines*, *surfaces*, and *solids*.

223. A line is an extension which has length only.

The units by which lines and lengths are measured are called **linear units.** The names and abbreviations of the units are *mile* (*mi.*), *rod* (*rd.*), *yard* (*yd.*), *foot* (*ft.*), and *inch* (*in.*).

Relation of the Units.

12 inches = 1 foot,
3 ft. = 1 yard,
5½ yd. = 1 rod,
320 rd. = 1 mile.

The following are sometimes used: 1 hand = 4 in., 1 pace = 3 ft., 1 fathom = 6 ft., 1 league = 3 mi., 1 barleycorn = ⅓ in., 1 furlong = ⅛ mi.

$$\left.\begin{array}{l}\text{60 geographical or}\\ \text{69⅙ statute miles}\\ \quad\text{(nearly)}\end{array}\right\} = \text{1 degree} \left\{\begin{array}{c}\text{of latitude on the meridian,}\\ \textit{or,}\\ \text{of longitude on the equator.}\end{array}\right.$$

224. Surveyors' linear units are also used in measuring lines and lengths, especially the boundaries of lands. They are *mile* (*mi.*), *chain* (*ch.*), *rod* (*rd.*), *link* (*l.*), and *inch* (*in.*).

Relation of the Units.

7.92 inches = 1 link,
25 l. = 1 rod,
4 rd. = 1 chain,
80 ch. = 1 mile.

The chain consists of 100 links, and is 4 rods or 66 feet long. Hence 1 rd. = .25 of a chain, and 1 l. = .01 of a chain.

EXERCISES (Oral).

225. *1.* How many inches are there in 2 ft.? In 5 ft.? In ½ ft.? In ⅓ ft.? In 1 yd.? In ¼ yd.?

2. How many feet in 24 in.? In 60 in.? In 1 yd.? In ⅔ yd.? In 5 yd.? In 7⅓ yd.? In 6 in.?

3. How many yards in 9 ft.? In 14 ft.? In 2 ft.? In 36 in.? In 108 in.? In 12 in.? In 10 rd.?

4. How many rods in 3 ch.? In 7 ch.? In ½ ch.? In 11 yd.? In 33 ft.? In ½ mi.? In ¼ mi.?

5. How many chains in 2 mi.? In ½ mi.? In ¼ mi.? In 325 l.? In 750 l.? In 10 rd.? In 17 rd.?

6. How many feet high is a horse that measures 15 hands? 16¼ hands?

7. A body of water is 4½ fathoms deep. What is its depth in yards?

8. What part of 1 ft. is 1 in.? What part of 1 yd. is 1 ft.?

What part:

9. Of 1 rd. is 1 yd.? Of 1 mi. is 1 rd.? Of 1 yd. is 1 in.?

10. Of 1 rd. is 1 ft.? Of 1 mi. is 1 ch.? Of 1 rd. is 1 ch.?

226. Reduction is the process of changing a denominate number from one unit to another without altering its value.

227. Reduction descending is changing from a higher to a lower unit.

Thus, 2 yd. may be changed to 6 ft., and 6 ft. to 72 in. This is reduction descending.

228. Reduction ascending is changing from a lower to a higher unit.

Thus, 60 in. may be changed to 5 ft., and 5 ft. to 1⅔ yd. This is reduction ascending.

229. A denominate fraction is a fractional part of a standard unit.

Thus, ¾ of a yard, which is written ¾ yd., is a denominate fraction.

230. ***To perform reduction descending.***

Reduction descending is performed by multiplication.

Reduce 2 yd. 1 ft. 9 in. to inches.

Operation.

$$\begin{array}{l} 2\text{ yd. } 1\text{ ft. } 9\text{ in.} \\ 3 \\ \hline 6 \\ 1 \\ \hline 7 \\ 12 \\ \hline 84 \\ 9 \\ \hline 93 \end{array}$$

Explanation.—Since there are 3 feet in 1 yard, in 2 yards there are 2 times 3 feet = 6 feet; and 6 feet + 1 foot = 7 feet.

Since there are 12 inches in 1 foot, in 7 feet there are 7 times 12 inches = 84 inches; and 84 inches + 9 inches = 93 inches.

Hence 2 yd. 1 ft. 9 in. = 93 inches.

RULE.—*Multiply the units of the highest denomination given by that number of the next lower denomination which equals one of this higher, and to the product add the units of the lower denomination.*

In like manner, reduce this result to units of the next lower denomination; and so continue until the given number is reduced to units of the required denomination.

EXERCISES (Written).

231. Reduce to yards:

1. 6 rd. 2 yd. *2.* 8 rd. 1 yd.
3. 5 rd. 1 yd. *4.* 2 mi. 46 rd. 1 yd.
5. 5 mi. 13 yd. *6.* 3 mi. 101 rd. $2\frac{1}{2}$ yd.

Reduce to feet:

7. 4 rd. 2 yd. 2 ft. *8.* 7 rd. $1\frac{1}{2}$ yd.
9. 8 rd. 1 yd. 1 ft. *10.* 14 rd. 2 ft.
11. 2 mi. 4 yd. 2 ft. *12.* 20 mi. 3 yd.

Reduce to inches:

13. 5 yd. 1 ft. 7 in. *14.* 2 rd. 4 yd. 1 ft. 9 in.
15. 10 rd. 2 ft. 5 in. *16.* 9 rd. $3\frac{1}{2}$ yd. 2 ft. 3 in.
17. 9 rd. 1 yd. 4 in. *18.* 3 mi. 70 rd. 5 yd. 1 ft.
19. 12 mi. 3 rd. 1 ft. *20.* 10 mi. 4 yd. 1 ft. 11 in.

Reduce to links:

21. 4 ch. 2 rd. 12 l. *22.* 3 mi. 60 ch. 3 rd. 20 l.
23. 5 mi. 45 ch. 3 rd. *24.* 10 mi. 1 rd. 15 l.

25. Reduce $\frac{3}{4}$ rd. to units of lower denominations.

SOLUTION.

$\frac{3}{4}$ of 1 rd. = $\frac{3}{4}$ of $5\frac{1}{2}$ yd. = $\frac{33}{8}$ yd. = $4\frac{1}{8}$ yd.
$\frac{1}{8}$ of 1 yd. = $\frac{1}{8}$ of 3 ft. = $\frac{3}{8}$ ft.
$\frac{3}{8}$ of 1 ft. = $\frac{3}{8}$ of 12 in. = $\frac{9}{2}$ in. = $4\frac{1}{2}$ in.
Hence $\frac{3}{4}$ rd. = 4 yd. $4\frac{1}{2}$ in.

Reduce to units of lower denominations:

26. $\frac{3}{4}$ yd.	**27.** $\frac{7}{9}$ yd.	**28.** $\frac{3}{11}$ mi.	**29.** $\frac{3}{8}$ ch.
30. $\frac{3}{7}$ rd.	**31.** $\frac{5}{9}$ mi.	**32.** $\frac{3}{16}$ ch.	**33.** $\frac{7}{12}$ mi.

34. Reduce .625 yd. to feet and inches.

SOLUTION.

.625 of 1 yd. = .625 of 3 ft. = 1.875 ft.
.875 of 1 ft. = .875 of 12 in. = 10.500 in.
Hence .625 yd. = 1 ft. 10.5 in.

$$\begin{array}{r} .625 \\ 3 \\ \hline 1.875 \\ 12 \\ \hline 10.500 \end{array}$$

Reduce to units of lower denominations:

35. .375 yd.	**36.** .725 yd.	**37.** .98 rd.	**38.** .275 rd.
39. .625 ch.	**40.** .875 mi.	**41.** .645 yd.	**42.** .8045 mi.

232. ***To perform reduction ascending.***

Reduction ascending is performed by division.

1. Reduce 917 inches to rods, etc.

EXPLANATION. — Since there are 12 inches in 1 foot, in 917 inches there are as many feet as 12 in. is contained times in 917 inches, or 76 ft. and 5 in.

Since there are 3 feet in 1 yard, in 76 feet there are as many yards as 3 feet is contained times in 76 feet, or 25 yards and 1 foot.

Since there are $5\frac{1}{2}$ yards in 1 rod, in 25 yards there are as many rods as $5\frac{1}{2}$ yards is contained times in 25 yards, or as many as 11 half-yards is contained times in 50 half-yards, or 4 rods and 6 half-yards, or 3 yards.

Hence 917 in. = 4 rd. 3 yd. 1 ft. 5 in.

OPERATION.

$$\begin{array}{r|l} 12 & 917 \\ \hline 3 & 76 + 5 \text{ in.} \\ \hline 5\frac{1}{2} & 25 + 1 \text{ ft.} \\ & 2 \\ \hline 11 & 50 \\ \hline & 4 + 6 \text{ half-yds.} \end{array}$$

ANS.—4 rd. 3 yd. 1 ft. 5 in.

RULE.—*Divide the given number by that number of the same denomination which equals one of the next higher, writing the quotient as units of the higher denomination and the remainder as units of the denomination divided.*

In like manner, reduce this quotient to units of the next *higher denomination; and so continue until the given number is reduced to units of the required denomination.*

The last quotient, with the several remainders, is the required result.

EXERCISES (Written).

233. Reduce to higher denominations:

1. 147 in.	*2.* 332 in.	*3.* 48121 in.
4. 275 l.	*5.* 337 ft.	*6.* 19870 ft.
7. 564 yd.	*8.* 443 ch.	*9.* 70287 in.
10. 6388 yd.	*11.* 1264 l.	*12.* 641558 in.

13. Reduce $\frac{5}{8}$ in. to the fraction of a rod.

EXPLANATION.

Since 1 in. $= \frac{1}{12}$ ft., $\frac{5}{8}$ in. $= \frac{5}{8}$ of $\frac{1}{12}$ ft. $= \frac{5}{96}$ ft.
Since 1 ft. $= \frac{1}{3}$ yd., $\frac{5}{96}$ ft. $= \frac{5}{96}$ of $\frac{1}{3}$ yd. $= \frac{5}{288}$ yd.
Since 1 yd. $= \frac{2}{11}$ rd., $\frac{5}{288}$ yd. $= \frac{5}{288} \times \frac{2}{11}$ rd.
$= \frac{5}{1584}$ rd.

OPERATION.

$$\frac{2}{11} \times \frac{1}{3} \times \frac{1}{12} = \frac{5}{1584}.$$

Reduce:

14. $\frac{3}{4}$ in. to the fraction of a yd.
15. $\frac{3}{8}$ ft. to rods.
16. $\frac{3}{5}$ in. to the fraction of a rd.
17. $\frac{5}{7}$ ft. to rods.
18. $\frac{11}{16}$ ft. to the fraction of a mi.
19. $\frac{33}{40}$ yd. to miles.
20. $\frac{3}{8}$ ch. to the fraction of a mi.
21. $\frac{202}{4}$ ft. to miles.

22. Reduce 4 yd. 1 ft. 6 in. to the decimal of a rod.

EXPLANATION.

6 in. $= \frac{6}{12}$ ft. $= .5$ ft.
1.5 ft. $= \frac{1.5}{3}$ yd. $= .5$ yd.
4.5 yd. $= \frac{4.5}{5\frac{1}{2}}$ yd. $= \frac{9}{11}$ yd. $= .8181+$ yd.

OPERATION.

$$\begin{array}{r|l}
12 & \underline{6.0} \\
3 & \underline{1.5} \\
5\frac{1}{2} & 4.5 \\
 & \underline{2} \\
11 & \underline{9.0} \\
 & .8181+
\end{array}$$

Reduce:

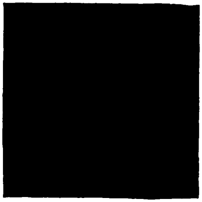
23. 2 yd. 2 ft. 9 in. to the decimal of a rod.
24. 3 yd. 1 ft. 6 in. to the decimal of a rod.
25. 36 rd. 2 yd. 1 ft. to the decimal of a mile.
26. 25 ch. 2 rd. 5 l. to the decimal of a mile.
27. 120 rd. 1 yd. 2 ft. 3 in. to the decimal of a mile.

234. A **surface** is an extension which has length and breadth only; as the face of a blackboard.

235. A **square** is a flat surface having four equal sides and four square corners.

236. A **unit of surface** is a square each side of which is a linear unit.

Thus, a square inch is a square each of whose sides is one inch long.



A SQUARE INCH.

237. The units by which surfaces are measured are called **square units.** They are *square mile* (*sq. mi.*), *acre* (*A.*), *square rod* (*sq. rd.*), *square yard* (*sq. yd.*), *square foot* (*sq. ft.*), and *square inch* (*sq. in.*).

Relation of the Units.

144 square inches = 1 square foot,
9 sq. ft. = 1 square yard,
30¼ sq. yd. = 1 square rod,
160 sq. rd. = 1 acre,
640 A. = 1 square mile.

238. Surveyors' square units are also used in measuring surfaces, especially in Government surveys. They are *township* (*Tp.*), *square mile* (*sq. mi.*), *acre* (*A.*), *square chain* (*sq. ch.*), *square rod* (*sq. rd.*), and *square link* (*sq. l.*).

Relation of the Units.

625 square links = 1 square rod,
16 sq. rd. = 1 square chain,
10 sq. ch. = 1 acre,
640 A. = 1 square mile,
36 sq. mi. = 1 township.

A square mile of land is also called a *section.*

EXERCISES (Oral).

239. *1.* How many sq. chains are there in 3 A.? In 5 A.? In ⅖ A.? In 32 sq. rd.? In 80 sq. rd.? In 50 sq. rd.?

2. How many sq. rods in 2 A.? In 10 A.? In ¾ A.?

3. How many sq. in. in 2 sq. ft.? In 5 sq. ft.? In ½ sq. ft.? In ⅓ sq. ft.?

4. How many sq. ft. in 288 sq. in.? In 72 sq. in.? In 36 sq. in.? In 3 sq. yd.? In ⅔ sq. yd.?

5. How many sq. rd. in 5 sq. ch.? In 7 sq. ch.? In ½ sq. ch.? In ⅜ sq. ch.?

6. How many acres in 40 sq. ch.? In 75 sq. ch.? In 2 sq. ch.? In 80 sq. rd.? In ½ sq. mi.? In ¼ sq. mi.? In ⅛ sq. mi.? In 1/16 sq. mi.?

7. How many sq. mi. in 2 Tp.? In ⅚ Tp.? In 320 A.? In 160 A.? In 40 A.?

8. What part of an acre is 1 sq. ch.? 1 sq. rd.? 8 sq. ch.? 100 sq. rd.?

EXERCISES (Written).

240. Reduce to square yards :

1. 2 A. 40 sq. rd.
2. 6 A. 60 sq. rd. 24 sq. yd.
3. 8 A. 10 sq. yd.
4. 1 sq. mi. 100 A. 16 sq. rd.

Reduce to square feet:

5. 9 sq. yd. 15 sq. ft.
6. 8 sq. rd. 4 sq. yd. 6 sq. ft.
7. 12 sq. rd. 18 sq. ft.
8. 3 A. 80 sq. rd. 10 sq. yd.
9. Reduce 6 sq. yd. 4 sq. ft. 10 sq. in. to square inches.
10. Reduce 2 sq. mi. 120 A. 9 sq. ch. to square chains.

Reduce to higher units :

11. 6040 sq. in.
12. 15270 sq. in.
13. 108000 sq. in.
14. 2216 sq. ft.
15. 10365 sq. ft.
16. 113240 sq. yd.
17. 5260 sq. yd.
18. 12300 sq. rd.
19. 540320 sq. ch.

Reduce to lower units:

20. $\frac{3}{4}$ sq. rd. *21.* $\frac{5}{6}$ sq. yd. *22.* $\frac{7}{12}$ sq. ch. *23.* $\frac{4}{9}$ sq. mi.
24. .7625 A. *25.* .84 sq. rd. *26.* .365 Tp. *27.* .0675 A.

28. Reduce $\frac{3}{8}$ of a sq. yd. to sq. rd.

SOLUTION.—1 sq. rd. = $30\frac{1}{4}$ or $\frac{121}{4}$ sq. yd.
1 sq. yd. = $\frac{4}{121}$ sq. rd.
$\frac{3}{8}$ sq. yd. = $\frac{3}{8}$ of $\frac{4}{121}$ sq. rd. = $\frac{3}{242}$ sq. rd.

29. Reduce $\frac{5}{12}$ sq. ft. to sq. rods.

30. Reduce $\frac{13}{18}$ sq. in. to sq. yards.

31. Reduce 8 sq. ch. 4 sq. rd. to the decimal of an acre.

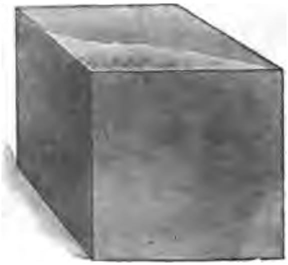
32. Reduce 1 sq. ft. 18 sq. in. to the decimal of a sq. yd.

241. A **solid** is an extension which has length, breadth, and thickness; as a box, a room, or a book.

242. A **cube** is a solid bounded by six equal squares, called faces. The sides of these faces are called **edges** of the cube.

243. A **unit of solids** is a cube, each edge of which is a linear unit.

Thus, a cubic inch is a cube each edge of which is 1 inch in length.



A CUBIC INCH.

244. The units by which solids are measured are called **cubic units.** They are *cord* (*cd.*), *cubic yard* (*cu. yd.*), *cubic foot* (*cu. ft.*), and *cubic inch* (*cu. in.*).

Relation of the Units.

1728 cubic inches = 1 cubic foot,
27 cu. ft. = 1 cubic yard,
128 cu. ft. = 1 cord.

A pile of wood 8 feet long, 4 feet wide, and 4 feet high is a cord. One foot in length of this pile, or 16 cu. ft., is a cord foot.

EXERCISES (Oral).

245. *1.* How many cu. ft. in 2 cu. yd.? In 5 cu. yd.? In $\frac{1}{3}$ cu. yd.? In $\frac{2}{9}$ cu. yd.?

2. How many cu. yd. in 81 cu. ft.? In 9 cu. ft.? 45 cu. ft.?

3. What part of cu. yd. is a cu. ft.? Of cu. ft. is a cu. in.?

4. What part of a cd. is 64 cu. ft.? 32 cu. ft.? 80 cu. ft.?

EXERCISES (Written).

246. Reduce to cubic inches:

1. 12 cu. ft. 100 cu. in.
2. 3 cu. yd. 20 cu. ft. 90 cu. in.
3. 40 cu. ft. 124 cu. in.
4. 5 cu. yd. 8 cu. ft. 135 cu. in.
5. 25 cu. yd. 860 cu. in.
6. 100 cd. 24 cu. ft. 745 cu. in.

Reduce to units of higher denominations:

7. 600 cu. ft.
8. 158342 cu. in.
9. 824 cu. ft.
10. 5876543 cu. in.

Reduce to lower units:

11. $\frac{5}{12}$ cu. ft.
12. $\frac{3}{4}$ cu. yd.
13. $\frac{5}{8}$ cd.
14. .675 cu. ft.
15. .875 cu. yd.
16. .62125 cd.

17. Reduce $\frac{9}{13}$ cu. ft. to a fraction of a cu. yard.

18. Reduce $\frac{24}{36}$ cu. in. to a fraction of a cu. foot.

19. Reduce 1 cu. ft. 459 cu. in. to the decimal of a cu. yard.

20. Reduce 92 cu. ft. 138.24 cu. in. to the decimal of a cord.

Units of Capacity.

247. Capacity signifies extent of room or capability of holding.

Units of capacity are of two kinds: *units of liquids* and *units of dry substances.*

248. Liquid units are used for measuring all kinds of liquids. They are *gallon* (*gal.*), *quart* (*qt.*), *pint* (*pt.*), and *gill* (*gi.*).

Relation of the Units.

4 gills = 1 pint,
2 pt. = 1 quart,
4 qt. = 1 gallon.

In measuring cisterns, etc.:
31½ gal. = 1 barrel (bar. or bbl.).
63 gal. = 1 hogshead (hhd.).

A gallon contains 231 cubic inches. The barrel and hogshead are not fixed quantities, but vary when used for commercial purposes.

EXERCISES (Oral).

249. *1.* How many gills in 2 pt.? In 5 pt.? In 1 qt.? In 3 qt.? In 1 gal.? In 4 gal.? In ½ gal.? In 3½ gal.? In ¾ qt.?

2. How many pints in 12 gills? In 20 gi.? In 3 qt.? In 7 qt.? In ½ qt.? In 3 gal.? In ½ gal.? In 2 gi.?

3. How many quarts in 4 pt.? In 20 pt.? In 2 gal.? In 10 gal.? In ½ gal.? In 32 gi.? In 12 gi.?

4. How many gallons in 64 gi.? In 20 qt.? In 30 pt.? In 2 bbl.? In 1/9 hhd.? In 3/7 hhd.?

What part:

5. Of 1 pt. is 1 gi? Of 1 pt. is 3 gi.? Of 1 pt. is 2 gi.?

6. Of 1 qt. is 1 pt.? Of 1 qt. is 1 gi.? Of 1 qt. is 3 gi.?

7. Of 1 gal. is 1 qt.? Of 1 gal. is 1 pt.? Of 1 bar. is 1 gal.?

250. Apothecaries' fluid units are used for measuring all liquids that enter into the composition of medical prescriptions. They are *gallon* (*cong.*), *pint* (*O*), *fluid ounce* (*f*℥), *fluid drachm* (*f*ʒ), and *minim* (♏).

Relation of the Units.

60 minims = 1 fluid drachm,
8 fʒ = 1 fluid ounce,
16 f℥ = 1 pint,
8 O = 1 gallon.

251. Dry units are used for measuring quantities that are not liquid; such as grain, potatoes, etc.

The units are *bushel* (*bu.*), *peck* (*pk.*), *quart* (*qt.*), and *pint* (*pt.*).

Relation of the Units.

2 pints = 1 quart,
8 qt. = 1 peck,
4 pk. = 1 bushel.

A bushel is 18½ inches in diameter and 8 inches deep, and contains 2150.42 cubic inches.

EXERCISES (Oral).

252. *1.* How many pints in 3 qt.? In 5 qt.? In ½ qt.? In 4 pk.? In ½ pk.? In ¾ pk.? In 1 bu.? In 1 pk. 2 qt.?

2. How many quarts in 8 pt.? In 20 pt.? In 11 pt.? In 2 pk.? In 7 pk.? In ¼ pk.? In 3½ pk.? In 2 bu.? In ¾ bu.?

3. How many pecks in 16 qt.? In 36 qt.? In 16 pt.? In 48 pt.? In 2 bu.? In 5¼ bu.? In 2 bu. 3 pk.?

4. How many bushels in 16 pk.? In 40 pk.? In 2 pk.? In 32 qt.? In 30 pk.? In 128 pt.?

EXERCISES (Written).

253. ***Reduce to gills, and also to the decimal of a gallon:***

1. 3 qt. 1 pt. 3 gi.
2. 1 qt. 1 pt. 1 gi.
3. 2 qt. 1 pt. 1 gi.
4. 3 qt. 1 pt. 2 gi.

Reduce to pints, and also to the decimal of a bushel:

5. 3 pk. 5 qt. 1 pt.
6. 2 pk. 3 qt. 1 pt.
7. 1 pk. 7 qt. 1 pt.
8. 3 pk. 4 qt. 1 pt.

Reduce to lower units:

9. $\frac{13}{16}$ gal.
10. $\frac{7}{10}$ qt.
11. .875 gal.
12. $\frac{5}{12}$ bu.
13. ⅜ pk.
14. .635 bu.

15. How many gallons in 15 gal. 3 qt. 1 pt.?

16. How many bushels in 24 bu. 2 pk. 5 qt.?

Reduce to higher units (liquid):

17. 1980 gi. *18.* 1236 pt. *19.* 2305 gi.
20. 1073 gi. *21.* 971 qt. *22.* 1101 gi.

Reduce to higher units (dry):

23. 105 pk. *24.* 1027 qt. *25.* 2353 pt.
26. 623 pk. *27.* 7250 qt. *28.* 8456 pt.

29. Reduce $\frac{8}{9}$ pt. to the fraction of a gallon.
30. Reduce $\frac{12}{25}$ pt. to the fraction of a bushel.

Units of Weight.

254. Weight is the measure of the force which attracts bodies to the earth.

255. Avoirdupois weight is used in weighing all coarse and heavy articles, such as coal, grain, and cattle, and all metals except gold and silver. The units are *ton* (*T.*), *pound* (*lb.*), and *ounce* (*oz.*).

Relation of the Units.

16 ounces = 1 pound,
2000 lb. = 1 ton.

The avoirdupois pound contains 7000 Troy grains. The long ton of 2240 pounds has almost gone out of use in the United States.

A hundredweight (cwt.) is 100 pounds.

EXERCISES (Oral).

256. *1.* How many ounces in 2 lb.? In 5 lb.? In $\frac{1}{2}$ lb.? In $2\frac{1}{4}$ lb.?

2. How many pounds in 48 oz.? In 64 oz.? In 4 oz.?

3. How many tons in 4000 lb.? In 12000 lb.? In 500 lb.?

257. Troy weight is used in weighing precious stones, gold, silver, etc. The units are *pound* (*lb.*), *ounce* (*oz.*), *pennyweight* (*pwt.*), and *grain* (*gr.*).

Relation of the Units.

24 grains = 1 pennyweight,
20 pwt. = 1 ounce,
12 oz. = 1 pound.

The Troy pound contains 5760 grains.

The carat, which = 3.168 grains, is a weight used in weighing diamonds and other jewels. The term carat is also used to indicate the fineness of gold. Thus, gold that is "18 carats fine" is $\frac{18}{24}$ gold and $\frac{6}{24}$ alloy.

EXERCISES (Oral).

258. *1.* How many grains in 3 pwt.? In $\frac{1}{2}$ pwt.? In $\frac{5}{8}$ pwt.?

2. How many pennyweights in 48 gr.? In 12 gr.? In 8 gr.? In 3 oz.? In $\frac{1}{4}$ oz.? In $2\frac{1}{2}$ oz.? In $1\frac{3}{4}$ oz.?

3. How many ounces in 60 pwt.? In 80 pwt.? In 10 pwt.? In 4 pwt.? In 3 lb.? In $\frac{2}{3}$ lb.?

4. How many pounds in 24 oz.? In 50 oz.? In 18 oz.?

259. Apothecaries' weight is used in prescribing and compounding dry medicines. The units are: ***pound*** (*lb.* or ℔), ***ounce*** (℥ or *oz.*), ***scruple*** (℈ or *scr.*), ***dram*** (ʒ or *dr.*), and ***grain*** (*gr.*).

Relation of the Units.

20 grains = 1 scruple,
3 ℈ = 1 dram,
8 ʒ = 1 ounce,
12 ℥ = 1 pound.

The pound, ounce, and grain of this weight are the same as those of Troy weight.

Medicines are bought and sold in large quantities by avoirdupois weight.

EXERCISES (Oral).

260. *1.* How many grains in 2 ℈? In 7 scr.? In $\frac{1}{2}$ ℈? In $\frac{4}{5}$ ℈? In $2\frac{1}{4}$ ℈?

2. How many scruples in 60 gr.? In 40 gr.? In 70 gr.? In 15 gr.? In 2 ʒ? In $3\frac{1}{3}$ ʒ? In 5 dr. 2 scr.?

3. How many drams in 4 ℈? 24 ℈? In 2 ℥? In 5 ℥? In ½ ℥? In ¾ ℥?

4. How many ounces in 16 ʒ? In 30 ʒ? In 5 lb.? In 4⅔ lb.? In 5 lb. 3 ℥?

5. How many pounds in 48 ℥? In 72 ℥? In 9 ℥?

EXERCISES (Written).

261. Reduce to ounces, and to the decimal of a ton:

1. 5 T. 16 lb. 12 oz.
2. 14 lb. 14 oz.
3. 8 T. 54 lb. 10 oz.
4. 72 lb. 11 oz.

Reduce to grains, and to the decimal of a pound:

5. 9 oz. 10 pwt. 12 gr.
6. 8 oz. 6 pwt.
7. 11 oz. 18 pwt. 6 gr.
8. 10 oz. 15 gr.
9. 6 ℥ 5 ʒ 2 ℈ 8 gr.
10. 8 ℥ 1 ℈ 6 gr.
11. 4 ℥ 3 ʒ 1 ℈ 12 gr.
12. 5 ʒ 2 ℈ 8 gr.

Reduce to higher units (Troy):

13. 1233 oz.
14. 2025 pwt.
15. 9327 gr.

Reduce to higher units (avoirdupois):

16. 1233 cwt.
17. 3561 lb.
18. 9327 oz.

Reduce to higher units (apothecary):

19. 573 ʒ.
20. 1265 ℈.
21. 9327 gr.

262. ***Comparison of weights.***

1 lb. Troy = 5760 gr. Troy.
1 lb. Avoir. = 7000 gr. Troy.
1 oz. Troy = 480 gr. Troy.
1 oz. Avoir. = 437½ gr. Troy.
175 lb. Troy = 144 lb. Avoir.

Time.

263. Time is a limited portion of duration.

264. The **units** by which time is measured are the *century* (*Cen.*), the *common year* (*yr.*), the *leap year* (*L. yr.*), the *month* (*mo.*), the *week* (*wk.*), the *day* (*da.*), the *hour* (*hr.*), the *minute* (*min.*), and the *second* (*sec.*).

Relation of the Units.

60 sec. = 1 min.
60 min. = 1 hr.
24 hr. = 1 da.
7 da. = 1 wk.

52 wk. 1 da., or 365 da. = 1 yr.
366 da. = 1 L. yr.
100 yr. = 1 Cen.

265. The **civil year** is divided into twelve calendar months, thus :

January (Jan.)	1st mo..31 da.	July (July)	7th mo..31 da.
February (Feb.)	2d mo..28 da.	August (Aug.)	8th mo..31 da.
March (Mar.)	3d mo..31 da.	September (Sep.)	9th mo..30 da.
April (Apr.)	4th mo..30 da.	October (Oct.)	10th mo..31 da.
May (May)	5th mo..31 da.	November (Nov.)	11th mo..30 da.
June (June)	6th mo..30 da.	December (Dec.)	12th mo..31 da.

In most business transactions, 30 days are counted as a month, and 12 months as a year.

A solar year, or the time of one revolution of the earth around the sun, is 365 da. 5 hr. 48 min. 46 sec., *nearly*. Now it is clearly most convenient to reckon by years which contain an exact number of days. Hence to some years are assigned 365 days and to others 366 days, in such a manner that the average length of a day is equal to that of the solar year. This end is nearly attained by the following

266. RULE.—*Centennial years exactly divisible by* 400, *and other years exactly divisible by* 4, *are leap years, and all other years are common years.*

Thus, taking the 400 years from 1601 to 2000, inclusive, every fourth year, except 1700, 1800, 1900, is a leap year, and the others, including 1700, 1800, 1900, are common years. That is, of the 400 years, 97 are leap years and 303 are common years. Hence there are in all

$$97 \times 366 \text{ days} + 303 \times 365 \text{ days} = 146097 \text{ days}.$$

Dividing this by 400, we get the average length of one civil year for that time ; viz., 365 da. 5 hr. 49 min. 12 sec., which is only 26 sec. more than the solar year.

The civil day begins and ends at 12 o'clock, midnight. A.M. denotes before noon, M. noon, and P.M. after noon.

EXERCISES (Oral).

267. *1.* How many hours from 7:30 A.M. to 6:45 P.M.?

2. How many seconds in 3 min.? In 5 min.? In $\frac{3}{4}$ min.?

3. How many minutes in 120 sec.? In 90 sec.? In $2\frac{1}{2}$ hr.?

4. How many hours in 180 min.? In 640 min.? In $1\frac{3}{4}$ da.?

5. How many days in 96 hr.? In 54 hr.? In 5 wk.?

What part:

6. Of 1 min. is 1 sec.? Of 1 hr. is 1 min.? Of 1 da. is 1 hr.?

7. Of 1 min. is 10 sec.? Of 1 hr. is 15 min.? Of 1 day is 3 hr.?

8. Of 1 yr. is 1 day? Of 1 L. yr. is 1 day? Of 1 mo. is 6 da.?

9. When are the days the longer, when the sun rises at 7 and sets at 5, or rises at 5 and sets at 7?

10. Which of the following were leap years: 1800, 1884, 1892, 1898, 1900, 1902?

EXERCISES (Written).

268. Reduce to lower units:

1. 4 hr. 20 min. 12 sec.
2. 5 wk. 3 da.
3. 7 hr. 32 min. 45 sec.
4. 3 wk. 5 hr.
5. 4 yr. 140 da. 25 min.
6. 4 hr. 8 sec.
7. $\frac{5}{8}$ of an hour.
8. .745 of a day.

Reduce to higher units:

9. 375 min.
10. 461 da.
11. 564375 sec.
12. 843 hr.
13. 500 hr.
14. 903117 sec.

15. Reduce $\frac{3}{8}$ of a minute to the fraction of a day.

16. Reduce 6 hr. 15 min. 45 sec. to the decimal of a day.

17. Reduce 8 min. 15 sec. to the decimal of an hour.

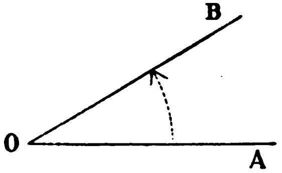
18. How many days, hr., etc., in .4715625 of a week?

19. What is the length of the day (1) when the sun rises at 5:7 A.M. and sets at 7:5 P.M.? (2) When the sun rises at 7:5 A.M. and sets at 5:7 P.M.

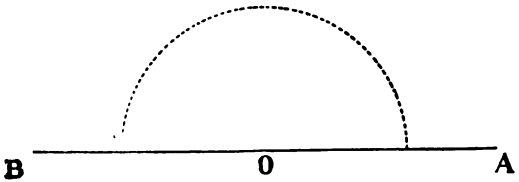
Angular or Circular Units.

269. Two lines starting from the same point form an **angle.** The lines are called **sides** of the angle.

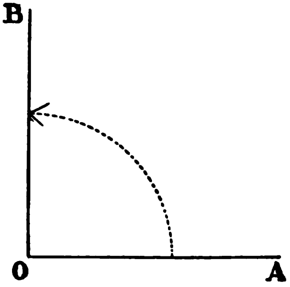
270. *The size of an angle depends on the extent of the opening between its sides.*



Thus, let OA and OB be the sides of the angle AOB; and suppose OB, starting from coincidence with OA, to revolve about the point O (that is, turn upon O as the leg of a pair of dividers turns upon its hinge). During this revolution the *opening* or *angle* between the moving side OB and the fixed side OA increases with the amount of turning, or rotation; and by continuing the revolution an angle of any magnitude may be described or generated.

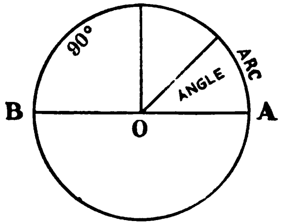


When the line OB has rotated from OA until its direction is opposite to that of OA, the angle described (AOB) is called a **straight angle.**



When the line OB has turned from OA through $\frac{1}{2}$ of a straight angle, the angle described (AOB) is called a **right angle,** and the line BO is said to be **perpendicular** to the line OA.

When the line OB has turned from OA through $\frac{1}{90}$ of a right angle, the angle described is called an angle of **one degree.**



CIRCLE.

271. A **circle** is a plane figure bounded by a curved line, called the circumference, every point of which is equally distant from a point within, called the center.

272. An **arc** is a part of the circumference.

273. The **diameter** of a circle is a straight line drawn through its center and terminated both ways by the circumference; as the line AB.

274. The **radius** of a circle is half its diameter; as OA.

275. A **degree** is 1 of the 360 equal arcs into which the circumference of a circle may be divided.

276. An **angle of one degree** is 1 of the 360 equal angles that exactly fill the surface about a common point in a plane.

An angle of 1 degree is constant, but the degree itself varies with every change in the size of the circle.

277. Angular or **circular units** are used in measuring angles, determining latitude and longitude, etc.

The names of the units are *circumference* (*cir.*), *degree* (°), *minute* ('), and *second* (").

Relation of the Units.

60 seconds = 1 minute,
60′ = 1 degree,
360° = 1 circumference.

A **sign** is an arc of 30° ; a **sextant** is an arc of 60° ; a **quadrant** is an arc of 90° ; and a **semi-circumference** is an arc of 180°. A **right angle** is an angle of 90°, and a **straight angle** is an angle of 180°.

EXERCISES (Oral).

278. *1.* How many minutes in 3°? In 8°? In $\frac{3}{8}$°? In 90″?

2. How many seconds in 5′? In 7′? In $\frac{5}{8}$′? In $2\frac{1}{2}$′?

3. How many degrees in 2 cir.? In $\frac{1}{2}$ cir.? In 180′? In 3 quad.?

4. How many signs in 1 cir.? In 1 quadrant? In 100°?

EXERCISES (Written).

279. Reduce:

1. 14′ 25″ to seconds. *2.* 5° 20′ 35″ to seconds.
3. 10° 30″ to seconds. *4.* 4 cir. 18′ to minutes.

5. 30′ 18″ to the decimal of a degree.
6. 19° 18′ 36″ to the decimal of a circumference.
7. $\frac{7}{8}$ of a degree to min., etc.
8. $\frac{5}{64}$ of a sign to degrees, etc.
9. .1255 of a quadrant to degrees, etc.

Reduce to higher units:

10. 4021″. *11.* 2547′. *12.* 596250″.
13. 3520°. *14.* 9875′. *15.* 809341″.

Units of Value.

280. **Value** is the worth of a commodity or property.

281. Value is measured by money.

Money is adopted for the sake of convenience to facilitate the exchange of one kind of wealth or property for another, and as a standard of value. Money, which is generally called *currency*, is therefore a circulating medium of exchange.

Coin or **specie** is stamped pieces of metal having a value fixed by law.

Paper money is notes and bills issued by a government or bank, and authorized to be used as money.

282. United States money, or Federal money, is the currency of the United States. The units are *eagle* (*e.*), *dollar* ($), *dime* (*d.*), *cent* (*ct.*), and *mill* (*m.*).

Relation of the Units.

10 mills = 1 cent,
10 ct. = 1 dime,
10 d. = 1 dollar,
$10 = 1 eagle.

The following coins are now issued by the U. S. Government:

Gold: the double eagle ($20), the eagle ($10), the half-eagle ($5), and the quarter-eagle ($\$2\frac{1}{2}$).

Silver: the dollar ($1), the half-dollar ($\$\frac{1}{2}$), the quarter-dollar ($\$\frac{1}{4}$), and the dime ($.10 or $\$\frac{1}{10}$).

Nickel: the 5-cent piece ($\$\frac{1}{20}$ or $.05).

Bronze: the 1-cent piece ($\$\frac{1}{100}$ or $.01).

There are various other coins of the United States in circulation, such as the 3-dollar and 1-dollar gold pieces, and the 5-cent silver piece, but they are not now coined.

The standard weight of a gold dollar is 25.8 grains, and the weight of the silver dollar is $412\frac{1}{2}$ grains.

The standard purity of the gold and silver coins is by weight 9 parts of pure metal and 1 part alloy. Hence a gold dollar contains 23.22 grains of pure gold, and a silver dollar contains 371.25 grains of pure silver.

283. English or Sterling money is the currency of Great Britain. The units are the *pound* or *sovereign* (*£.*), the *shilling* (*s.*), the *penny* (*d.*), and the *farthing* (*far.* or *qr.*).

Relation of the Units.

4 farthings = 1 penny,
12d. = 1 shilling,
20s. = 1 pound.

The pound sterling is represented by a gold coin, called the sovereign (sov.), whose value in U. S. money is $4.8665.

284. French money is expressed in *francs* and *centimes;* a franc being 100 centimes.

A franc has the value of $0.193, and about 5.18 francs are equivalent to a dollar.

285. The **money of the German Empire** is expressed in *marks* (reichmarken) and *pennies* (pfennige); a mark being 100 pennies.

A mark is equivalent to $0.238, and 4 marks are about 95 cents.

EXERCISES (Oral).

286. *1.* How many farthings in 3d.? In 5d.? In 1s.? In $\frac{3}{4}$d.?

2. How many pence in 12 far.? In 17 far? In 2s.? In £1?

3. How many shillings in 36d.? In 9d.? In £3? In £$\frac{4}{6}$?

4. How many pounds in 40s.? In 70s.? In 5s.? In 12s.?

5. How many centimes in 3 francs? In 5$\frac{1}{2}$ francs?

6. How many francs in 600 centimes? In 275 centimes?

EXERCISES (Written).

287. Reduce:

1. 10s. 8d. to far. *2.* £5 12s. 7d. to d.
3. 4s. 3 far. to far. *4.* £6 8s. 3$\frac{1}{4}$d. to far.
5. £5 2$\frac{3}{4}$d. to far. *6.* £2 6s. 3 far. to far.
7. 7s. 9d. to the decimal of a pound.
8. 5s. 6d. 3 far. to the decimal of a pound.
9. $\frac{5}{18}$ of £ to shillings, etc.
10. .440625 of £ to shillings, etc.

Reduce to higher units:

11. 321d. *12.* 137s. *13.* 25364 far.
14. 485s. *15.* 877d. *16.* 83507 far.

Find the value of the following in U. S. money:

17. £5 8s. 6d.

Solution.—£5 8s. 6d. = £5.425.
£1 = $4.8665.

Therefore, £5 8s. 6d = 5.425 × $4.8665 = $26.40.

18. £10 8s. *19.* 6s. 3d. *20.* £20 12s. 3d. 3 far.
21. 5d. 1 far. *22.* £7 6d. *23.* £18 14s. 7d. 2 far.
24. 20 francs 15 centimes. *25.* 12 marks 5 pennies.
26. 16 francs 50 centimes. *27.* 25 marks 40 pennies.

Find the value of the following in English money:

28. $126.164. *29.* $152.422. *30.* $89.868.
31. $1000. *32.* $249.651. *33.* $615.86.

Find the value of the following in French and German money:

34. $624.50. *35.* $27.185. *36.* $1000.

288. Miscellaneous Units.

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

The old **French linear** and **land measure** is still partly used in Louisiana, and in other French settlements of the United States.

Table.

12 lines = 1 inch.
12 inches = 1 foot.

6 feet = 1 toise.
32 toises = 1 arpent.

1024 sq. toises = 1 sq. arpent.

The French foot equals 12.79 English inches.

The arpent is the old French name for acre, and is equal to about $\frac{5}{6}$ of an English acre.

ADDITION.

289. ***To add two or more denominate numbers:***

Add 5 yd. 1 ft. 5 in., 2 yd. 2 ft. 7 in., 3 yd. 2 ft. 9 in., and 1 ft. 6 in.

OPERATION.

5 yd.	1 ft.	5 in.
2	2	7
3	2	9
	1	6
2 rd. 1. yd.	2 feet	3 in.

Write the parts with like-named numbers in the same columns, and begin at the right to add. The sum of the inches is 27 in., or 2 ft. 3 in.; write the 3 inches below. The sum of the 2 ft. and the numbers in the column of feet is 8 ft., or 2 yd. 2 ft.; write the 2 ft. below. The sum of the 2 yd. and the numbers in the column of yards is 12 yd., or 2 rd. 1 yd., which write below.

EXERCISES (Written).

290. Find the sum of:

1.

rd.	yd.	ft.
20	3	1
18	2	2
32	4	2

2.

s.	d.	far.
16	9	3
25	0	2
8	5	1

3.

lb.	℥	ʒ
12	8	7
14	5	0
21	6	5

4.

mi.	rd.	yd.	ft.
10	100	4	2
18	182	5	1
24	16	0	1
33	108	3	0

5.

mi.	ch.	rd.	l.
24	70	3	20
36	65	2	18
	40	0	12
21	37	3	0

6. Find the sum of £8 15s. 8d., £5 12s. 9d., 16s. 6d., £3 11d. 3 far., and 7s. 2d. 2 far.

7. Find the sum of 5 yr. 124 da. 17 hr. 25 min., 7 yr. 265 da. 11 hr. 30 min. 31 sec., 175 da. 42 min. 37 sec.

8. What is the sum of 4 bu. 2 pk. 7 qt., 5 bu. 3 pk. 1 pt., 6 bu. 5 pk. 4 qt. 1 pt., and 10 bu. 5 qt.?

9. Add 3 T. 10 cwt. 25 lb., 4 T. 13 cwt. 54 lb. 11 oz., 18 cwt. 7 lb. 8 oz., 6 T. 43 lb., and 1 T. 7 cwt. 13 oz.

10. Find the sum of $\frac{5}{9}$ lb., $\frac{1}{6}$ oz., and $4\frac{1}{2}$ pwt.

RULE. — *Reduce each fraction to integers of lower denominations, and add the results.*

OPERATION.

$\frac{5}{9}$ lb.	=	6 oz.	13 pwt.	8 gr.
$\frac{1}{6}$ oz.	=		3	8
$4\frac{1}{2}$ pwt.	=		4	12
Sum	=	7 oz.	1 pwt.	4 gr.

Find the sum of:

11. £$\frac{5}{6}$, $\frac{3}{8}$s., and $5\frac{1}{2}$d.

12. $2\frac{3}{4}$ rd. and $1\frac{3}{4}$ ft.

13. $3\frac{3}{8}$ T., $4\frac{4}{5}$ cwt., $\frac{3}{4}$ lb.

14. $\frac{3}{8}$ mi. and $17\frac{1}{6}$ rd.

15. $\frac{5}{6}$ wk., $\frac{7}{8}$ da., and $\frac{3}{4}$ hr.

16. $7\frac{1}{6}$ wk. and $\frac{5}{8}$ da.

17. $\frac{3}{5}$ C., $\frac{3}{8}$ S., and $\frac{5}{6}$°.

18. .365 da. and .75 hr.

19. A lad walked $2\frac{2}{3}$ mi. one day, $3\frac{5}{8}$ mi. the next day, and $120\frac{3}{8}$ rd. the next. How far did he walk in all ?

20. A farmer has wheat in three bins: the first contains $10\frac{3}{8}$ bu., the second 12 bu. $3\frac{5}{16}$ pk., and the third 9 bu. 2 pk. $7\frac{1}{2}$ qt. How much wheat has he in all, and what is it worth at 75 cts. a bushel ?

21. A jeweler has three pieces of pure gold: the first weighs $3\frac{5}{8}$ oz., the second 2 oz. $1\frac{2}{3}$ pwt., and the third 5 oz. $7\frac{3}{8}$ pwt. How many dollars would they make, there being 23.22 gr. of pure gold in each dollar ?

SUBTRACTION.

291. ***To subtract one denominate number from another.***

1. What is the difference between 9 yd. 1 ft. 7 in. and 3 yd. 2 ft. 3 in.?

OPERATION.

9 yd.	1 ft.	7 in.
3	2	3
5 yd.	2 ft.	4 in.

EXPLANATION.—Write the parts of the subtrahend under like-named parts of the minuend, and begin at the right to subtract. 3 in. from 7 in. leave 4 in., which write below. 2 ft. cannot be subtracted from 1 ft., but the 9 yd. 1 ft. of the minuend are 8 yd. 4 ft.; and 2 ft. from 4 ft. leave 2 ft., which write below. 3 yd. from the remaining 8 yd. of the minuend leave 5 yd., which write below.

EXERCISES (Written).

292. Subtract:

1.	lb.	oz.	gr.
	25	2	13
	18	11	7

2.	lb.	℥	ʒ
	40	1	1
	37	9	5

3.	℥	ʒ	℈.
	31	2	0
	16	5	2

4.	da.	hr.	min.	sec.
	9	11	32	15
	8	17	25	42

5.	cir.	°	′	″
	42	75	42	18
	24	144	57	10

6. From 18 yr. 116 da. 12 min. take 5 yr. 100 da. 13 hr. 46 sec.

7. From 47 bu. 2 pk. take 25 bu. 1 pk. 5 qt. 1 pt.

8. From 4 mi. 200 rd. 2 yd. take 2 mi. 250 rd. 3 yd. 1 ft.

9. From 30 cu. yd. subtract 21 cu. yd. 11 cu. ft. 525 cu. in.

10. A barrel whose capacity is 36 gal. contains 18 gal. 3 qt. 1 pt. 2 gi. of syrup. How much more would it hold?

11. From $2\frac{3}{5}$ lb. (Troy) subtract $9\frac{5}{8}$ oz.

RULE.—*Reduce the fractions to integers of lower denominations, and subtract the results.*

OPERATION.

$2\frac{3}{5}$ lb.	=	2 lb.	7 oz.	4 pwt.	0 gr.
$9\frac{5}{8}$ oz.	=		9	12	12
Dif.	=	1 lb.	9 oz.	11 pwt.	12 gr.

Find the difference between:

12. $\frac{5}{9}$ wk. and $\frac{2}{15}$ da.

13. 8.75 lb. and $9\frac{5}{8}$ ℥.

14. $\frac{3}{4}$ rd. and $2\frac{1}{2}$ ft.

15. .675 gross and $\frac{2}{5}$ doz.

16. .8 bu. and 6.5 qt.

17. $8\frac{3}{5}$ cwt. and $16\frac{4}{7}$ lb.

18. A man starts on a journey of 100 miles; after traveling 77 mi. $120\frac{4}{9}$ rd., how far will he have to go?

19. From a cistern containing 500 gal. of water, a family used 43 gal. 3 qt. 1 pt. daily for 6 days. How much remained in the cistern?

20. From a body of land containing $637\frac{3}{11}$ A. a father gave his son 250 A. 100 sq. rd. 18 sq. yd. How much had the father left?

21. How many years, months, and days from June 17, 1880, to March 11, 1885?

Operation.

1885 yr.	3 mo.	11 da.
1880	6	17
4 yr.	8 mo.	24 da.

Explanation.—Write the later of the two dates for the minuend, and the earlier for the subtrahend, writing the *number* of the year, month, and day in order.

Subtract as in other compound numbers, allowing 30 da. to the mo. and 12 mo. to the yr.

Find the time:

22. From July 12, 1850, to Sep. 3, 1857.
23. From Oct. 5, 1863, to Dec. 17, 1894.
24. From Feb. 3, 1870, to Nov. 1, 1900.
25. From May 29, 1874, to March 13, 1880.

26. How old is a person who was born June 16, 1844?

27. A note dated June 23, 1878, was paid Sep. 18, 1891. How long did it remain unpaid?

28. George Washington was born Feb. 22, 1732, and died Dec. 14, 1799. How old was he when he died?

MULTIPLICATION.

293. ***To multiply a denominate number by a number.***

Multiply 5 gal. 3 qt. 1 pt. by 5.

Operation.

5 gal.	3 qt.	1 pt.
		5
29 gal.	1 qt.	1 pt.

Explanation.—Write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multiply. 5 times 1 pt. are 5 pt., or 2 qt. 1 pt.; write the 1 pt. below. 5 times 3 qt., plus the 2 qt. of the first partial product, are 17 qt., or 4 gal. 1 qt.; write the 1 qt. below. 5 times 5 gal., plus the 4 gal. of the second partial product, are 29 gal., which write below.

EXERCISES (Written).

294. Multiply:

1. £	s.	d.
12	9	4
		7

2. lb.	oz.	pwt.
9	13	15
		9

3. rd.	yd.	ft.
40	4	2
		8

4. A.	sq. ch.	sq. rd.
60	8	12
		10

5. sq. yd.	sq. ft.	sq. in.
15	7	43
		6

6. Multiply 7 bu. 3 pk. 5 qt. 1 pt. by 6.

7. Multiply 5 lb. 7℥ 3ʒ 2℈ by 7.

8. Multiply 12 A. 90 sq. rd. 20 sq. yd. 6 sq. ft. by 12.

9. Multiply 6 cd. 18 cu. ft. 1400 cu. in. by 5.

10. How much land is there in 11 building lots, each lot containing 6 sq. rd. 18 sq. yd.?

11. How much water will a family use in a year, at the rate of 20 gal. 2 qt. 1 pt. per day.?

12. A solar year contains 365 da. 5 hr. 48 min. 46 sec. How much time is there in 30 solar years?

DIVISION.

295. ***To divide a denominate number by a number.***

Divide 47 bu. 2 pk. 7 qt. by 6.

OPERATION.

6)47 bu. 2 pk. 7 qt.

7 bu. 3 pk. 6 qt. 1 pt.

EXPLANATION.—Write the dividend and divisor as in integers, and begin at the left to divide. 1 sixth of 47 bu. is 7 bu. and 5 bu. remainder; write the 7 bu. below. The 5 bu. remainder plus the 2 pk. of the dividend are 22 pk.; 1 sixth of 22 pk. is 3 pk. and 4 pk. remainder; write the 3 pk. below. The 4 pk. remainder plus 7 qt. are 39 qt.; 1 sixth of 39 qt. is 6½ qt., or 6 qt. 1 pt., which write below.

EXERCISES (Written).

296. Divide :

1.	£	s.	d.
	5)22	2	6

2.	cwt.	lb.	oz.
	12)37	26	12

3.	gal.	qt.	pt.
	10)389	1	1

4.	bu.	pk.	qt.
	18)35	2	7

5.	lb.	oz.	pwt.	gr.
	12)3	6	7	12

6. Divide 526 rd. 4 ft. 9 in. by 12.

7. Divide 39 lb. 11 oz. 10 pwt. by 7.

8. Divide 60 T. 16 cwt. 84 lb. by 8.

9. Divide 14 wk. 2 da. 14 hr. 9 min. 36 sec. by 42.

10. If 26 bu. 4 qt. of corn be divided equally among 10 persons, how much will each receive ?

11. If 14 acres produce 404 bu. 3 pk. 5 qt. of corn, what is the average yield per acre ?

12. If an army marched 153 mi. 313 rd. 3 yd. 2 ft. in 11 days, what was the average rate per day ?

13. Divide £1 16s. by 4s. 6d.

SOLUTION.—£1 16s. = 432d.
4s. 6d. = 54d.
432d. ÷ 54d. = 8—Ans.

14. Divide 15 yd. 1 ft. 9 in. by 5 yd. 7 in.

15. Divide 35 mi. 158 rd. by 3 mi. 72 rd. 9 ft.

16. Divide 30 lb. 7 oz. 8 pwt. 8 gr. by 3 lb. 9 oz. 18 pwt. 13 gr.

17. If 1 man can do a piece of work in 11 wk. 6 da. 18 hr., how many men will it take to do it in 1 wk. 2 da. 7 hr. 20 min. ?

18. If one sack will hold 3 bu. 3 pk. 4 qt. of corn, how many sacks will be needed to hold 34 bu. 3 pk. 4 qt. ?

19. If one suit of clothes cost £4 5s. 7d. 3 far., how many suits can be bought with £47 2s. 1d. 1 far. ?

20. How many hours will it take a freight train to run 150 mi. 178 rd. 3 yd. at the rate of 16 mi. 233 rd. 2 ft. 10 in. per hour ?

Parallel Problems.

297. *1.* If 1 pk. of corn cost 10 cts., what cost $1\frac{1}{3}$ bu. ?

2. If 1 pt. of berries cost $4\frac{1}{2}$ cts., what cost 3 pk. 5 qt. ?

3. If 1 bu. of peas cost 60 cts., what cost $2\frac{1}{3}$ pk. ?

4. If $3\frac{1}{2}$ bu. of corn cost $1.54, what cost 5.65 pk. ?

5. A boy picked 6 qts. of berries, and after giving his mother $\frac{1}{4}$ of them, sold the remainder at 5 cts. a pint. How much did he receive ?

6. A farmer gathered 580 bu. 2 pk. of corn, and after reserving .56 of it, sold the remainder at $2\frac{1}{2}$ cts. a qt. How much money did he receive ?

7. How many gills in $\frac{7}{8}$ of a gallon ?

8. How many seconds in $\frac{13}{28}$ of a week ?

9. How many rods will a man travel in $1\frac{2}{3}$ hr., if he goes 4 chains in each minute ?

10. How far will a horse travel in 5 wk. 3 da. at the rate of 6 miles an hour, allowing 6 days to the wk. and 9 hr. to the da. ?

11. Reduce $\frac{3}{5}$ of a lb. to oz. pwt., etc.

12. Reduce $\frac{7}{9}$ of a mile to rd. yd., etc.

13. Reduce £3.275 to £ s. d., etc.

14. Reduce 7.696875 lb. to lb. oz. pwt. etc.

15. How many times greater is 2 yd. 1 ft. than 1 ft. 2 in. ?

16. How many rings, each weighing 3 pwt. 20 gr., can be made from a bar of gold weighing 1 lb. 13 pwt. ?

17. How many kegs, each containing 5 gal. 1 qt., can be filled from a cask holding 63 gal. ?

18. How many times is 1d. 1 far. contained in $\frac{5}{6}$s. ?

19. A wheel is 15 ft. 10 in. in circumference. How many rotations will it make in rolling $\frac{17}{18}$ of a mile ?

20. How many axes, each weighing 3 lb. 2 oz., can be made from $\frac{3}{8}$ of a ton of iron ?

21. A farmer sold 3 boxes of wheat weighing 200, 250, and 180 pounds, at 80 cts. per bushel. How much did he receive for the wheat?

22. What is the value of 4 loads of corn, weighing 12 cwt. 54 lb. 5 oz., 13 cwt. 72 lb. 8 oz., 11 cwt. 15 oz., 10 cwt. 66 lb. 12 oz., at $.65 per bushel?

23. What is the value, in U. S. money, of 1 shilling? 1 penny?

24. How much U. S. money is equal to £12 15s. 6d.?

25. A cu. ft of water weighs 1000 oz. How many cu. ft. will it take to weigh 1 ton?

26. How many gold dollars will it take to weigh 100 pounds?

27. 140 dozen are equal to how many scores?

28. A tank that will hold 12,000 gallons of water will hold how many bushels of wheat?

29. If a horse eats 1 pk. 2 qt. of corn a day, how many days will 6 bu. 1 pk. last him?

30. If 1 pk. of apples will make 3 qt. 1 pt. of cider, how many bushels will be required to make 395 gal. 2 qt.?

31. A man killed 5 sheep, and the total weight was 241 lb. 9 oz. What was their average weight?

32. A farmer raised 488 bu. 1 pk. of oats from 14 acres of land. What was the yield per acre?

33. A stationer, by selling paper at a profit of 6 cts. a quire, made $3.60. How many reams did he sell?

34. A druggist, retailing medicine at a profit of 5 cts. a scruple, made $430. How many pounds did he sell?

35. A merchant lives 241 rd. $2\frac{1}{2}$ yd. from his store, and attends the store daily for 52 wk. of 6 da. each, going home to dinner every day. How much time does he consume in going to and fro, if he travels 65 yards per minute?

Find the value of:

36. $3\frac{4}{5}$ lb. + $4\frac{5}{6}$ oz. + $16\frac{1}{3}$ pwt. — ($4\frac{3}{5}$ oz. + $1\frac{5}{8}$ pwt.).

37. $3\frac{4}{5}$ lb. — $4\frac{5}{6}$ oz. + $16\frac{1}{3}$ pwt. — ($4\frac{3}{5}$ oz. — $1\frac{5}{8}$ pwt.).

38. 6 × ($\frac{3}{5}$ wk. + $3\frac{2}{5}$ da. — 5.675 hr.).

39. Bought a cask of oil, containing 68½ gallons, at 72 cents a gallon; ⅜ having leaked out, the remainder was sold at 90 cents a gallon. Did I make or lose, and how much?

40. At $½ a foot, what is the cost of 4 rd. of fencing?

41. At $⅔ a foot, what is the cost of 7 rd. 2½ yd. of pipe?

If land is worth $8 an acre, what is the value:

42. Of a lot containing 65 sq. ch.?

43. Of a field containing 2040 sq. rd.?

44. If a lad steps 2 ft. at a time, how many steps will he make in going 2 rd. 1 yd.?

45. How often will a wheel 9 ft. 3 in. in circumference turn over in rolling a distance of 12 miles?

What will it cost to build a fence:

46. Two-thirds of a chain long, at 2 cts. per foot?

47. $\frac{7}{11}$ of a mile long, at $$\frac{3}{16}$ per yard?

48. What is the value of 352 square rods of land, at $6⅔ an acre?

49. What is the value of 1600 cu. ft. of wood, at $3⅛ a cord?

At 3 cents a pint:

50. What will be the cost of 3½ qt. of plums?

51. What will be the cost of 5 gal. 3 qt. of cherries?

What is the price of a quart of potatoes:

52. If 1 pk. 3 qt. cost $1.43?

53. If 3 bu. 2 pk. 5 qt. cost $9.36?

How many times can a cup which holds 2 pt. 1 gill be filled:

54. From a bucket containing 4 qt. 1 pt. of water?

55. From a barrel containing 38 gal. 1 qt. of whisky?

56. A boy bought ¾ of a peck of chestnuts for 30 cts., and sold them at 7 cts. a quart. How much did he make?

57. A dealer bought 36 gal. of syrup for $20.15, and retailed ⅝ of it at 9 cts. a pint, and the balance at 8½ cts. a pint. How much did he make?

How many spoons, each weighing :

58. 2 oz., can be made from 3 lb. 4 oz. of silver ?

59. 2 oz. 10 pwt. 12 gr., can be made from 8 lb. 5 oz. of silver ?

If your pulse beats 75 times in a minute :

60. How many times will it beat in an hour ?

61. How many times will it beat in a leap year ?

62. How many minutes were there in the month of February, 1888 ? In February, 1900 ?

63. A boy is 10 yr. 7 da. old. How many days has he lived, allowing 365 days to the year ?

64. A man is 60 yr. 17 da. old. How many days has he lived, allowing 365¼ days to the year ?

65. How long would it take a locomotive, running at the rate of 45 miles an hour, to go from the earth to the moon, a distance of 240,000 miles ?

66. The population of the earth is about 1,500,000,000. How long would it take to count them, counting 75 persons each minute, and 12 hours each day ?

67. How many hours in the century beginning with the year 1801 and ending with the year 1900 ?

68. How many days would it take a person, traveling at the rate of 2 seconds of distance in 1 second of time, to go round the earth ?

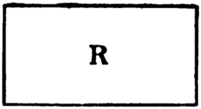
ORDINARY PRACTICAL MEASUREMENTS.

298. If a surface is such that a straight edge will fit it, in whatever position the straight edge may be placed on it, the surface is called a **plane.**

This may be illustrated by applying the edge of a straight rule to the face of a blackboard, or top of a desk, etc.

A plane, in ordinary language, is a *flat surface.*

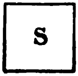
299. Any part of a plane bounded by straight or curved lines is a **plane figure.**



A **rectangle** is a plane figure bounded by four straight lines and having four right angles; as the figure R.

A **square** is a rectangle whose four sides are equal to one another; as S.

The **dimensions** of a rectangle are its *length* and *breadth.*



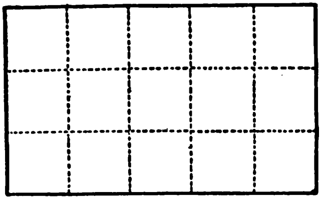
300. The **area** of a surface is the number of square units it contains.

Thus, if S is a square unit, as a square inch, and R is equivalent to 9 pieces each the size of S, then the area of R is 9 S or 9 sq. inches.*

301. *To find the area of a rectangle.*

Find the area of a rectangle whose dimensions are 5 in. and 3 in.

SOLUTION.—The rectangle may be divided into 3 rows, each containing 5 sq. inches. Therefore the entire area is 3 × 5 sq. inches, or 15 sq. inches.



A RECTANGLE, 5 BY 3.

PRINCIPLE.—*The area of a rectangle is equal to the number of units in its length multiplied by the number of like units in its breadth.*

This principle may be briefly stated thus: *Area of a rectangle is equal to its length multiplied by its breadth,* or = *the product of the two dimensions.*

The dimensions must always be expressed in like units.

EXERCISES (Oral and Written).

302. Find the area of a rectangle whose length and breadth are:

1. 5 in. and 4 in. *2.* 2 yd. and 4 ft.
3. 9 rd. and 2 ch. *4.* 6⅔ ft. and 9 ft.

---

* Strictly speaking, the abstract number 9 is the area—being the ratio of the surface to the unit.

Find the area when the dimensions are:

5. 11 ft. and 1⅓ yd.
6. 1¾ ch. and 2¼ rd.
7. 2½ ft. and 5⅓ in.
8. 5 yd. 1 ft. and 2 ft. 3 in.

9. How many square yards are there in a carpet 27 in. wide and 42 ft. long?

Solution.—27 in. = 2¼ ft. 42 × 2¼ = 94½. Hence the carpet contains 94½ sq. ft. = (94½ ÷ 9) = 10½ sq. yd.

10. How many sq. ft. in a city lot 85½ ft. front by 164 ft. deep? How many square yards?

11. How many sq. rd. in a field 7 ch. 2 rd. long and 12 rd. wide? How many acres?

12. What is the area of a garden 176 yd. by 165 yd?

13. What is the area of a field 45 ch. by 68 rd?

What is the cost of a piece of land:

14. 352 yd. by 110 yd., at $12 per acre?

15. 75 rd. by 352 yd., at $8½ per acre?

16. 32.8 ch. by 21.05 ch., at $6.80 per acre?

Students should be required to measure the length and width of rooms, boards, tables, etc., and compute the areas from their measurements.

17. One side of the roof of a house is 30 ft. by 12 ft. How many boards, 6 in. wide, will it take to cover both sides, if 8 in. of the length of each board is exposed?

Remember that the bottom course is doubled, which is equivalent to adding 8 in. to the width (12 ft.) of the roof. Thus:

2 × (30 × 12⅔) ÷ (½ × ⅔) = the required number.

18. How many boards are required to cover a house, if one side of the roof is 42 ft. by 18 ft., and the part of the board exposed is 9 in. by 7 in.?

19. One side of the roof of a church is 55 ft. by 22 ft., and is to be covered with shingles 4 in. wide, laid 4 in. to the weather. What will the shingles cost at $12 per M.?

As shingles are usually made and laid, 1000 are estimated to cover 100 sq. ft., called a *square*, or 10 to the sq. foot.

20. How many shingles must be bought to cover a house 60 ft. long, supposing the rafters to be 18 ft. long and the roof to project 1 ft. over the ends of the house?

303. ***To find the area of the walls and ceiling of a room.***

The ceiling, sides, and ends of a room are rectangular. The dimensions of the ceiling are the length and width of the room. The dimensions of the sides are the length and height of the room. The dimensions of the ends are the width and height of the room.

1. How many square feet in the walls and ceiling of a room 20 ft. long, 15 ft. wide, and 10 ft. high?

SOLUTION.

2 sides, 2 (20 × 10), = 400 sq. ft.
2 ends, 2 (15 × 10), = 300 sq. ft.
Ceiling, 20 × 15, = 300 sq. ft.
Total, 1000 sq. ft.

EXERCISES (Written).

304. How many sq. yards in the walls and ceiling:

1. Of a room 18 ft. long, 16 ft. wide, and 12 ft. high?

2. Of a room 20 ft. long, 16 ft. wide, and 11½ ft. high?

3. Of a room 24 ft. long, 20 ft. wide, and 14 ft. high, deducting for 3 doors, each 6½ ft. by 3 ft., and 8 windows, each 5 ft. 4 in. by 2 ft. 3 in.?

305. ***Plastering, papering, and painting.***

4. How much will it cost to plaster the walls and ceiling of a room 18 ft. long, 16 ft. wide, and 11 ft. high, at 33⅓ cts. per sq. yd., no allowance being made for doors and windows?

5. At 8 cts. a sq. yd., what will it cost to kalsomine the walls and ceiling of a dining-room 28 ft. long, 15 ft. wide, and 12 ft. 6 in. high, allowing 5½ sq. yd. for doors and windows?

6. What will it cost to paint the walls of a parlor 17½ ft. long, 16 ft. wide, and 11¼ ft. high, at 30 cts. per sq. yd., allowing for 2 doors, each 6¾ ft. by 3⅓ ft.?

7. What will be the cost of papering the walls of a room 22 ft. 6 in. long, 16 ft. 9 in. wide, and 12 ft. high, at the price of 12 cts. per sq. yd., allowing for 6 windows, each 5 ft. by 3 ft., and 3 doors, each 6 ft. 9 in. by 3 ft. 4 in.?

306. ***To find the number of yards to carpet a room.***

The number of yards of carpeting required for a room is found by multiplying the length of the strips required by the necessary number of strips.

If the carpet is to be laid lengthwise, the length of the strips must be equal to the length of the room plus the waste in matching the pattern; and the number of strips is found by dividing the width of the room by the width of the carpet, regarding the fraction, if any, as 1.

If the carpet is to be laid crosswise, the length of the strips must be equal to the width of the room plus the waste in matching pattern; and the number of strips is found by dividing the length of the room by the width of the carpet, regarding the fraction, if any, as 1.

1. How many yards of carpet 2¼ ft. wide will cover a floor 20 ft. by 15¾ ft., if the strips run lengthwise?

Solution.—20 ft. = length of strips; 15¾ ÷ 2¼ = 7, number of strips. 7 × 20 ft. = 140 ft. = 46⅔ yd.—Ans.

2. In the same problem, how many yards would be required if the strips ran crosswise?

Solution.—15¾ ft. = length of strips; 20 ÷ 2¼ = 8⁸⁄₉; hence 9 strips will be required. 9 × 15¾ ft. = 3 × 15¾ yd. = 47¼ yd.—Ans.

EXERCISES (Written).

307. *1.* A room is 17 ft. long and 15 ft. wide. How much matting a yard wide should be bought to cover the floor (1) if the strips are to be laid lengthwise? (2) If the strips are to be laid crosswise?

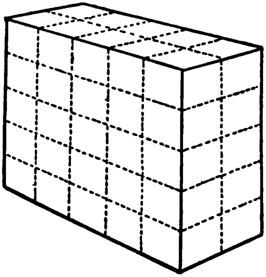
2. How many yards of carpet ¾ of a yard wide will be required for a floor 6¼ yd. long and 15 ft. wide, if the strips run lengthwise and there is a waste of ¼ of a yard in each strip in matching patterns?

3. A floor 18 ft. by 14 ft. is to be covered with a carpet 30 in. wide. What will the carpet cost at $1.25 per yard, if the strips are laid (1) lengthwise? (2) Crosswise?

4. I wish to purchase a carpet 27 in. wide, for a room 15 ft. by 13½ ft. How much more will the carpet cost at $1.60 per yard, if laid crosswise than lengthwise?

Solids.

308. A **rectangular solid** is a solid bounded by six rectangles; as the figure P.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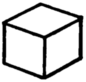
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The bounding rectangles are called **faces,** and the lines bounding the faces are called **edges.**

A brick or a box has the shape of a rectangular solid, either of which has 6 faces and 12 edges.

The **dimensions** of a rectangular solid are its *length, breadth,* and *height.*

A **cube,** as Q, is a rectangular solid whose edges or dimensions are all equal to one another.

309. The **volume** or **contents** of a solid is the number of cubic units it contains.

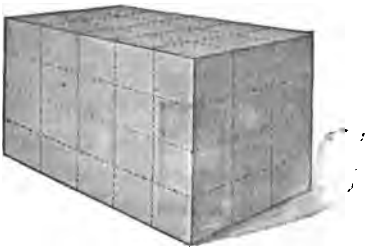
Thus, if Q is a cubic unit, as a cu. inch, and P is equivalent to 50 pieces each the size of Q, then the volume of P is 50 Q, or 50 cu. inches.*

310. *To find the volume of a rectangular solid.*

Find the volume of a rectangular solid whose dimensions are 5 in., 4 in., and 3 in.

SOLUTION.—The solid may be divided into 4 segments, each 5 in. long, 3 in. wide, and 1 in. high or thick. Each segment contains as many cu. inches as there are sq. inches in a rectangle 5 in. by 3 in.; that is, 5 × 3 cu. inches. Hence the entire volume is

4 × 5 × 3 cu. in., or 60 cu. in.


A SOLID, 5 BY 4 BY 3.

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* Strictly speaking, the abstract number 50 is the volume—being the ratio of the solid to the unit.

Principle.—*The volume of a rectangular solid is equal to the product of the numbers expressing its length, breadth, and height.*

This principle may be briefly expressed thus: *The volume of a rectangular solid is equal to the product of its three dimensions.*

The dimensions must be expressed in like units.

EXERCISES (Oral and Written).

311. Find the volume of a rectangular solid whose dimensions are:

1. 4 ft., 3 ft., and 2 ft. *2.* 4 ft., 4 ft., and 2½ ft.
3. 2 yd., 7 ft., and 5 ft. *4.* 5 ft., 3 ft., and 4⅓ ft.
5. 2 yd., 5 yd., and 3⅕ ft. *6.* 6 yd., 1 yd. 2 ft., and 5 ft.

7. How many cubic yards of earth are there in an embankment 11 ft. long, 8 ft. wide, and 6 ft. high?

Solution.—(11 × 8 × 6) ÷ 27 = 19⁵⁄₉. Hence, 19⁵⁄₉ cu. yd.

What is the cost of digging:

8. A trench 120 yd. by 4 ft. by 2 ft., at $.60 per cu. yd.?
9. A ditch 330 yd. by 3 ft. by 18 in., at $.50 per cu. yd.?
10. A cellar 20 ft. by 16½ ft. by 11⅓ ft., at $⅖ per cu. yd.?

Note.—A cubic foot holds about ⅕ of a bu. of "unshucked" corn.

11. A farmer has a crib 18 ft. long, 12 ft. wide, and 10½ ft. high, which is ⅔ full of corn in the shuck. How much corn is in the crib?

12. A box, 3 ft. long, 2 ft. 3 in. wide, and 1 ft. 4 in. deep, is full of water. What is the weight of the water, 1 cu. foot of water weighing 62½ pounds?

13. How many pounds of water will a cubical box hold whose edge is 2 ft. 6 in.?

14. How many cords of wood in a pile 32 ft. long, 8 ft. wide, and 5 ft. high?

15. How many cords of wood in two piles, the first being 20 ft. long, 4 ft. wide, and 9 ft. high; and the second 30 ft. long by 5 ft. wide and 8 ft. high?

16. A wagon contains a load of wood 11 ft. long, 4 ft. wide, and 5 ft. deep. What is the wood worth at $3.20 per cord?

312. ***To find the capacity of tanks, etc., in gallons.***

1. How many gallons will a box hold which is 5 ft. long, 3 ft. wide, and 2 ft. deep?

EXACT METHOD.

$5 \times 3 \times 2 \times \frac{1728}{231} = 224.4 +$

PRACTICAL METHOD.

$5 \times 3 \times 2 \times 7\frac{1}{2} = 225.$

NOTE.—$\frac{1728}{231}$ = *about* $7\frac{1}{2}$.

RULE.—*Divide the number of cu. in. by* 231, *or multiply the number of cu. ft. by* $7\frac{1}{2}$.

EXERCISES (Oral and Written).

About how many gallons will a box hold:

1. Which is 4 ft. long, 3 ft. wide, and 1 ft. deep?

2. Which is 3 ft. long, 3 ft. wide, and $\frac{1}{2}$ ft. deep?

3. Which is 4 ft. long, $2\frac{1}{2}$ ft. wide, and 3 ft. deep?

4. How many gallons *exactly* will the last box hold?

Solve the following by the practical method:

5. A cistern, 6 ft. by 6 ft. by 8 ft., is full of water. How long will it supply a family which uses 30 gallons per day?

6. A tank, 6 ft. square and 9 ft. high, is $\frac{3}{4}$ full of water. How many gallons are in it?

313. ***To find the capacity of bins, etc., in bushels.***

1. How many bushels of wheat will a box hold which is 6 ft. long, 5 ft. wide, and 2 ft. deep?

EXACT METHOD.

$6 \times 5 \times 2 \times \frac{1728}{2150.42} = 48.21+$

PRACTICAL METHOD.

$6 \times 5 \times 2 \times .8 = 48.$

NOTE.—$\frac{1728}{2150.42}$ = *about* .8.

RULE.—*Divide the number of cu. in. by* 2150.42, *or multiply the number of cu. ft. by* .8.

EXERCISES (Oral and Written).

About how many bushels will a bin hold :

1. Which is 6 ft. long, 3 ft. wide, and 1 ft. high ?

2. Which is 5 ft. long, 3 ft. wide, and 2 ft. high ?

3. Which is 4 ft. long, 2½ ft. wide, and 5 ft. high ?

4. How many bushels *exactly* will the last bin hold ?

About how many bushels will a bin hold :

5. Which is 6 ft. long, 4½ ft. wide, and 2⅔ ft. deep ?

6. Which is 5⅓ ft. long, 3¾ ft. wide, and 6½ ft. deep ?

7. A bin, 9 ft. long, 6 ft. 8 in. wide, and 3 ft. 11 in. deep, is two thirds full of shelled corn. What is the corn worth at $.60 per bushel ?

Small grain is usually sold by weight, hence the measurement of bins is of little practical value.

314. ***To find the number of board feet in boards, planks, etc.***

A board 1 foot square and 1 inch thick is called a *board foot,* or a foot *board measure* (Bd. ft.).

When lumber is more than 1 inch thick, the number of board feet depends upon the thickness.

1. How many Bd. ft. in a plank 15 ft. long, 8 in. wide, and 1½ in. thick ?

OPERATION.

$$\frac{15 \times 8 \times 1\frac{1}{2}}{12} = 15$$

RULE.—*Express the length in feet, and the width and thickness in inches; multiply the three together, and divide by* 12.

If a board or plank is less than 1 in. thick, the calculation is made as if it were 1 in. thick.

EXERCISES (Oral and Written).

315. How many Bd. ft.:

1. In 2 planks, each 10 ft. by 6 in. by 2 in.?

2. In 5 boards, each 9 ft. by 4 in. by ½ in.?

3. In 10 rafters, each 12 ft. by 4 in. by 2 in.?

4. How many Bd. ft. in a load of 45 boards, each 16 ft. long, 9 in. wide, and 1½ in. thick ?

5. What will be the cost of 75 planks, each 9 ft. by 8 in. by ¾ in., at $19 per thousand Bd. ft.?

In lumber bills an expression like 8 — 2 × 6 — 12, means 8 pieces of lumber, 2 in. thick and 6 in. wide and 12 ft. long. The price of lumber is always the price per thousand board feet.

6. Copy, extend the items, and find the footing of the following lumber bill :

TROY, GA., *May* 2, 1902.

J. A. Thigpen,

Bought of Jackson Lumber Co.

10 — 2 × 8 — 12 (Pine) @ $16		
12 — ½ × 6 — 10 (Pine) @ $12		
20 — 1 × 8 — 16 (Cypress) @ $20		
18 — 3 × 4 — 12 (Pine) @ $12		
15 — 2 × 6 — 10 (Cypress) @ $18		

316. To measure round logs. Round logs are sold by the amount of square lumber that can be cut from them, for the computation of which various rules have been suggested. The following rule, which is in very general use throughout the States, is the law in Louisiana for determining the number of board feet in a round log :

Take four from the number of inches in the diameter of the log, multiply the square of half the remainder by the number of feet in the length of the log, and divide the product by 4*; and the quotient will be the number of board feet contained in the log.*

FORMULA : $$\left(\frac{\text{Diameter} - 4}{2}\right)^2 \times \frac{\text{length}}{4}.$$

This rule gives the table on the calipers known as "Doyle's Scale."

By "diameter" is usually meant the diameter of the smaller end of the log.

1. Find the number of board feet in a log 12 ft. long and 18 in. in diameter.

SOLUTION.—18 − 4 = 14; 14 ÷ 2 = 7; 7 × 7 = 49;
12 × 49 = 588; 588 ÷ 4 = 147 Bd. ft.

EXERCISES (Written).

317. Find the number of board feet in:

1. A log 16 ft. long and 16 in. in diameter.
2. A log 20 ft. long and 14 in. in diameter.
3. A log 15 ft. long and 13 in. in diameter.
4. A log 18 ft. long and 12 in. in diameter.
5. A log 24 ft. long and 11 in. in diameter.

318. *To find the number of bricks in walls, etc.*

There is no uniformity in the size of bricks. A brick 8 in. by 4 in. by 2 in. is called "an ordinary brick," of which it takes 27 to make a cubic foot. But on account of breakages and mortar, 22 are estimated to the cubic foot.

The bricks in most general use in the Southern States are, after shrinkage, 8½ inches long, 4 inches wide, and 2½ inches thick; and of these, 20 are estimated to the cubic foot. This kind of bricks is supposed to be used in all the following examples.

In estimating amount of work, masons measure the length of walls on the outside, thus counting the corners twice, and deductions are usually made for one half of openings. This is considered just on account of the extra labor of construction.

In estimating amount of material the corners are not counted twice, and usually deductions are made for openings. The following computations are made in this manner.

1. How many bricks will be required to build the walls of a house 30 ft. long, 18 ft. wide, 12 ft. high, and 18 in. thick, allowing for 7 openings, each 6 ft. by 3 ft.?

SOLUTION.

2 × (30 + 18) = 96, number of linear feet around the walls.
96 × 12 × 1½ = 1728, no. cu. ft. in walls, 8 corners, and openings.
4 × 12 × $\frac{3}{2}$ × $\frac{3}{2}$ = 108, the number of cu. ft. in the four corners.
7 × 6 × 3 × $\frac{3}{2}$ = 189, the number of cu. ft. in the seven openings.
1728 − (108 + 189) = 1431, no. cu. ft. in walls and corners.
1431 × 20 = 28620, the number of bricks required.

2. In the same example, what would be the cost of laying the brick, at $5 per thousand bricks ?

SOLUTION.—We have found the number of cu. ft. in the walls, *twice* the corners, and openings, to be 1728; from this we subtract ½ of the cu. feet in the 7 openings, and obtain 1633½, which multiplied by 20 gives 32670, the number of bricks to be estimated in the cost.

32.67 × $5 = $163.35—Ans.

EXERCISES (Written).

319. *1.* The walls of a cellar are to be 44 ft. long, 30 ft. wide, 10 ft. high, and 2 ft. thick, no openings. What will the bricks cost at $7 per M.; and what will the work of laying the brick cost, at $4.50 per M.?

2. How many bricks will be required to build the walls of a house 36 ft. long, 24 ft. wide, 15 ft. high, and 16 in. thick, deducting 96 cu. ft. for openings ?

3. In the same problem, how much should a mason receive for building the walls of the house, if he charges $5 per M.?

4. What will it cost to build the walls of a house 40 ft. long, 30 ft. wide, 20 ft. high, and 18 in. thick, when brick cost $8 per M. and laying the bricks is paid for at the rate of $5½ per M., there being 2 doors each 7 ft. by 3 ft. and 10 windows each 6 ft. by 3 ft.?

Parallel Problems.

320. *1.* How many acres in a field 15 ch. long and 6 ch. wide ? In a field 180 rd. by 140 rd. ? 860 yd. by 528 yd. ?

2. A garden is 6 ch. long. How wide must it be to contain 2 A. ?

3. A farmer desires to lay off a field that shall contain 32 A., and be 80 rd. long. How wide must it be ?

4. What length of a board, 9 in. wide, will make a box cover 2 ft. 3 in. square ?

5. How many yd. of carpeting, 3 ft. 7 in. wide, are required to carpet a room 17 ft. 11 in. square ?

6. What will it cost to carpet a room 18 ft. long and 15 ft. wide, with carpeting 3 ft. wide, at 60 cts. per yard ?

7. What will it cost to carpet the same room with carpeting 4 ft. wide and worth $.75 per running yard, if the carpeting is laid (1) lengthwise ? (2) Crosswise ?

8. If the same room is 11 ft. high, how much will it cost to plaster the walls and ceiling, at 40 cts. per sq. yd., deducting for 2 doors each $6\frac{1}{4}$ ft. by $3\frac{1}{6}$ ft., and 3 windows each $5\frac{1}{3}$ ft. by $2\frac{5}{8}$ ft. ?

9. How many bricks 8 in. long and 4 in. wide will pave a walk 12 ft. by 5 ft. ? A banquette 125 yd. by $5\frac{1}{2}$ ft. ?

10. The walls of a house are 40 ft. long, and the rafters 15 ft. How many shingles are required to cover it, allowing the roof to project 18 in. over the ends ; and what will they cost at $7 per thousand ?

11. How long will it take a family, using 5 cu. ft. of water per day, to exhaust the water in a tank 6 ft. long, $2\frac{1}{3}$ ft. wide, if the water is $4\frac{1}{3}$ ft. deep ?

12. In a schoolroom, 40 ft. long, 30 ft. wide, and 12 ft. 6 in. high, are 44 pupils, each breathing $12\frac{1}{2}$ cu. ft. of air per hour. In what time will they breathe as much air as the room contains ?

13. What is the cost of digging a cellar 21 ft. by 15 ft. by 12 ft., at $\$\frac{2}{7}$ per cu. yd. ? $\frac{21 \times 15 \times 12}{3 \times 3 \times 3} \times \$\frac{2}{7} = ?$

14. How many bricks are required to build the walls of a house 20 ft. by 18 ft. by $11\frac{2}{3}$ ft. ; and what will they cost at $8.50 per thousand ?

15. At $3.75 a cord, what is the value of a pile of wood 15 ft. long, 8 ft. wide, and 6 ft. high ?

16. About how many gallons of water will a tank hold which is 5 ft. by 4 ft. by 6 ft. ?

17. A cistern, whose base is 7 ft. square and whose height is 10 ft. 6 in., is $\frac{2}{3}$ full of water. How long will the water supply a family that uses 25 gal. 3 qt. a day (prac. meth.)?

18. A box 5 ft. by 4 ft. by 2½ ft. is full of shelled corn. About how many bushels are in the box?

19. A granary 8 ft. by 3 ft. by 4½ ft. is ⅚ full of wheat. About how much is the weight of the wheat?

20. What cost 60 planks 10 ft. long, 8 in. wide, and ¾ in. thick, at $1½ per hundred board feet?

21. Find the cost of flooring a three-story house, the floors being 42 ft. by 30 ft., and the plank 1½ in. thick, supposing the carpenters to charge 12½ cts. per sq. yd., and the sawyer $23 per M. Bd. ft.?

THE METRIC SYSTEM.

321. The **metric system** is a decimal system of standard units for measuring quantities in general.

322. The **meter** (m.) is the principal unit for measuring lengths; and from it are derived the units for measuring *surfaces, solids, capacity,* and *weight.* Hence the name, *metric system.*

323. The principal unit:

For measuring capacity is the **liter,** pronounced *lē'ter;*
For measuring weight is the **gram;**
For measuring land is the **are,** pronounced *air;*
For measuring wood is the **stere,** pronounced *stair.*

324. The other units for length, capacity, and weight are decimal multiples or sub-multiples of the principal units. The names of the multiples are formed by prefixing to the name of the principal unit the Greek prefixes, *deka* (*D.*) for 10, *hekto* (*H.*) for 100, *kilo* (*K.*) for 1000; *myria* (*M.*) for 10000; and the names of the sub-multiples are formed by prefixing to the name of the principal unit the Latin prefixes *deci* (*d.*) for 10th, *centi* (*c.*) for 100th., *milli* (*m.*) for 1000th. Hence the form and relations :

10 milli units = 1 centi unit,
10 centi units = 1 deci unit,
10 deci units = 1 unit,
10 deka units = 1 hekto unit,
10 hekto units = 1 kilo unit,
10 kilo units = 1 myria unit,
10 units = 1 deka unit.

325. By substituting *meter, liter,* and *gram* for *unit* in the above table, we get, respectively, the following three tables :

Linear Units.

10 millimeters (mm.) = 1 **centimeter** (cm.),
10 cm. = 1 decimeter (dm.),
10 dm. = 1 **meter** (m.),
10 m. = 1 dekameter (Dm.),
10 Dm. = 1 hektometer (Hm.),
10 Hm. = 1 **kilometer** (Km.),
10 Km. = 1 myriameter (Mm.).

Units of Capacity.

10 milliliters (ml.) = 1 **centiliter** (cl.),
10 cl. = 1 deciliter (dl.),
10 dl. = 1 **liter,**
10 l. = 1 dekaliter (Dl.),
10 Dl. = 1 **hektoliter** (Hl.),
10 Hl. = 1 kiloliter (Kl.),
10 Kl. = 1 myrialiter (Ml.).

Units of Weight.

10 milligrams (mg.) = 1 **centigram** (cg.),
10 cg. = 1 decigram (dg.),
10 dg. = 1 **gram** (g.),
10 g. = 1 dekagram (Dg.),
10 Dg. = 1 hektogram (Hg.),
10 Hg. = 1 **kilogram** (Kg.),
10 Kg. = 1 myriagram (Mg.).

326. As in U. S. money *dollar* and *cent* are the units most generally used, so in the metric system the units usually employed in business and science are those in the tables that are printed in bold-face type, like centimeter, meter, kilometer, etc. The approximate values of these units, and the purposes for which they are used, should be carefully noted by the student.

327. The length of a meter is 39.37079 in., or nearly the ten-millionth part of the distance on a meridian from the equator to the pole.

The approximate length of a meter is 1.1 yd. ; of a centimeter, ⅖ of an inch ; of a kilometer, ⅝ of a mile.

The meter, like the yard, is used in measuring short distances ; the kilometer, like the mile, in measuring long distances ; and the centimeter and millimeter are used for minute measurements.

The following represents the exact length of 10 centimeters :

328. The liter is equal in volume to a cube whose edge is a decimeter, or $\frac{1}{10}$ of a meter. Its value is 1.0567 liquid quarts, or .908 dry quart.

The approximate value of a liter is 1 quart ; and of the hektoliter, 26½ gal. or 2⅚ bu.

The liter is used in measuring milk, wine, etc., and the hektoliter in measuring grain, liquids in casks, etc.

329. The gram is equal to a cubic centimeter of distilled water at the temperature of 4° Centigrade or 39.2° Fahrenheit, and its value is 15.43248 + troy grains.

The approximate value of a gram is 15½ grains ; of the kilogram, often called kilo, 2⅕ pounds, and of the tonneau or ton (= 100 myriagrams), 2200 pounds.

The gram is used in weighing gold, silver, jewels, letters, etc., and in mixing medicines ; the kilogram in weighing common articles like sugar, butter, etc., and the ton in weighing heavy articles.

Units of Surface.

330. The units of surface are squares whose edges are the corresponding linear units; hence it requires 10 times 10, or 100, of one unit to make one of the next higher.

Thus, a square meter is a square each of whose sides is 1 meter, or 10 decimeters. Hence 1 sq. meter is equal to 100 sq. decimeters.

100 sq. millimeters (sq. mm.) = 1 sq. centimeter (sq. cm.),
100 sq. cm. = 1 sq. decimeter (sq. dm.),
100 sq. dm. = 1 sq. meter (sq. m.),
100 sq. m. = 1 sq. dekameter (sq. Dm.),
100 sq. Dm. = 1 sq. hektometer (sq. Hm.),
100 sq. Hm. = 1 sq. kilometer (sq. Km.).

In measuring land a square meter is called a centare (ca.), a square dekameter an are (a.), and a square hektometer a hectare (Ha.). Hence 100 ca. = 1 a., and 100 a. = 1 Ha.

The approximate area of an are is ¼ sq. ch., or 4 sq. rd.; and of a hectare, 2½ acres.

Units of Solids.

331. The units of solids are cubes whose edges are the corresponding linear units. Hence it requires 10 × 10 × 10, or 1000 of one unit to make a unit of the next higher order.

Thus, a cubic meter is a cube each of whose edges is 1 meter, or 10 decimeters. Hence 1 cu. meter is equal to 1000 cu. decimeters.

1000 cu. millimeters (cu. mm.) = 1 cu. centimeter (cu. cm.),
1000 cu. cm. = 1 cu. decimeter (cu. dm.),
1000 cu. dm. = 1 cu. meter (cu. m.),
1000 cu. m. = 1 cu. dekameter (cu. Dm.),
1000 cu. Dm. = 1 cu. hektometer (cu. Hm.),
1000 cu. Hm. = 1 cu. kilometer (cu. Km.).

In measuring wood, stone, etc., a cubic meter is called a stere, the approximate value of which is ¼ of a cord.

Reduction.

332. A denominate number in the metric system can be reduced to higher or lower units by simply *removing the decimal point.*

In measures of solids, since it requires 1000 of each unit to make one of the next higher, *the decimal point must be removed three places at a time.*

Thus, 4.37564 cu. m. = 4375.64 cu. dm. = 4375640 cu. cm.

In measures of surfaces, since it requires 100 of each unit to make one of the next higher, *the decimal point must be removed two places at a time.*

Thus, 4.37564 sq. m. = 437.564 sq. dm. = 43756.4 sq. cm.

In other measures, *remove the decimal point one place at a time.*

325 l. = 32.5 Hl. = 3.25 Dl. = .325 Kl.
24 Hg. = 240 g. = 2400 dg. = 24000 cg.

EXERCISES (Oral).

333. Reduce :

1. 18 Dm. to meters.
2. 13 dm. to meters.
3. 15 Hg. to grams.
4. 20.5 cg. to grams.
5. 425 ml. to liters.
6. 34 Kl. to liters.
7. 17 g. to centigrams.
8. 250 m. to hektometers.
9. 58.91 l. to milliliters.
10. .6584 g. to kilograms.

11. Change 24 sq. m. to sq. dm. ; to sq. Hm.

12. Change 37.5 cu. m. to cu. cm. ; to cu. Dm.

13. There are 39.37079 in. in a m. What is the length of a Dm. ? Hm. ? Km. ? dm. ? cm. ? mm. ?

14. The liter is equal to 1.0567 qt. How many quarts in a dl. ? cl. ? ml. ? Dl. ? Hl. ? Kl. ?

15. One gram is equal to 15.432 grains. How many grains in a Kg. ? In a Hg. ? Dg. ? mg. ? cg. ? dg. ?

334. For convenience, the ratio of the corresponding principal units are here given correctly to four figures, which is sufficiently accurate for all ordinary business and scientific purposes.

1 m.	= 1.094 yd.	1 cm.	= .3937 in.	1 Km.	= .6214 mi.
1 yd.	= .9144 m.	1 in.	= 2.540 cm.	1 mi.	= 1.609 Km.
1 g.	= 15.43 gr.	1 Kg.	= 2.679 lb.[2]	1 Kg.	= 2.205 lb.[4]
1 gr.	= .0648 g.	1 lb.	= .3733 Kg.[2]	1 lb.	= .4536 Kg.[4]
1 l.	= 1.057 qt.[1]	1 l.	= .9081 qt.[3]	1 Hl.	= 2.838 bu.
1 qt.	= .9462 l.[1]	1 qt.	= 1.101 l.[3]	1 bu.	= .3524 Hl.
1 sq. m.	= 1.196 sq. yd.	1 cu. m.	= 1.308 cu. yd.	1 Ha.	= 2.471 A.
1 sq. yd.	= .8361 sq. m.	1 cu. yd.	= .7645 cu. m.	1 A.	= .4047 Ha.

EXERCISES (Written).

335. *1.* What will 45 yards of cloth cost at $1.25 per meter?

2. What will 30 meters of rope cost at $.06 per yard?

3. How many acres in a field 8 Hm. long and 5 Hm. wide?

4. How many hectares in a field 400 yd. by 300 yd.?

5. A barrel of flour weighs 196 pounds. What is its weight in kilograms?

6. A man bought 250 bu. wheat at $.65 a bushel and sold it at $2.10 a hektoliter. How much did he gain?

7. A tank is 10 m. long, 32 dm. wide, 142 cm. deep. How long will it take a water-pipe to fill it, if 2 cu. yd. flow into it per hour?

8. A room is 6 meters long, 5 meters wide, and 4 meters high. What will it cost to plaster it at 40 cts. a sq. yard?

9. How many liters of water will a tank hold which is 2.4 m. long, 1.8 m. wide, and 8 dm. deep?

10. What cost 4.8 Hl. of wine, at $2.75 per gallon?

11. Reduce 30 lb. 12 oz. avoirdupois to hektograms.

12. How many square feet are there in 15 sq. meters?

13. If a man's height is 5 ft. 9 in., what is his height expressed in decimeters?

[1] Liquid. [2] Troy. [3] Dry. [4] Avoirdupois.

14. The distance around the earth is about 25,000 mi. What is the distance in kilometers ?

15. What would I make by buying 50 meters of cloth at $1.20 a meter, and selling it at the same price per yard ?

16. The capacity of a certain bin is 50 cu. meters and it is full of grain. What is the grain worth at 75 cts. per bushel ?

17. How much will you weigh in kilograms, if your weight is 120 pounds ?

LONGITUDE AND TIME.

336. The **meridian** of any place is a line on the surface of the earth passing from the North to the South Pole through that place.

337. The **longitude** of any place is the number of degrees, minutes, and seconds, reckoned on the equator, between a standard meridian (marked 0°) and the meridian of the given place.

All places are in east or west longitude according as they are east or west of the standard meridian—180°, or half the circumference of the earth, being the limit.

The English and Americans generally reckon longitude from the meridian of Greenwich, England. The following is the longitude, from Greenwich, of 14 important cities :

Berlin, 13° 23′ 43″ E.
Boston, 71° 3′ 30″ W.
Chicago, 87° 37′ 45″ W.
Cincinnati, 84° 29′ 45″ W.
London, 5′ 38″ W.
New Orleans, 90° 3′ 49″ W.
New York, 74° 24″ W.
Paris, 2° 20′ 22″ E.
Pekin, 116° 27′ 30″ E.
Philadelphia, 75° 10″ W.
Rome (Italy), 12° 27′ E.
San Francisco, 122° 26′ 45″ W.
St. Louis, 90° 12′ 14″ W.
Washington, 77° 0′ 36″ W.

338. *To find the difference in longitude of two places.*

RULE.—*When the longitudes are both east or both west, subtract the less from the greater.*

When one longitude is east and the other west, add them.

EXERCISES (Written).

Find the difference in longitude between:

1. Boston and New Orleans.
2. Rome and New York.
3. Rome and Cincinnati.
4. Paris and St. Louis.
5. London and Washington.
6. Pekin and Chicago.
7. Washington and San Francisco.
8. Cincinnati and Philadelphia.

Relation of Longitude and Time.

339. The earth rotates east on its axis once in 24 hours; hence the sun appears to pass west over 360° in 24 hours; or 15° in 1 hr., or 60 min.; or 15′ in 1 min.; or 15″ in 1 sec. That is,

A difference in longitude of		A difference in time of
15°,	makes	1 hr.
15′,	"	1 min.
15″,	"	1 sec.

PRINCIPLE.—*There are* 15 *times as many degrees, minutes, and seconds in the difference of longitude as there are hours, minutes, and seconds in the difference of time, respectively.*

As the sun appears to move from east to west, places east of a given meridian have earlier or more time, and places west have later or less time.

340. ***Difference in time given, to find the difference in longitude.***

1. The difference in time between two places is 4 hr. 10 min. 25 sec. What is the difference in longitude?

EXPLANATION.—In accordance with the preceding principle, we multiply by 15, and obtain 62° 36′ 15″.

OPERATION.

4 hr.	10 min.	25 sec.
		15
62°	36′	15″ Ans.

RULE.—*Multiply the difference in time by* 15.

EXERCISES (Written).

Find the difference in longitude between two places, the difference in time being:

1. 33 min.
2. 42 sec.
3. 4 min. 12 sec.
4. 26 min. 36 sec.
5. 2 hr. 7 min. 12 sec.
6. 7 hr. 16 min. 47 sec.

Find the longitude of a place whose time, compared with Greenwich time, is:

7. 42 min. fast.
8. 2 hr. 6 min. slow.
9. 3 hr. 2 sec. fast.
10. 3 hr. 32 sec. slow.
11. 5 hr. 21 min. 7 sec. fast.
12. 8 hr. 12 min. 56 sec. slow.

13. A traveler found on reaching Washington that his watch, which indicated the correct time of the place he left, was 1 hr. 20 min. too fast. Required the longitude of the place he left.

14. An astronomer in New Orleans observed the beginning of an eclipse of the sun at 9 o'clock A.M., and another astronomer, at a different place, observed it at 1½ o'clock P.M. Required the longitude of the latter place.

341. ***Difference in longitude given, to find the difference in time.***

1. The difference in longitude between two places is 32° 2′ 15″. What is the difference in time?

Explanation.—In accordance with the principle of Art. 339, we divide by 15 as in Art. 295, and obtain 2 hr. 8 min. 9 sec.

Operation.

$$\begin{array}{r|l} 15 & 32^\circ \quad 2' \quad 15'' \\ \hline & 2 \text{ hr. } 8 \text{ min. } 9 \text{ sec.} \end{array}$$

Rule.—*Divide the difference in longitude by* 15.

EXERCISES (Written).

Find the difference in time between:

1. St. Louis and Chicago.
2. Cincinnati and Washington.

Find the difference in time between:

3. Paris and New York.
4. New Orleans and Rome.
5. New Orleans and San Francisco.

What time is it in Washington, when the time:

6. At New York is 8 hr. 10 min., A.M.?
7. At Philadelphia is 11 hr. 40 min., A.M.?
8. At London is 3 hr., P.M.?
9. At Paris is 2 hr. 20 min. 32 sec., P.M.
10. At New Orleans is 12 M.?

PERCENTAGE.

342. *1.* If a yardstick be divided by marks into 100 equal divisions, what part of the yardstick is 5 of the divisions? 11 of the divisions? 23 divisions? 71? 85? 12½? 100?

2. Using *per cent* instead of *hundredths*, what part of the yardstick is 7 of the divisions? 13 of the divisions? 25 divisions? 33⅓? 54? 62½? 91? 100?

3. A man earned $100 and spent $8. What per cent of his earnings did he spend?

4. John had 100 marbles and lost 13 of them. What per cent of his marbles did he lose?

5. In a pile of walnuts 15 of every hundred are bad. What per cent is bad? What per cent is good?

6. In a load of apples there are as many good as bad apples. How many good ones are there in every hundred apples? How many bad ones in every hundred? What per cent, then, is good? What per cent is bad?

7. In a pile of nuts there are four times as many good as bad ones. How many of each kind are there on an average in every hundred nuts? What per cent, then, is good? What per cent is bad?

343. Per cent means hundredths or by the hundred.

344. The **sign of per cent** is %.

Thus, 8% is read 8 per cent, and is the same as $\frac{8}{100}$ or .08.

EXERCISES (Oral and Written).

345. *1.* What decimal is the same as 9%?

SOLUTION.—9% = $\frac{9}{100}$ = .09.

Express decimally :

2. 7 per cent. *3.* 12%. *4.* 25%. *5.* 125%.
6. 3 per cent. *7.* 31%. *8.* 54%. *9.* 140%.
10. 10 per cent. *11.* 70%. *12.* 80%. *13.* 160%.
14. $\frac{1}{2}$ per cent. *15.* $6\frac{3}{5}$%. *16.* $33\frac{1}{3}$%. *17.* $210\frac{1}{2}$%.

---

18. What common fraction is the same as 60%.

SOLUTION.—60% = $\frac{60}{100}$ = $\frac{6}{10}$ = $\frac{3}{5}$.

Express as a common fraction in its lowest terms :

19. 4 per cent. *20.* 5%. *21.* 20%. *22.* 120%.
23. 10 per cent. *24.* 8%. *25.* 75%. *26.* 125%.
27. 15 per cent. *28.* 25%. *29.* 50%. *30.* 280%.
31. $6\frac{1}{4}$ per cent. *32.* $8\frac{1}{2}$%. *33.* $16\frac{2}{3}$%. *34.* $287\frac{1}{2}$%.
35. $\frac{1}{2}$ per cent. *36.* $\frac{2}{5}$%. *37.* $\frac{5}{8}$%. *38.* $\frac{25}{17}$%.

---

39. What per cent is the same as $\frac{3}{8}$?

SOLUTION.—$\frac{3}{8} = \frac{3 \times 12\frac{1}{2}}{8 \times 12\frac{1}{2}} = \frac{37\frac{1}{2}}{100} = 37\frac{1}{2}$%.

Or, we may say, since 1 = 100%, $\frac{3}{8}$ = $\frac{3}{8}$ of 100% = $37\frac{1}{2}$%.

Reduce the following to equivalent per cents :

40 $\frac{1}{2}$. *41.* $\frac{1}{3}$. *42.* $\frac{1}{4}$. *43.* $\frac{1}{5}$. *44.* $\frac{1}{6}$.
45. $\frac{2}{3}$. *46.* $\frac{3}{4}$. *47.* $\frac{2}{5}$. *48.* $\frac{5}{6}$. *49.* $\frac{6}{5}$.
50. $\frac{5}{4}$. *51.* $\frac{3}{5}$. *52.* $\frac{4}{5}$. *53.* $\frac{5}{8}$. *54.* $\frac{7}{8}$.

346. Percentage is the process of computing in which the standard of comparison is a hundred.

For example, if one man loses 7 sheep out of every 20, and another 9 sheep out of every 25, their losses may be conveniently compared by reducing them to per cents. Thus, 7 of every 20 is the same as 35 of every 100, and 9 of every 25 is the same as 36 of every 100; that is, one man loses 35 per cent of his sheep and the other 36 per cent. Hence, one loses 1 per cent, or one of every hundred, more than the other.

Again, suppose an investment of $360 yields a profit of $63. Now, we can get a clearer idea of the relation of these sums by reducing them to a scale of 100. This we do by finding the corresponding profit on an investment of $100. Thus, since $360 yields a profit of $63, each $1 yields a profit of $\frac{1}{360}$ of $63, or $$\frac{7}{40}$, and each $100 yields a profit of 100 × $$\frac{7}{40}$, or $17½. That is, a profit of $63 on an investment of $360 is equivalent to a profit of $17½ on every $100 invested, which is a profit of 17½ per cent.

347. The quantities involved in percentage are the ***base***, the ***rate***, the ***percentage***, and the ***amount*** or ***difference***.

348. The **base** is the number of which the per cent is to be found.

349. The **rate** is the number of hundredths taken.

350. The **percentage** is the result of taking that per cent of the base expressed by the rate.

351. The **amount** is the base plus the percentage, and the **difference** is the base less the percentage.

Thus, 5% of $400 is $\frac{5}{100}$ of $400 = $20. Here $400 is the base, 5% the rate, $20 the percentage; $400 + $20 = $420 the amount, and $400 − $20 = $380 the difference.

352. ***Relation of the quantities.***

(1) *Percentage = Base × Rate.*

(2) *Amount = Base + Percentage = Base × (1 + Rate).*

(3) *Difference = Base − Percentage = Base × (1 − Rate).*

The amount or difference may be regarded as the result of taking that per cent of the base expressed by the sum or difference of 1 and the given rate, respectively. In this sense amount and difference are percentages.

353. ***Base and rate given, to find the percentage.***

EXERCISES (Oral).

1. Is 25% of 80 the same as ¼ of 80? Why? What, then, is 25% of 80?

2. Is 7% of $300 the same as .07 of $300? Why? What, then, is 7% of $300?

Find the following:

3. 9% of $400; 8% of 50 bu.
4. 50% of 640; 25% of $720.
5. 20% of 75 lb.; 33⅓% of 69.
6. 5% of 560 ft.; 12½% of 84.

7. A drover had 180 horses and sold 20% of them. How many horses did he sell?

8. How many sq. feet are there in 33⅓% of a sq. yd.?

9. How many sq. inches are there in 66⅔% of a sq. foot?

10. James had 60 apples and gave his sister 15% of them. How many apples did he give his sister?

11. A wife's age is 80% of her husband's age. If the husband is 35 years old, how old is the wife?

12. A owned $\frac{6}{11}$ of a mill and sold 33⅓% of his share. What part of the mill did he sell?

13. A barrel that will hold 40 gallons is 87½% full of water. How many gallons of water are in the barrel?

14. A rope is 600 ft. long when dry. How many feet shorter will it be when wet, if it shrinks 3⅓%?

354. *1.* Find 7½% of 4326.

Operation.

$$\begin{array}{r} 4326 \\ \underline{.07\frac{1}{2}} \\ 2163 \\ \underline{30282} \\ 324.45 \end{array}$$

Explanation.—7½% = $\frac{7\frac{1}{2}}{100}$ = .07½.
Hence, 7½% of 4326 = 4326 × .07½.

Rule.—*Multiply the base by the rate.*

EXERCISES (Written).

Find:

1. 6 % of $7340. *2.* 9 % of $540.60.
3. 4½ % of $486. *4.* 5⅔ % of $681.9.
5. 7.2 % of 550. *6.* ¾ % of 365 yds.
7. 6⅓ % of 75.6. *8.* 81 % of $54.30.

9. In an army of 7650 soldiers 24 % of the men were killed in battle. How many men were killed ?

10. A farmer who had 3240 acres of land, after reserving 32 % of it, sold the balance at $5 an acre. How much did he receive for the part sold ?

11. A merchant bought a bill of goods for $1850, and sold them at a gain of 12 %. How much did he make ?

12. A speculator bought land for $5284.50, and sold it at a loss of 16 %. How much did he lose ?

13. In a school of 350 pupils, 44 % are boys. How many boys are there in school ?

14. How much can a man save yearly whose salary is $1240 and whose expenses are 65 % of his salary ?

15. A cask contains 38 gallons of wine. If 15 % of the wine were drawn out, how many gallons would be left ?

16. James and John together bought a bicycle for $55, of which James paid 56% and John the rest. What did each pay ?

17. An agent sold 16 buggies at $58.50 each, and was allowed 12½ % for his services. How much did he receive ?

18. An estate is worth only 65 %. What is the worth of a claim against the estate amounting to $650.40 ?

19. A farmer raises 1650 bushels of grain, of which 28 % was wheat, 46 % rye, and the rest oats. How many bushels of each did he raise ?

20. A man having $34,000 invested 14 % in bank stock, 12½ % in bonds, 33⅓ % in town property, and the rest in a farm. How much did the farm cost ?

355. ***Base and percentage given, to find the rate.***

EXERCISES (Oral).

1. What part of 10 is 2? Is $\frac{1}{5} = \frac{20}{100}$? What per cent, then, of 10 is 2?

2. 9 is what part of 36? The *part* is the same as what %? What per cent, then, is 9 of 36?

3. What part of 25 is 4? Is the *part* the same as $\frac{16}{100}$? What %, then, is 4 of 25?

4. Is 3 of every 20 the same as 5 × 3 of every 5 × 20? What %, then, is 3 of 20?

What per cent:

5. Is 5 of 10? Is 3 of 15?
6. Is 7 of 28? Is 9 of 27?
7. Is 4 of 40? Is 4 of 50?
8. Is 5 bu. of 45 bu.? Is 7 lb. of 70 lb.?
9. Is 5 min. of 6 min.? Is $\frac{1}{3}$ sec. of $\frac{1}{2}$ sec.?

10. A farmer had 44 pigs and sold 11 of them. What per cent did he sell? What per cent did he not sell?

11. Of 45 marbles 15 were lost. What per cent was lost? What per cent was not lost?

12. A man rents a house worth $600 for $63. At what rate per cent is the property rented?

13. What is the sum of 9, 18, and 63? What % of the sum is 9? 18? 63? What is the sum of the three per cents? Why should it be so?

14. A miller takes 4 qt. of corn out of every bushel for toll. What per cent is taken for toll?

15. What per cent should be deducted from the price of milk, when 7 pints of milk are sold for a gallon?

16. In an examination 40 questions were asked, of which A answered 30, B 24, and C 18. What % did each make?

EXERCISES (Written).

356. *1.* What per cent of 36 is 9 ?

SOLUTION.—9 is $\frac{9}{36}$ or $\frac{1}{4}$ of 36, and $\frac{1}{4} = \frac{25}{100}$ = 25 per cent. Therefore, 9 is 25 % of 36.

2. What per cent of $450 is $31.50 ?

OPERATION.

4.50)31.50(7
31 50

Ans.—7 per cent.

EXPLANATION.—1 % of $450 = $4.50. Therefore, $31.50 is as many per cent of $450 as $4.50 is contained times in $31.50, or 7 %.

RULE.—*Divide the percentage by the base, and reduce the quotient to hundredths; or take such a part of* 100 % *as the percentage is of the base.*

What per cent :

1. Of 450 is 75 ? Of 980 min. is 49 min. ?
2. Of 725 is 15 ? Of 560 sec. is 42 sec. ?
3. Of 9.3$\frac{1}{3}$ is 3.5 ? Of 6400 rd. is 288 rd. ?
4. Of $12 is $.15 ? Of 520 men is 13 men ?
5. Of 433$\frac{1}{3}$ is 7$\frac{2}{9}$? Of 16 bu. is 3 qt. 1 pint ?
6. Of .62$\frac{1}{2}$ is .1$\frac{7}{8}$? Of 10 lb. is 2 lb. 10 pwt. ?

7. A farmer had 175 sheep and sold them all but 7. What % of the flock did he sell ?

8. From a hogshead of sugar containing 1450 pounds, 220 pounds were sold at one time, and $\frac{2}{3}$ of the remainder at another time. What % of the whole remained ?

9. If a boy sells a gun which cost $12 for $3 more than cost, what % of the cost does he gain ?

10. A merchant sold goods which cost $2400 for $2640. What % of the cost did he gain ?

11. Out of 720 words a student spelled 120 correctly, What per cent of the words did he spell ? What % did he miss ?

12. A farmer raised 260 bushels of oats, and sold all but 65 bushels. What % of his crop did he sell ?

13. Of 1950 sheep $\frac{1}{3}$ were sold at one time, and $\frac{1}{5}$ of the remainder at another time. What % of the sheep remained unsold ?

14. A merchant bought a barrel of flour for $4.50 and sold it at a profit of 54 cts. What per cent did he make ?

15. A boy bought a gun for $12 and sold it for $16. What per cent of the cost did he make ?

16. A speculator bought a lot for $4800 and sold it for $5180. What per cent did he make ?

17. If a man sleeps $7\frac{1}{5}$ hours out of 24, what % of his time does he sleep ?

18. A, B, and C built a schoolhouse for $3240, of which A paid $1398.60, B paid $35\frac{1}{2}$ per cent, and C the remainder. What per cent of the cost did C pay ?

357. ***Rate and percentage given, to find the base.***

EXERCISES (Oral).

1. How much is 100 % of 44 ? Why ?

2. How much is 100 % of any number ?

3. If 1 % of a number is 3, what is 100 % of the number ?

4. If 1 % of a number is 5, what is the number ?

5. If 6 % of a number is 30, what is 1 % of the number ? What is 100 % of the number ?

6. If 7 % of a number is 63, what is 1 % of the number ? What is the number itself ?

7. 8 % of a number is 40. What is the number ?
8. 5 % of a number is 60. What is the number ?
9. 9 % of a number is 72. What is the number ?
10. $\frac{3}{4}$ % of a number is 18. What is the number ?

Find the number of which :

11. 15 is 20 % ; 40 is $12\frac{1}{2}$ %.
12. 30 is 12 % ; 18 is $37\frac{1}{2}$ %.

13. Henry lost 10 marbles, which was $16\frac{2}{3}$ % of all he had. How many marbles had he at first ?

14. A barrel contains 12 gal. of water, which is 30% of its capacity. How many gallons will the barrel hold?

15. What is the value of a bale of cotton if 6% of it is worth $3? If 8% of it is worth $3.50?

16. A lady having been asked her age, said: "2 years is 8% of my age." How old was she?

17. In an examination, Moses answered 36 questions and made 75%. How many questions were asked in all?

EXERCISES (Written).

358. *1.* 156 is 24 per cent of what number?

EXPLANATION.

24% of the number = 156;

1% of the number = $\frac{156}{24}$.

100% of the number = 100 × $\frac{156}{24}$ = $\frac{15600}{24}$.

OPERATION.

```
24)15600(650
   144
   ---
    120
    120
    ---
     00
```

RULE.—*Divide the percentage by the rate; or, take the percentage as many times as* 100% *contains the rate.*

Find the number of which:

2. 680 is 12½%.
3. 434 is 58⅓%.
4. 837 is 56¼%.
5. 6400 is 4½%.
6. 1535 is 35%.
7. $625 is 75%.
8. 896 is 112%.
9. $7.875 is ⅜%.
10. 840 is 175%.
11. 12 ft. is $\frac{4}{7}$%.

12. A farmer sold 650 bu. of corn, which was 91% of his crop. How many bushels did he raise?

13. A man spends $1260 a year, which is 84% of his salary. What is his salary?

14. 32% of a lot of land sells for $1000. What is the value of the entire lot at the same rate?

15. A lady having been asked her age, said: "3 years is 7½% of my age." What was her age?

16. What is the length of a rope, if 29⅗ ft. is 16 % of its length ?

17. A farmer sold 3150 bushels of grain, and had 30 % of his entire crop left. What was his entire crop ?

18. A lad having gathered a number of chestnuts, sold 20 % of them, gave his sister 50 % of the remainder, and had 52 chestnuts left. How many had he at first ?

19. A man owning 45 % of a steamboat sells 16⅔ % of his share for $6510. What is the value of the boat ?

20. A man has 13⅓ % of his capital in bank, and 68½ % in merchandise. If the latter is $4634 more than the former, what is the amount of his capital ?

21. James, Frank, and Henry together bought a pony; James paid 63 % of the cost, Frank 27 %, and Henry the remainder, and Frank paid $3.40 more than Henry. What did the pony cost ?

359. ***Rate and amount or difference given, to find the base.***

1. The result of increasing a number by 12 % of itself is 560. What is the number ?

SOLUTION.

100 % of the number = the number.
12 % " " " = the increase.
112 % " " " = the amount, 560.
1 % " " " = $\frac{560}{112}$ = 5.
100 % " " " = 100 × 5 = 500.

2. The result of decreasing a number by 8 % of itself is 368. What is the number ?

SOLUTION.

100 % of the number = the number.
8 % " " " = the decrease.
92 % " " " = the difference, 368.
1 % " " " = $\frac{368}{92}$ = 4.
100 % " " " = 100 × 4 = 400.

RULES.—*Divide the amount by* 1 *plus the rate.*
Divide the difference by 1 *minus the rate.*

EXERCISES (Oral).

360. What number increased by:

1. 10% of itself = 66? *2.* 25% of itself = 15?

3. $12\frac{1}{2}$% of itself = 27? *4.* 20% of itself = 36?

5. 40% of itself = 28? *6.* 30% of itself = 65?

What number decreased by:

7. 20% of itself = 48? *8.* 25% of itself = 9?

9. 50% of itself = 35? *10.* $16\frac{2}{3}$% of itself = 25?

11. $37\frac{1}{2}$% of itself = 55? *12.* 25% of itself = $4\frac{1}{2}$?

13. Henry has 24 marbles, which is 20% more than John has. How many marbles has John?

14. Ann and Mary together have $35, and Mary's part is 75% of Ann's. How many dollars has Ann?

15. A clerk's salary was decreased 10%, and now it is $360. What was it at first?

16. 25% of a pole was broken off, and it is now 15 ft. long. How long was it at first?

17. Charles is 16 years old, and is $33\frac{1}{3}$ per cent. older than Moses. How old is Moses?

18. A piece of cloth, after shrinking 5% of its length, was 19 feet long. What was its length before shrinking?

19. What fraction increased by 20% of itself equals $\frac{8}{15}$?

20. What fraction decreased by $16\frac{2}{3}$% of itself equals $\frac{1}{4}$?

21. In selling a cow for $55, I made 10% of the cost. What did the cow cost?

22. In selling a gun for $21, I lost 30% of the cost. What was the cost of the gun?

EXERCISES (Written).

361. What number increased by:

1. 5% of itself = 168.00? *2.* 15% of itself = $207.80?

3. $4\frac{1}{2}$% of itself = 250.8? *4.* 6% of itself = $382.13?

5. 7% of itself = $58.85? *6.* 8% of itself = $486.81?

7. 5% of itself = $\frac{18}{25}$ sec.? *8.* $7\frac{1}{2}$% of itself = $2\frac{3}{16}$ days?

What number decreased by:

9. 5% of itself = 133.0?
10. 5¼% of itself = 189½ bu.?
11. 16% of itself = $\frac{147}{200}$?
12. 40% of itself = 432 yd.?
13. 7% of itself = $\frac{31}{50}$ bu.?
14. 8% of itself = 258.06 pk.?

15. After increasing a number 6% it is 360.4. What is the number?

16. After decreasing a number 6% it is 169.2. What is the number?

17. A clerk, after spending 65% of his salary, had $315 left. What was his salary?

18. A grocer, after pricing his groceries 12% above cost, asks $84 for a hhd. of sugar. What did the sugar cost?

19. The price of cotton to-day is 10 cents a pound, and is 8% less than it was yesterday. What was the price yesterday?

20. Your watch, which is valued at $45, is worth 12½% more than mine. What is the value of my watch?

21. A merchant sold a stock of goods for $10,811, and gained 13⅘%. What was the cost of the goods?

22. What is the price of cotton, if advancing it ⅔% makes it 6.795. cts?

23. What is the price of sugar, if discounting it ¾% makes it 8.734 cts.?

24. Sold a horse for $76, at a loss of 5%. What % would I have gained, if I had received $10 more?

25. Sold a cow for $42, at a gain of 5%. What % would I have lost, if I had received $12 less?

26. A man deposited 74% of his money in a bank, and afterward drew out 25% of the sum deposited, and then had $2375.40. How much money had he at first?

27. A farmer bought three mules. The first cost 6% more than the second, and the second 5% less than the third. For the first he paid $120.84. What did he pay for the other two?

PROFIT AND LOSS.

362. PRINCIPLE.—*In buying or selling, the gain or loss is always some % of the cost.*

This principle is established by business usage.

1. A farm was bought for $5463.85 and sold at a gain of 6 %. What was received for it ?

Since our smallest coin is 1 cent, we give the answer to the nearest cent. 5 or more mills is counted 1 ct.; less than 5 mills is neglected.

$5463.85 = cost.
.06
$327.8310 = gain.
5463.85 = cost.
$5791.68 = sale.

2. A dealer lost $22.40 in selling a horse at a loss of 16 %. What did the horse cost ?

SOLUTION.	PROOF.
16 % of the cost = the loss, $22.40.	$140 = cost.
1 % of the cost = $\$\frac{22.40}{16}$ = $1.40.	.16
100 % of the cost = 100 × $1.40 = $140.	$22.40 = loss.

3. Bought a piano for $350 and sold it for $308. What % was lost ?

SOLUTION.	OPERATION.
$350 − $308 = $42 = loss.	$350 = cost.
1 % of the cost = $3.50.	308
Hence $42 is as many % of the cost as $3.50 is contained times in $42; that is, 12 %.	3.50)42.00(12

4. A cow was sold for $48.60 at a gain of 8 %. What was the cost of the cow ?

SOLUTION.	PROOF.
100 % of the cost = the cost.	$45 = cost.
8 % of the cost = the gain.	.08
108 % of the cost = the sale, $48.60.	$3.60 = gain.
1 % of the cost = $\$\frac{48.60}{108}$ = $.45.	45.
100 % of the cost = 100 × $.45 = $45.	$48.60 = sale.

EXERCISES (Oral and Written).

363. *1.* How many dollars are made in selling a cow which cost $22 at a profit of 25% ?

2. What is the gain in selling a horse which cost $97.20 at a profit of 13½% ?

3. If flour which cost $6 a barrel is damaged 33⅓%, for how much should it be sold ?

4. A man paid $3250 for a farm, and sold it at a loss of 16%. What did he receive for it ?

5. What % does a boy make who buys a gun for $40 and sells it so as to make $8 ?

6. If a farm cost $8560 and is sold for $1712 more than cost, what per cent is made ?

7. What % does a man make who buys a gun for $12 and sells it for $16 ? What % does he lose if he buys it for $16 and sells it for $12 ?

8. What % would a dishonest dealer gain by using a false weight of 15 oz. instead of a pound ? What % would the customer lose ?

9. A girl gave 5% of her money for a doll that cost 45 cts. How much money had she at first ?

10. What is the value of a house if it rents for $400 at 6⅔% ?

11. In selling a watch for $48 I gained 20% of the cost. What did the watch cost ?

12. A man sold a horse for $126.60 and gained 5½%. Find the cost of the horse.

13. If in selling a cow for $60 there is a loss of 25%, what did the cow cost ?

14. What did a drover pay for a mule which he sold for $85.05 at a loss of 5½% ?

15. James and John together buy a baseball. James paid 18 cents, which was 40% of what both paid. How much did John pay ?

16. A farmer raised two crops of corn. The second year he gathered 3689 bu., which was 58⅓% of what he raised in both years. How much was the first crop?

17. Bought a horse for $80. What must I ask for him, so that 20% of the price may be deducted and still leave a profit of 50%?

Solution.

Selling price = (100% + 50%) of cost = $120.
(100% − 20%) of asking price = selling price.
80% " " " = $120.
1% " " " = $1.50.
100% " " " = $150.

18. In 1900 a farmer raised 640 bu. corn. In 1901 he raised 15% more than he did in 1900, and 8% less than in 1902. How many bushels did he raise in 1902?

19. A man sold a cow for $55 and made 10% on the cost. What % would he have lost if he had sold the cow for $46?

20. A man has three farms, of which the first is worth $2816 and the third $3424. If the third is worth 7% more than the second, what % is the value of the first less than that of the second?

The value of the second farm is, in both instances, the base.

21. A lad bought two sheep, and sold one for $12 at a profit of 33⅓%, and the other for $8 at a loss of 20%. What did he make or lose by the transaction?

22. A drover sold one horse for $134.40, at a profit of 12%, and another for $138 at a loss of 8%. How much did he make or lose?

23. How must cloth, costing $4 a yard, be marked that a merchant may deduct 16⅔% from the marked price and still make 25% profit?

24. How must a watch that cost $50 be marked, so that 12% may be deducted and a profit of 10% made?

25. Bought 2 horses for $96 each; sold one at a gain of 33⅓% and the other at a loss of 25%. What did I gain or lose by the trade?

26. Bought 2 houses for $2060 each; sold one at an advance of 23 % and the other at a discount of 31 %. What did I make or lose by the transaction?

27. If 20 % of a number is equal to 33⅓ % of 48, what is the number?

28. A merchant bought a wagon for $48, and after marking it observed that 16 % of the cost was equal to 12 % of the marked price. What was the price?

29. A boy had 60 apples, gathered 25 % more, and sold them all at 2 cents each. How much did he receive?

30. A drove of 140 sheep, after increasing in number 15 %, was sold at $2¼ a head. What was received for all?

31. A boy sold 8 melons for 60 cts., at a profit of 50 %. What was the cost per melon?

32. A grocer sold 82 barrels of apples for $250.10, at 22 % profit. What was the cost per barrel?

33. What is the result of decreasing $60 by 33⅓ %, and then increasing the result by 50 %?

34. A merchant paid $580 for a lot of goods; 12½ % of them proved to be damaged, and he sold the remainder at an advance of 14$\frac{2}{7}$ %. How much did he gain or lose?

35. A man sold a saddle for $10 and lost 33⅓ %. If he had sold it at 20 % profit, what would he have received?

36. Sold a piano for $240, and lost 20 %. What selling price would have given a profit of 17⅓ %?

37. Sold hats for $1 less than cost, and lost 12½ %. What did they cost?

38. If corn, selling for 10⅖ cts. a bushel less than cost, gives a loss of 15 %, what did it cost?

39. A merchant sold goods for $228 and lost 5 %, whereas he should have gained 15 %. How much were they sold under their proper value?

40. A drover bought 24 mules at $80 apiece. If 6 of them should die, at what % profit must he sell the remainder in order to realize a clear gain of 15 % by the speculation?

COMMISSION AND BROKERAGE.

364. Commission or **brokerage** is a fee allowed an **agent** who transacts business for another, called the **principal.**

An agent who buys and sells merchandise is called a **commission merchant,** and one who collects debts, rents, etc., a **collector.**

Commission is the fee allowed a collector or commission merchant for transacting business.

The **consignor** or **shipper** is the person who sends goods to be sold; a **consignment,** the goods sent; and the **consignee** or **correspondent,** the person to whom the goods are sent.

The **net proceeds** of a transaction is the sum of money due the **principal** or **consignor** after all expenses of commission, etc., are paid.

An **account sales** is a written statement made by a commission merchant to his principal, containing an account of goods sold, their price, the expenses, and the net proceeds.

365. PRINCIPLES.—I. *If an agent sells, his commission is some % of the sales.*

II. *If an agent buys, his commission is some % of the purchase.*

1. Find the amount of sales when an agent receives $92.50 at a commission of 5%.

SOLUTION.— 5% of the sales = $92.50.
1% " " " = $\frac{1}{5}$ of $92.50 = $18.50.
100% " " " = 100 × $18.50 = $1850.—Ans.

2. A commission merchant receives $189, with instruction to invest it in flour after deducting his commission at 5%. Find the cost of the flour.

SOLUTION.—100% of the purchase = the cost of the flour.
5% " " " = the commission.
105% " " " = the amount, $189.
1% " " " = $\frac{\$189}{105}$ = $1.80.
100% " " " = 100 × $1.80 = $180, cost of flour.

EXERCISES (Oral and Written).

366. *1.* A collector collects for me $650, and charges 4% commission. What is his commission?

2. A commission merchant sold cotton to the amount of $3240.16. What was his commission at 2¾%?

3. An agent collects for Mr. Jones a debt of $125 at 8% commission. How much should the agent retain, and what should go to Mr. Jones?

4. A commission merchant received a consignment of 4 bales of cotton, averaging 456 lb. to the bale, and sold it at 7⅔ cts. a pound. How should the proceeds be divided between the agent and the principal, commission being 1½%?

5. What is the rate of commission when $5 is charged for collecting $125?

6. An agent sold $1400 of bank stock, and charged $17½ for his services. What was the rate of commission?

7. An agent sold a gun for $20 and remitted $19 to the principal. What was the rate of commission?

8. What is the rate of commission if an agent remits the principal $1212.50 from a sale of $1250?

9. A collector collected a debt on a commission of 12½%, and received $15 for his services. What was the amount of the debt?

10. What is the amount of sales when an agent, who charges 2¾% commission, receives a commission of $160.60?

11. An agent receives $44, to be invested in flour after 10% of the cost of the flour is deducted for commission. Find the amount paid for flour.

12. An agent receives $420, with instructions to invest it in sheep at $8 per head. How many sheep does he purchase, commission being 5%?

13. What amount of wool, at $.52 a pound, can be bought for $3109.60, after taking out a commission of 4%?

14. If an agent receives $63 to be invested in sheep, what will be his commission at 12½%?

15. A consignor sent an agent $2846.25 to be invested in flour. What will be the fee of the agent at 3½%?

16. An agent in New Orleans received $414 to be invested in prints, at 12½ cts. a yard. After taking out his commission of 3½%, how many yards did he purchase?

17. A commission merchant, who charges 2% for selling, and 2% for buying, receives 16 bales of cotton, averaging 510 pounds, with instructions to sell it and invest the proceeds in flour. If he sells the cotton at 8 cts. a pound and buys the flour at $4 a barrel, how many barrels does he buy?

INSURANCE.

367. Insurance is a contract by one party to indemnify another party for loss or damage. It is of two kinds, viz.: **property** and **personal.**

Property insurance includes **fire insurance, marine insurance,** and **live stock insurance.**

Fire insurance is indemnity for loss of property by fire; **marine insurance,** for loss of vessel or cargo; and **live stock insurance,** for loss of horses, cattle, etc.

Personal insurance includes **life insurance, health insurance,** and **accident insurance.**

The **insurer** or **underwriter** is the party who takes the risk or makes the contract, and the **insured** or **policy-holder** is the party to be indemnified in case of loss. The **policy** is the written contract between the insurer and the insured.

The **premium** is the sum of money paid by the insured for the insurance.

PRINCIPLE.—*The premium is some % of the amount insured.*

A house worth $6000 was insured for ¾ of its value at 2%. What was the premium?

SOLUTION.—¾ of $6000 = $4500, the amount insured.
$4500 × .02 = $90, the premium.

EXERCISES (Oral and Written).

368. *1.* What will it cost to have a house insured for $1500, at 3%?

2. A man owning a mill worth $1436, had it insured at 3¼%. What was the premium?

What is the premium for insuring:

3. 5 bales of cotton worth $45 each, at 2%?

4. A drove of 23 horses, valued at $97 each, at ¾%?

5. What will be the charges for insuring a house and furniture for $3750, at 3%?

6. What will it cost to have a ship, valued at $32,560, insured ⅝%?

7. A man has his horse insured for $150, at 3%. If the horse should die, what would be the net loss of the insurer?

8. What net loss is sustained by the underwriters in the destruction of the cargo of a ship, which they had insured for $10,250, at ¾%?

9. A man paid $3 to have his horse insured for $150. What was the rate of insurance?

10. A man had his dwelling insured for $1240, and paid $43.40 premium. What was the rate of insurance?

What is the rate of insurance:

11. If $4 premium is paid for an insurance of $700?

12. If $8⅖ premium is paid for an insurance of $480?

What is the amount of the annual premium:

13. At $25 per $1000, on a life policy of $3200?

14. At $42 per $1000, on a life policy of $8600?

15. A farmer paid $6 for having his barn insured at the rate of 2%. What was the barn valued at?

16. What is a mill valued at, if a premium of $16.50 is paid to insure it, at 1⅜%?

What is the amount of insurance:

17. If the premium is $3, and the rate ⅔%?

18. If the premium is $⅔, and the rate 7⅑%?

19. A man paid $3 to have his horse insured. The horse died, and the total loss of the insurers was $117. What was the rate of insurance?

20. A ship was insured for ¾ of its value at a premium of $285.168. The ship was destroyed, and the insurers sustained a net loss of $17,537.832. What was the rate of insurance, and the value of the ship?

21. One fourth of a piece of property was insured at 1¼%, ⅔ of the remainder at 1⅕%, ⅖ of the last remainder at 1½%, and the sum of the premiums was $425. What was the value of the property?

TAXES.

369. A **tax** is a sum of money assessed on the person, property, or business of an individual for any public purpose.

A tax on the person of an individual is called a **poll** or **capitation tax**; a tax on property, a **property tax**; and a tax on business, an **excise tax.**

Property is of two kinds: **real** and **personal.**

Real property, or **real estate,** is immovable property; as lands and houses.

Personal property is movable property; as money, tools, ships, furniture, horses, etc.

An assessor is a person appointed to estimate the value of property, and apportion the taxes accordingly.

370. PRINCIPLE.—*The collector usually receives some % of the amount collected.*

1. What sum must be raised to build a schoolhouse at a cost of $8037, and pay 5% for collection?

SOLUTION.

100% of the sum collected = the total cost.
5% " " " " = the commission.

95% " " " " = the cost of house, $8037.
1% " " " " = $\$\frac{8037}{95}$ = $84.60.
100% " " " " = 100 × $84.60 = $8460.

EXERCISES (Written).

371. *1.* What sum must be raised to construct a bridge at a cost of $420, and pay 5 % for collection ?

2. An assessor is required to collect, for public purposes, $12,050. What amount should be assessed, his commission being $2\frac{1}{2}$ % ?

To find the tax of an individual, we have the following general

RULE.—*From the sum to be raised, deduct the poll tax, if any, and divide the remainder by the assessed value of the taxable property, real and personal; the quotient will be the rate of tax.*

Multiply the assessed value of each man's property by the rate, and to the product add his poll tax, if any; the result will be the whole amount of his tax.

3. The tax levied in a certain city was 3 thousand dollars, and the taxable property was 150 thousand dollars. What was the rate, and what were the taxes of A, B, and C, whose properties were valued at $600, $750, and $975 respectively ?

4. In a township where the amount of tax to be raised was $5,184.72, the assessor made the following appraisements : the taxable property, $345,648 ; A's property, $5,240 ; B's property, $4,307.50; and C's property, $874.15. Required the rate of tax, and the taxes of A, B, and C ?

5. A tax of $46,714 is assessed upon a certain parish containing 5360 taxable polls, at $1.25 each; the real estate is valued a $3,474,840, and the personal property at $80,960. What will be

(1) The rate of tax ?

(2) The tax of A, whose property is valued at $5,420, and who pays for 4 polls ?

(3) The tax of B, whose property is valued at $6,275, and who pays for 3 polls ?

INTEREST.

SIMPLE INTEREST.

372. *1.* If I borrow $300, what should I pay for the use of it at the rate of $4 for every $100 (or 4 per cent) ?

2. If I borrow $500, what should I pay for the use of it at 5 % ?

3. If money is loaned at the rate of 6 % per year, how much must be paid for the use of $150 for 1 year ? For 2 years ? 4 years ? 7 years ?

4. If 5 % is charged annually for the use of money, how much must be paid for the use of $400 for 1 year ? For 1½ years ? 2¾ years ? 3⅔ years ?

5. At the rate of 6 % annually, how many dollars must be paid for the use of $200 for 1 yr.? For 1 yr. 6 mo.? 2 yr. 4 mo.? 3 yr. 2 mo.? 4 yr. 9 mo.? 5 yr. 7 mo.?

373. Interest is money paid for the use of money.

374. The **principal** is the money for the use of which interest is paid.

375. The **rate of interest** is the per cent to be taken of the principal for its use one year or other specified time.

Rate *per year* is to be understood in this book when no other time is specified.

376. The **amount** is the sum of the principal and interest.

377. Simple interest is the sum paid for the use of money of which the accrued interest forms no part.

Legal interest is the interest at the rate fixed by law. Usury is interest at a higher rate than legal interest.

In some of the States a higher rate than the legal is allowed, if specified in writing. When no rate is specified in accounts, notes, etc., the legal rate is always understood.

In computing interest it is usual to regard a year as 12 months, and a month as 30 days.

378. ***To find the interest for any number of years and months.***

Since 12 months are 1 year, any number of months may be reduced to the fraction of a year by dividing by 12.

Thus, 8 mo. = $\frac{8}{12}$ yr. = $\frac{2}{3}$ yr. 3 yr. 4 mo. = $3\frac{1}{3}$ yr.

1. What is the amount of $340.60 for 2 yr. 4 mo., at 7%?

Explanation.—Since the interest of any sum at 7% for 1 yr. is .07 of the principal, the interest of $340.60 for 1 yr. at 7% is .07 of $340.60, or $23.842; the interest for 2 yr. 4 mo. is $2\frac{1}{3}$ times the interest for 1 yr., or $55.6313; and $55.6313 + $340.60 = $396.2313, the amount.

Operation.

$340.60	Principal.
.07	Rate.
$23.8420	Int. for 1 yr.
$2\frac{1}{3}$	No. of yr.
$47.6840	Int. for 2 yr.
7.9473	Int. for $\frac{1}{3}$ yr.
$55.6313	Int. for 2 yr. 4 mo.
340.60	Principal.
$396.2313	Amount.

Rule.—*Multiply the principal by the rate, and the product is the interest for* 1 *year.*

Multiply the interest for 1 *year by the time in years, and the fraction of a year; the product is the required interest.*

Add the principal to the interest for the amount.

EXERCISES (Written).

379. Find the interest and amount of:

1. $200 for 2 yr. 6 mo., at 5%.
2. $864 for 3 yr. 9 mo., at 8%.
3. $150 for 4 yr. 4 mo., at 6%.
4. $250 for 4 yr. 5 mo., at 8%.
5. $360 for 1 yr. 8 mo., at 5%.
6. $750 for 5 yr. 8 mo., at 4%.
7. $5.50 for 1 yr. 9 mo., at 8%.
8. $94.08 for 1 yr. 11 mo., at 9%.
9. $540.60 for 7 yr. 7 mo., at 7%.
10. $702.24 for 3 yr. 5 mo., at 5%.
11. $437.28 for 4 yr. 1 mo., at 11%.
12. $6.318 for 2 yr. 10 mo., at $5\frac{1}{2}$%.

13. Find the interest of $325 from May 16, 1900, to Jan. 16, 1904, at 6%.

SOLUTION.

				325
1904 yr.	1 mo.	16 da.		.06
1900	5	16		19.50
3	8		= 3⅔ yr.	3⅔
				71.50

Find the amount of:

14. $221.40 from June 26, 1899, to Feb. 26, 1902, at 5%.

15. $108.36 from March 9, 1888, to June 9, 1900, at 6%.

16. $323.04 from April 7, 1901, to Dec. 7, 1901, at 7½%.

17. What is the interest of $300 from May 12, 1898, to July 12, 1901, at 6%?

18. What is the amount of $20.15 from Dec. 17, 1900, to March 17, 1905, at 8%?

19. A man borrowed $165.60, Feb. 16, 1881, and settled the account Oct. 16, 1885. How much did he owe, the rate of interest being 8%?

20. Find the interest *orally* of $180 for 2 yr. 6 mo., at 8%.

SOLUTION.—2 yr. 6 mo. = 2½ yr. Since the rate for 1 yr. is 8% for 2½ yr., it is 2½ times 8%, or 20%. 20% = ⅕, and ⅕ of $180 = $36—Ans.

Find the interest *orally* of:

21. $256 for 1 yr. 8 mo., at 6%.

22. $408 for 6 yr. 3 mo., at 4%.

23. $523 for 1 yr. 3 mo., at 8%.

24. $808 for 2 yr. 6 mo., at 5%.

25. $9.60 for 4 yr. 2 mo., at 8%.

26. $420.40 for 5 yr., at 5%. *27.* $630.10 for 8⅓ yr., at 6%.

28. $177.30 for 2 mo., at 6%. *29* $923.15 for 6 da., at 6%.

380. ***To find the interest and amount for any number of years, months, and days.***

Since 30 days are reckoned as a month, 3 days is $\frac{1}{10}$ or .1 of a month; hence any number of days may be reduced to tenths of a month by dividing by 3. Thus, 6 da. = .2 mo.; 18 da. = .6 mo.; 22 da. = .7⅓ mo.;

5 mo. 12 da. = 5.4 mo.; 2 yr. 7 mo. 27 da. = 31.9 mo.; 1 yr. 2 da. = 12.0⅔ mo.

Rate per year may be reduced to rate per month by dividing by 12. Thus, 7% per yr. = $\frac{7}{12}$% per month; 8% per annum = ⅔% per month; 6% yearly = ½% monthly.

1. Find the interest of $516.60 for 2 yr. 3 mo. 18 da., at 8%.

EXPLANATION.—Rate per mo. = 8% ÷ 12 = .00⅔. Hence the int. for 1 mo. is .00⅔ of the principal, which is $3.444, and for 2 yr. 3 mo. 18 da., or 27.6 mo., it is 27.6 times $3.444, which is $95.0544, the required int.

OPERATION.

$516.60	**Principal.**
.00⅔	**Rate per mo.**
$3.4440	**Int. for 1 mo.**
27.6	**No. mo.**
$95.05440	**Int. for 27.6 mo.**

RULE.—I. *Divide the rate by* 12; *the quotient is the monthly rate.*

II. *Multiply the principal by the rate per month; the product is the interest for* 1 *month.*

III. *Multiply the interest for* 1 *month by the time in months, and tenths of a month; the product is the required interest.*

EXERCISES (Written).

381. Find the interest of:

1. $200 for 1 yr. 3 mo. 15 da., at 6%.
2. $480 for 3 yr. 9 mo. 24 da., at 7%.
3. $900 for 2 yr. 6 mo. 18 da., at 8%.
4. $528.60 for 1 yr. 1 mo. 1 da., at 5%.
5. $1500 for 3 yr. 1 mo. 3 da., at 4%.
6. $38.40 for 2 yr. 7 mo. 17 da., at 4½%.
7. $400 for 3 mo. 27 da., at 9%.
8. $504.72 for 7 mo. 26 da., at 6%.
9. $240 for 21 da., at 5%.
10. $326.40 for 19 da., at 9%.
11. $300 for 4 yr. 12 da., at 8%.
12. $347.83 for 5 yr. 8 da., at 7%.
13. $834 for 9 yr. 10 mo. 15 da., at 6%.

Find the interest of:

14. $23.75 for 7 yr. 7 mo. 21 da., at 5%.

15. $.325 for 9 yr. 5 mo. 14 da., at 8%.

16. $2480 for 4 mo. 4 da., at 7½%.

17. $570.05 for 3 yr. 5 mo. 5 da., at 6½%.

What is the amount of:

18. $100 for 2 yr. 6 mo. 15 da., at 6% ?

19. $531.70 for 3 yr. 3 mo. 23 da., at 6% ?

20. $500 for 1 yr. 8 mo. 6 da., at 12% ?

21. $37.41 for 7 yr. 6 mo. 29 da., at 6⅜% ?

Find the amount of:

22. $346.85 from Sep. 15, 1901, to Apr. 13, 1902, at 5%.

23. $427.93 from Apr. 13, 1900, to Nov. 3, 1900, at 6%.

24. $318.60 from Aug. 7, 1901, to Dec. 1, 1905, at 7%.

25. A debt of $40.50 was paid May 21, 1890, with interest at 6% from Nov. 9. 1884. What was the amount?

26. A note of $150.75, dated June 15, 1897, was paid Jan. 1, 1900, with interest at 5%. What was the amount paid?

27. A note of $1630, dated April 1, 1878, was paid Oct. 10, 1882, with interest at 6%. What was the interest paid?

28. An account of $2150 was contracted May 10, 1877, and settled with interest at 6%, Jan. 1, 1881. What amount was paid?

382. ***A convenient method of computing interest when the time is expressed in days.***

EXERCISES (Oral).

1. Find the interest of $3465.72 for 6 da. at 6%.

EXPLANATION.—6 da. = $\frac{1}{60}$ yr.; hence the int. of $3465.72 for 6 da. at 6%
= $3465.72 × $\frac{6}{100}$ × $\frac{1}{60}$
= $3465.72 × .001 = $3.46572.

OPERATION.

3465.72 Principal.
3.46572 Interest.

Hence, to find the interest of any principal for 6 da., at 6%, *move the decimal point three places to the left.*

Tell instantly the interest for 6 da., at 6 %, of:

2. $283. *3.* $4385. *4.* $1375.60. *5.* $2.30

6. $2.45. *7.* $60.50. *8.* $40268.5. *9.* $.935.

Find the interest of $4040. for 93 da. at 6 %:

EXPLANATION.—Moving the decimal point three places to the left gives $4.04, which is the int. for 6 da. Therefore the int. for 93 da. will be as many times $4.04 as 6 da. is contained times in 93 da.; that is, 15½ times $4.04 is the required interest.

OPERATION.

$93 \div 6 = 15\frac{1}{2}$.

$4.04 \times 15\frac{1}{2} = 62.62$.

Hence, to find the interest of any principal for any number of days, at 6 %, *move the decimal point three places to the left, and multiply by $\frac{1}{6}$ of the number of days.*

This method may be employed for any rate by first finding the interest at 6 %, and then increasing or decreasing the interest by such a part of itself as will give the required interest.

Thus, for 5 % we decrease the interest at 6 % by $\frac{1}{6}$ of itself, because $5 = 6 - (\frac{1}{6} \text{ of } 6)$. Similarly, for 8 % we add on a third, $8 = 6 + (\frac{1}{3} \text{ of } 6)$.

EXERCISES (Written).

383. Find the interest of:

1. $2750 for 60 da., at 6 %.
2. $8750 for 45 da., at 6 %.
3. $20.40 for 30 da., at 6 %.
4. $94.60 for 80 da., at 6 %.
5. $6000 for 75 da., at 6 %.
6. $67.55 for 133 da., at 6 %.
7. $324.25 for 93 da., at 6 %.
8. $150,000 for 12 da., at 8 %.
9. $4365.20 for 33 da., at 8 %.
10. $7865.50 for 200 da., at 9 %.
11. $130.645 for 150 da., at 5 %.

EXERCISES (Written).

384. Solve the following by one of the preceding methods :

1. What is the interest of $367 for 17 da., at 6% ?

2. Amount of $865.60 for 83 da., at 8% ?

3. Interest of $24.60 for 3 yr. 5 mo. 18 da., at 9% ?

4. Amount of $8.84 for 1 yr. 3 mo. 23 da., at 7% ?

5. What amount is due on a note for $250 that has run 70 days, with 8% interest ?

6. What is the interest on a note for $600, dated June 10, 1900, and paid January 3, 1901, at 5% interest ?

7. What is the amount of $1600 for 7 mo. and 18 da., at 7½% ?

8. What is the amount of $63.96 for 1 yr. 7 mo. and 10 da., at 6¼% ?

9. A man bought a farm for $3260, and agreed to pay for it in 1 yr. and 8 mo., with 5% interest. What amount did he pay ?

10. A farmer sold a plantation on the following terms : $1800 in cash on delivery, $2400 in 10 mo., and $2375 in 2 yr. 3 mo., with 5% interest. What was the whole amount paid ?

Problems in Simple Interest.

385. ***Principal, interest, and time given, to find the rate.***

1. Find the interest of $300 for 4 yr. at 1%. At 5%.

2. How many times greater is interest at 6% than interest at 1%, the principal and time being the same ?

3. If the interest is $10 when the rate is 1%, what is the interest when the rate is 7% ?

4. If the interest is $6 when the rate is 1%, what is the per cent when the interest is $30 ?

5. The interest of $200 for 4 yr. at 1% is $8. What should the rate be to make the interest $56 ?

6. What should the rate per cent be in order that the interest of $360 for 3 yr. 4 mo. may be $66 ?

Explanation.—We first find the int. of $360 for 3 yr. 4 mo. at 1 %, which is $12. Then we say, if 1 % gives $12 int., it will take as many per cent to give $66 as $12 is contained times in $66, or 5½ %.

Operation.

$360 \times .01 \times 3\frac{1}{3} = 12.$

$66 \div 12 = 5\frac{1}{2}.$

Rule.—*Divide the given interest by the interest of the principal for the given time, at* 1 %.

EXERCISES (Oral and Written).

386. What is the rate per cent, when the interest :

1. Of $40 for 2 yr. 6 mo. is $15 ?
2. Of $64 for 3 yr. 9 mo. is $12 ?
3. Of $1500 for 3 yr. 4 mo. is $150 ?
4. Of $240 for 1 yr. 9 mo. is $29.40 ?
5. Of $100 for 2 yr. 3 mo. is $13.50 ?
6. Of $218 for 3 mo. 12 da. is $4.63 ?
7. Of $218 for 9 mo. 18 da. is $12.208 ?
8. Of $216.50 for 1 yr. 5 mo. 18 da. is $22.227⅓ ?

9. At what rate per cent must $453.20 be loaned for 2 yr. 8 mo. in order that the interest may be $60.42⅔ ?

10. A house which cost $1500 rents for $75 a year. What per cent does it pay on the investment ?

11. I invest $840 in a business that pays me $3.85 a month. What annual rate of interest do I realize on my investment ?

12. A loaned B $600, and in 12 yr. 6 mo. it amounted to twice $600. What was the rate ?

387. ***The principal, rate, and interest given, to find the time.***

1. Find the interest of $250, at 4 %, for 1 year. For 5 years.

2. How many times greater is interest for 7 years than for 1 year, the principal and rate being the same ?

3. If the interest for 1 yr. is $20, what is the interest for 3½ yr. ?

4. If the interest for 1 yr. is $8, for how many years will the interest be $42 ?

5. The interest of $300 for 1 yr., at 4 %, is $12. What should the time be to make the interest $33 ?

6. In what time will $516.60 produce $95.0544 interest at 8 % ?

EXPLANATION—We first find the int. of $516.60 for 1 yr. at 8 %, which is $41.328. Then we say, if 1 yr. gives $41.328 int., it will take as many years to give $95.0544 int. as $41.328 is contained times in $95.0544, or 2.3 yr.

OPERATION.

$$516.60 \times .08 \times 1 = 41.328$$
$$95.0544 \div 41.328 = 2.3$$
$$2.3 \text{ yr.} = 2 \text{ yr. } 3 \text{ mo. } 18 \text{ da.}$$

RULE.—*Divide the given interest by the interest of the principal for* 1 *year at the given rate.*

EXERCISES (Oral and Written).

388. In what time will:

1. $400 produce $50 interest at 5 % ?
2. $560 produce $42 interest at 6 % ?
3. $600 produce $35 interest at 7 % ?
4. $576 produce $260.48 interest at 8 % ?
5. $378 produce $101.43 interest at 6 % ?
6. $184.80 produce $18.48 interest at 9 % ?

7. A man borrowed $180 at 5 %, and kept it until the amount was $210. How long did he keep it ?

8. How long must $369.60 be on interest at 7 % to amount to $547.6856 ?

9. How long will it take $100 to gain $100, at 2 % ? 4 % ? 5 % ? 10 % ?

At 100 % any principal doubles itself in one year; hence, to double itself at 2 %, it will require as many years as 2 is contained in 100, etc.

10. How long will it take any principal to double itself at 2 % ? 3 % ? 4 % ? 5 % ? 10 % ? 12½ % ?

11. In what time will the interest of $375.20, at 7 %, equal the principal ? *Twice* the principal ?

12. How long must I keep $672.24 on deposit, at 6 %, to pay a debt of $762.4322 ?

13. The use of $800, at 8 %, for 3 yr. 4 mo. and 24 da. is equivalent to the use of $600, at 6 %, for what length of time ?

389. ***The rate, time, and interest given, to find the principal.***

1. What is the interest for 5 yr. at 6 % of $1 ? Of $7 ?

2. How many times greater is the interest of $8 than of $1, the time and rate being the same ?

3. If the interest of $1 is 12 cts., what is the interest of $9 ?

4. If $1 produces $.09 interest, how many dollars will produce $1.80.

5. What is the interest of $1 for 2 yr. 6 mo. at 6 % ? What sum, then, for the same time and at the same rate, will produce $3 interest ?

6. What sum of money at 6 % will produce $25.20 interest in 1 yr. 6 mo. ?

EXPLANATION.—We first find the int. of $1 for 1½ yr. at 6 %, which is $.09. Then we say, if $1 produces $.09 it will take as many dollars to produce $25.20 as $.09 is contained times in $25.20, or $280.

OPERATION.

$1 \times .06 \times 1\frac{1}{2} = .09$

$25.20 \div .09 = 280.$

RULE.—*Divide the interest by the interest of* $1 *for the given time at the given rate.*

If the amount is given instead of the interest, divide the amount by the amount of $1 for the given time and rate.

EXERCISES (Oral and Written).

390. What sum of money will produce :

1. $30 interest in 2 yr. 6 mo. at 6 % ?
2. $24 interest in 7 yr. 6 mo. at 5 % ?
3. $60 interest in 3 yr. 9 mo. at 4 % ?
4. $49 interest in 8 yr. 4 mo. at 7 % ?

What sum of money will amount to :

5. $60 in 2 yr. 6 mo. at 8 % ?
6. $848.9657 in 4 yr. 10 mo. 15 da. at 6 % ?
7. $1129.1952 in 3 yr. 10 mo. at 8 % ?
8. $1206.4344 in 3 yr. 7 mo. 18 da. at 5 % ?

9. A man borrowed some money at 6 %, kept it 2 yr. 2 mo. 25 da., then settled the whole amount, $762.4322. How much did he borrow ?

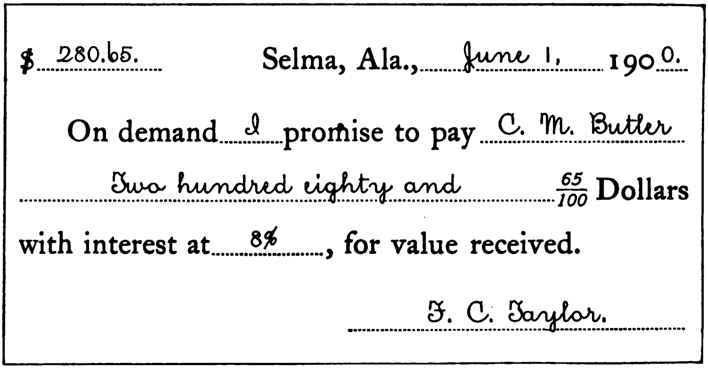
Partial Payments.

391. A promissory note is a written promise by one party to pay a certain sum of money to another party, on demand, or at a specified time.

A note should contain the place where it was made, the date, the time when payable, the amount or face of the note written in words, the words "for value received" and "with interest" if such is the contract.

Forms of Notes.

(*1*)



$ 280.65. Selma, Ala., June 1, 1900.

On demand I promise to pay C. M. Butler

Two hundred eighty and $\frac{65}{100}$ Dollars

with interest at 8%, for value received.

T. C. Taylor.

(2)

$ 300. Atlanta, Ga., Dec. 12, 1901.

Sixty days after date, for value received, I

promise to pay J. P. Cornwell or order,

Three Hundred $\frac{00}{100}$ Dollars

at The Lowry National Bank,

with interest at 5%.

No. 25. H. B. Brown.

(3)

$ 164.25. New Orleans, La., May 3, 1901.

Four months after date we promise to

pay Page Baker or bearer,

One hundred sixty-four and $\frac{25}{100}$ Dollars

value received, with interest at 6%.

F. F. Hansell & Bro.

The **face** of a note is the sum promised to be paid.

The **maker or drawer** of a note is the party who promises to pay, and who signs the note. The **payee** is the party to whom, or to whose order, the money is to be paid.

An **indorser** is a person who signs his name on the back of a note, and thereby makes himself responsible for its payment.

A **negotiable note** is one made payable to bearer, or to the order of the payee. When so made, it can be sold or transferred, otherwise the note is payable only to the payee.

392. Partial payments are part payments of notes, bonds, or other obligations.

Indorsements are acknowledgments of such payments written on the back of the note, bond, etc., stating the time and amount of the same.

For computing the balance due on an interest-bearing note on which partial payments have been made, two rules are in common use; viz., the *mercantile rule* and the *United States rule.* The mercantile rule is generally used when the note is settled in full within a year.

393. *To compute the balance due by the mercantile rule.*

1. On a note for $820 at 8%, dated Jan. 7, 1898, were the following indorsements: March 25, 1898, $400; June 7, 1898, $150. What was due Dec. 19, 1898?

Solution.—Amount, at 8% simple int., of:

$820 from Jan. 7, 1898, to Dec. 19, 1898 =		$882.32
$400 from Mar. 25, 1898, to " " " =	$423.46	
$150 from June 7, 1898, to " " " =	156.40	$579.86
Balance due " " " "		$302.45

Rule.—I. *Find the amounts of the principal and payments from their dates to the time of settlement.*

II. *Subtract the sum of the amounts of the payments from the amount of the principal; the result is the balance due.*

EXERCISES (Written).

1. Find the amount due on a note for $720, at 6%, dated Jan. 12, 1901, on which were the following indorsements: March 15, $180; May 18, $100; Aug. 23, $230; Oct. 3, $102. What was due Dec. 6, 1901?

2. A note was given for $2500, Feb. 5, 1900, at 8%, on which the following payments were made: April 8, $700; June 14, $550; Aug. 20, $640; Nov. 25, $480. What was due 1 year from date?

3. A bill of goods, amounting to $1008, was to be paid May 5, 1900. Received July 5, 1900, $360; Sep. 20, 1900, $450; Dec. 15, 1900, $180. What was due May 5, 1901, at 8% interest?

4. An account of $750, due Jan. 1, 1901, received the following payments that year: June 10, $145; Sep. 23, $465; Oct. 3, $23. What was due on the 31st of the next December, allowing 6% interest?

The mercantile rule works injustice to the holder of a note when the note runs longer than a year, as may be seen by an extreme case. Thus, if A owes B $500, at 6% interest, and only pays the interest annually, he will discharge the debt in about 24 years, without the payment of one cent of the principal.

394. ***To compute the balance due by the United States rule.***

1. A note for $600, dated June 7, 1898, payable on demand, bears the following indorsements: March 19, 1899, $140.20; Aug. 28, 1900, $31.50; Feb. 12, 1901, $208.70. What was due Jan. 5, 1902, reckoning interest at 6%?

SOLUTION.

Principal,	$600
Int. from June 7, '98, to Mar. 19, '99,	28.20
Amount,	$628.20
First payment,	140.20
Balance due (2d principal),	$488.00

Int. from Mar. 19, '99, to Aug. 28, '00, $42.21. This interest exceeds the corresponding payment, $31.50. If it were added to $488 and the payment subtracted, the new principal would be $498.71, and the borrower would be paying interest on $10.71 of interest. Hence we proceed as follows:

Int. from Mar. 19, '99, to Feb. 12, '01,	$55.55
Amount,	$543.55
Second payment and third payment,	240.20
Balance due (3d principal),	$303.35
Int. from Feb. 12, '01, to Jan. 5, '02,	16.83
Balance due Jan. 5, 1902,	$320.18

RULE.—*Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest.*

From this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal, and proceed as before.

EXERCISES (Written).

395. *1.* Let the student solve, by this method, Ex. 1 of the preceding article, and compare the results.

Demand Note.

2. $360. HOMER, LA., *June* 3, 1900.

For value received, I promise to pay Dr. J. C. Willis on demand, Three hundred sixty Dollars, with interest at 6%.

G. G. GILL.

Indorsements: Aug. 9, 1900, $140; Nov. 21, 1900, $92; Feb. 6, 1901, $75. Required the amount due May 15, 1902.

Time Note.

3. 475\frac{50}{100}$. BATON ROUGE, LA., *Sep.* 11, 1894.

Ninety days after date, I promise to pay Thos. D. Boyd, or bearer, Four hundred seventy-five $\frac{50}{100}$ Dollars, with interest at 8%, for value received. H. SKOLFIED.

Indorsements: Dec. 23, 1894, $100; Jan. 29, 1895, $120; May 8, 1895, $6; July 14, 1896, $64.50. Required the balance due Aug. 2, 1896.

Joint and Several Note.

4. $580.25. AUSTIN, TEXAS, *Feb.* 17, 1899.

Five months after date, we jointly and severally promise to pay G. N. Price, or order, Five hundred eighty $\frac{25}{100}$ Dollars, with interest at 7%, for value received.

N. P. CRAVENS.
J. H. WEBSTER.

Indorsements: April 23, 1899, $200; Sep. 2, 1899, $150; March 14, 1901, $84.20; April 6, 1902, $20.60. Required the balance due May 17, 1903.

Exact Interest.

396. When 12 months of 30 days each, or 360 days, is regarded as a year, the interest obtained for months and days is not strictly correct. To find the exact interest we must reckon 365 days to a year.

397. *To compute exact interest.*

1. Find the exact interest of $2568 for 93 da., at 6 %.

SOLUTION.

$2568 × .06 = $154.08, interest for 1 year.
$154.08 ÷ 365 = $.422+, interest for 1 day.
$.422+ × 93 = $39.26, interest for 93 days.

EXERCISES (Written).

Find the exact interest of:

2. $642.72 for 25 da., at 6 %.

3. $144.50 for 144 da., at 5 %.

4. $567.50 for 73 da., at 4 %.

5. $5200 for 123 da., at 7 %.

COMPOUND INTEREST.

398. Compound interest is interest on the principal and accrued interest for some specified time.

Interest is usually compounded annually, semi-annually, or quarterly, according to agreement.

Compound interest cannot usually be collected by law, even if it is specified in the contract.

399. *To compute compound interest.*

1. Find the compound interest of $200 for 2 yr. 6 mo., at 6 %.

SOLUTION.

$200	Principal.
10	Interest for 1st yr., at 5 %.
$210	Principal for 2d year.
10.50	Interest for 2d yr., at 5 %.
$220.50	Principal for 3d year.
5.5125	Interest for 6 mo., at 5 %.
$226.01+	Amount for 2 yr. 6 mo., at 5 %.
200.	Original principal.
$26.01	Comp. int. for 2 yr. 6 mo., at 5 %.

EXERCISES (Written).

400. What is the compound interest of:

1. $650 for 2 yr., at 6 % ?

2. $600 for 2 yr., at 5 % ?

3. $600 for 3 yr., at 8 % ?

4. $500 for 5 yr., at 4 % ?

Find the amount, at compound interest, of:

5. $400 for 2 yr., at 10 %.

6. $325 for 4 yr., at 8 %.

Find the compound interest of:

7. $240 for 2 yr. 6 mo., at 8 %.

8. $360 for 3 yr. 4 mo., at 6 %.

9. $250 for 2 yr. 3 mo., at 6 %.

10. $535 for 3 yr. 5 mo., at 7 %.

11. $580 for 3 yr. 8 mo. 20 da., at 6 %.

12. $800 for 3 yr. 6 mo. 24 da., at 6 %.

13. If interest be compounded semi-annually, what will be the amount of $450 for 2 yr. 3 mo., at 8 % ?

Compound interest may be readily computed by using the following compound interest table, page 226.

14. Find the amount at compound interest of $500 for 6 yr., at 5 %.

Solution.—By the table we find that the amount, at compound interest, of $1 for 6 yr., at 5 %, is $1.340096 ; therefore the amount of $500, for the same time and rate, is

500 × $1.340096 = $670.05.

15. Find by the table the compound interest of $524 for 3 yr. 6 mo., at $8 %, interest payable semi-annually.

Explanation.—3 yr. 6 mo. = 7 periods of 6 mo. each ; 8 % per annum = 4 % per pd. The amt. of $1 for 7 pds., at 4 %, is $1.315932, which, multiplied by 524, = $689.55, the amt.; and this minus the first principal = the comp. int.

Operation.

524 × $1.315932 = $689.55
$689.55 − $524 = $165.55

16. Find the compound interest of $800 for 2 yr. 3 mo. 15 da., at 7%.

SOLUTION.

$1.1449 Amount of $1 for 2 yr. at 7%.
.02337 Int. of $1.1449 for 3 mo. 15 da. at 7%.

$.16827 Interest of $1 for 2 yr. 3 mo. 15 da. at 7%.
$.16827 × 800 = $134.62, the required interest.

Find by the table the compound interest of:

17. $600 for 2 yr. 6 mo. at 6%.
18. $360 for 3 yr. 3 mo. at 5%.
19. $650 for 3 yr. 4 mo. 12 da. at 6%.
20. $889.37 for 6 yr. 9 mo. 21 da. at 7%.

21. To what will $450 amount in 2 yr., at 8%, if the interest be compounded semi-annually.

22. To what will $735.60 amount in 2 yr. 5 mo. 24 da., at 8%, if the interest be compounded quarterly?

23. What principal, at 7% compound interest, will amount to $52.432 in 4 years?

SOLUTION.—I find from the table that the compound amount of $1 for the given time and rate is $1.310796; hence the required principal is as many dollars as $1.310796 is contained times in $52.432, or $40.

24. What principal, at 6% compound interest, will produce $2370 in 10 years?

25. What sum will amount to $640.405 in 6 yr., at 8%, compounded semi-annually?

Compound Interest Table.

401. *Amount of $1, at various rates, compound int. from* 1 *to* 20 *periods of time.*

Pd.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Pd.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.269712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

Annual Interest.

402. Annual interest is simple interest on the principal, and on each year's interest of the principal, to the time of settlement.

To compute annual interest.

1. Find the annual interest and amount of $500 for 4 yr. at 8 %.

Solution.—The annual int. is $40, the first of which will draw int. 3 yr.; the second 2 yr.; the third 1 yr.

Int. of $500 for 4 yr., at 8 %,	$160
Int. of $40 for 3 yr., at 8 %,	9.60
Int. of $40 for 2 yr., at 8 %,	6.40
Int. of $40 for 1 yr., at 8 %,	3.20
Total int.,	$179.20

Amount, $500 + $179.20 = $679.20.

2. What is the annual interest of $450 for 5 yr. at 8 % ?

Explanation.—The annual interest is $36, which is to be taken 5 times; and 8 % of it is to be taken 4 times, 3 times, 2 times, 1 time, or 5 × (5 − 1) ÷ 2 times, = 10 times. Hence the annual interest is the sum of 5 × $36, and 10 × .08 of $36.

Operation.

$5 \times (5 - 1) = 20;\ 20 \div 2 = 10$

$\$450 \times .08 = \$36.$

$\$36 \times 5 = \$180.$

$\$36 \times .08 \times 10 = 28.80$

Int. $= \$208.80$

Rule.—*Compute the interest on the principal for the given time and rate, to which add the interest on each year's interest, for the time it has remained unpaid.*

To obtain the latter, multiply the interest on each year's interest by one-half the product of the number of years by that number less one.

EXERCISES (Written).

403. *1.* What will $3450 amount to in 10 yr. at 6 %, annual interest?

2. What is the annual interest of $40.50 for 3 yr. at 4 % ? For 5 yr. at 5 % ? For 8 yr. at 6 % ? For 15 yr. at 8 % ?

3. What is the difference between the annual interest and the compound interest of $1240 for 4 yr. at 7 % ?

4. What amount will be due on the following note at maturity?

$475. NEW ORLEANS, LA., *July* 2, 1901.

Five years after date, for value received, I promise to pay J. S. Richardson, or order, Four hundred seventy-five Dollars, with interest at seven per cent, payable annually.

WILLIAM G. JONES.

DISCOUNT.

404. Discount is the sum deducted from a debt or price.

There are three kinds of discount, viz.: *Commercial discount*, *true discount*, and *bank discount.*

Commercial Discount.

405. Commercial discount is a deduction from the price or value of a commodity.

Manufacturers and wholesale dealers generally have fixed price-lists of their goods, and when the market varies, instead of changing these prices, they change the rate of discount. The fixed price is called the **list price.**

The term "30 and 10 off," or "a discount of 30 % and 10 %," means that 30 % is first deducted, leaving 70 % of the price, from which a deduction of 10 % is made, leaving a remainder of 63 % of the price. Hence, "30 and 10 off" is equivalent to a single deduction of 37 %.

The term "¼ and 12 % off" means a deduction of ¼ of the price, and then a deduction of 12 % from the remainder.

The amount of a bill less the discounts is called the **net amount.**

1. Find the net amount of a bill of $400 with 10 and 5 off.

SOLUTION.

10% of $400 = $40; $400 − $40 = $360;
5% of $360 = $18; $360 − $18 = $342—Ans.

EXERCISES (Written).

406. Find the net amount of a bill of:

1. $350 with a discount of 8% for cash.
2. $400 with discounts of 12% and 5%.
3. $240 with 15 off.
4. $600 with $\frac{1}{6}$ and 10% off.
5. $840 with 25 and 33$\frac{1}{3}$ off.
6. $900 with 15, 10, and 5 off.

7. The gross amount of a bill of goods on a credit of 60 days is $1275. What is the cash value of the goods with 10 and 8 off, and a discount of 4% for cash?

8. A fine clock is listed $75 with $\frac{1}{5}$ and 10% off. For what can the clock be bought?

9. Find the net cash amount of a bill of $280.40, discounts being $\frac{1}{5}$, $\frac{1}{4}$, and 5%.

10. On a bill of $1200, find the difference between a discount of 25% and a discount of 15% and 10%.

11. What is the difference between the net costs of two pianos which are listed, respectively, $900 with $\frac{1}{3}$ and 16$\frac{2}{3}$ off and $835 with $\frac{1}{4}$ and 20 off?

12. Reduce "a discount of 40% and 10%" to a single discount.

SOLUTION.

1 − .40 = .6; 1 − .20 = .8; .6 × .8 = .48 = 48%—Ans.

13. Find a single discount equivalent to the successive discounts 20%, 16$\frac{2}{3}$%, and 25%.

TRUE DISCOUNT.

407. *1.* Is $100 cash worth as much as $100 plus its use one year from now? At 6%, what is its use worth? How much cash, then, should discharge a debt of $106 due 1 year from now?

2. At 4%, what is the use of $50 worth for 2 yr. 6 mo.? How much ready money, then, is equivalent to $55 payable in 2 yr. 6 mo. from now?

3. If the use of money is worth 5%, is $60 cash equivalent to $70 payable 3 yr. 5 mo. from now? Why?

The **present worth** of a sum of money due at the end of a fixed time is the sum that, put at interest for the fixed time, will amount to the given sum.

True discount is the difference between the whole debt and the present worth.

Thus, $100 will in 3 yr., at 5%, amount to $115. Therefore, $100 paid now is equivalent to the payment of $115 3 yr. from now. Here $100 is the present worth of $115 payable in 3 yr. at 5%, and $115 − $100 = $15 is the true discount.

To find the present worth and true discount.

1. What is the present worth and true discount of $762.14 payable in 7 yr. 3 mo. 18 da., at 6%?

Solution.—The amount of $1 for the given time and rate is $1.438. Hence the present worth of $1.438 due in 7 yr. 3 mo. 18 da., at 6%, is $1, and therefore the present worth of $762.14 for the same time and rate is as many dollars as $1.438 is contained times in $762.14, or $530. The true discount is $762.14 − $530, or $232.14.

The operation of finding the present worth of a sum of money at a given rate of interest is the same as the operation of finding the principal when the amount, time, and rate are given.

Rule.—*Divide the amount due by the amount of* $1 *for the given time and rate, and the quotient will be the present worth.*

Subtract the present worth from the amount due, and the remainder will be the true discount.

EXERCISES (Oral and Written).

408. Find the present worth of:

1. \$108 payable in 1 yr. 4 mo., at 6%.
2. \$234.80 due in 1 yr. 5 mo. 12 da., at 12%.
3. \$22 payable in 1 yr. 3 mo., at 8%.
4. \$451.50 due in 2 yr. 6 mo. 27 da., at 5%.
5. \$52 payable in 3 yr. 4 mo., at 9%.
6. \$216.66 due in 3 yr. 4 mo. 22 da., at 6%.

Find the true discount of a debt of:

7. \$125 payable in 6 yr. 3 mo., at 4%.
8. \$185.75 due in 5 yr. 4 mo. 24 da., at 9%.

9. A boy bought a gun for \$5.30 on a credit of 9 mo. If he should conclude to pay for it at once, what ought he to pay, the use of money being worth 8%?

10. A man owes \$1608, due in 1 yr. 3 mo. 20 da., which he wishes to pay immediately. How much should he pay, money being worth 5%?

If the use of money is worth 6%, what is the difference in cash:

11. Between buying flour for \$5.20 on 8 mo. time, and for \$6.30 on 10 mo. time?

12. Between selling cotton for \$640 on 7 mo. time, and for \$650 on 8 mo. time?

13. Bought a sheep for \$4.50, and sold the same for \$6 on 2 yr. 6 mo. time. How much ready money was made at 8%?

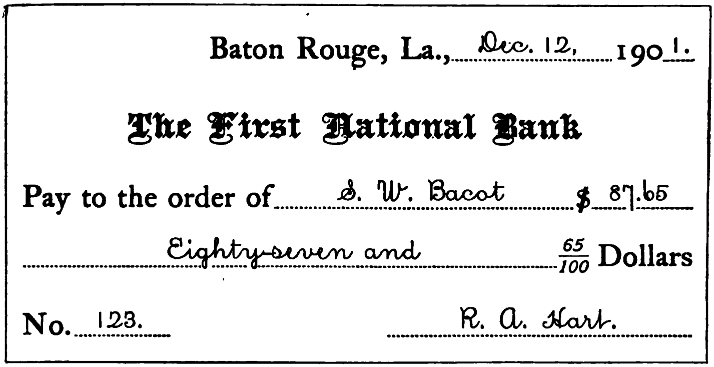
14. Bought a farm for \$2560, and sold it for \$3625, payable in 1 yr. 6 mo. 20 da. How much was gained in ready money, discounting at 8%?

BANK DISCOUNT.

409. A **bank** is a corporation chartered by law for the safe-keeping, loaning, or issuing of money.

410. A **check** is a written order of a depositor, directing a bank to pay a specified sum of money to a certain person, or to his order.

The following is a common form of a check :



Baton Rouge, La., Dec. 12, 1901.

The First National Bank

Pay to the order of S. W. Bacot $87.65

Eighty-seven and $\frac{65}{100}$ Dollars

No. 123. R. A. Hart.

If the check is made payable to the order of the payee, as above, the payee must indorse it by writing his name across the back of it.

411. A large part of the business of banks is the paying of notes before they are due.

A note is payable at the date specified in the note, but three days, called days of grace, are usually allowed before payment can be legally enforced. Some States have abolished days of grace, but in the following examples they are to be allowed.

A note matures on the last day of grace.

Banks do no business on Sundays or legal holidays, and notes falling due then are payable on the preceding or succeeding business day, according to the law or custom of the locality.

412. The **term of discount** is the whole time the note has to run ; that is, the time from the date of discount to the date of maturity.

In some localities the term of discount includes the day of discount and the day of maturity.

Bank discount is the sum retained by a bank in cashing a note before it is due, or on which it loans money. *It is simple interest for the term of discount upon the sum due on a note at its maturity.*

The **proceeds** of a note is the difference between the sum due on the note at its maturity and the bank discount.

A **protest** is a statement made by a notary public, giving legal notice to the maker and indorsers of a note of its non-payment. If a note is not protested on the third day of grace, the indorsers are released from all obligation to pay it.

413. *To find the bank discount and proceeds.*

1. Find the bank discount and proceeds of a note of $250, payable in 90 days, discounted at 8 %.

Solution.—The term of discount is 90 da. + 3 da. = 93 da.
$250 × .08 = $20, int. of $250 for 1 yr., or 360 da.
$ 20 ÷ 4 = $ 5, int. of $250 for 90 da.
$ 5 ÷ 30 = $ .17, int. of $250 for 3 da.
Hence, $5.17, int. of $250 for 93 da., is the discount,
and $250 − $5.17 = $244.83, is the proceeds.

Questions in bank discount are calculated like questions in simple interest, discount being used instead of interest, and rate of discount instead of rate of interest. Any convenient method of computing interest may be employed in finding the discount.

EXERCISES (Written).

414. Find the bank discount and proceeds of a note:

1. Of $150, payable in 60 days, discounted at 9 %.
2. Of $137, payable in 45 days, discounted at 7 %.
3. Of $386, payable in 100 days, discounted at 8 %.
4. Of $64.50, payable in 200 days, discounted at 6 %.
5. Of $36.20, payable in 80 days, discounted at 5 %.

6. A note for $350, payable in 5 months, was discounted at a bank at 8 %. What was the bank discount?

7. What are the proceeds of a note for $1500 due in 4 months, discounted at a bank at 6 %?

8. I sell a horse for $150, payable in 8 mo. and 18 da., with 6 % interest, for which I take a note. What will be the proceeds of the note when discounted at a bank at 8 %?

Note.—If a note is on interest, find the discount on the amount of the note at maturity.

9. What are the proceeds of a note for $1000 due in 90 da., bearing interest at 9 %, and discounted at a bank at the same rate?

415. ***To find the date of maturity, the term of discount, the discount, and the proceeds.***

$800. NASHVILLE, TENN., *May* 12, 1901.

Sixty days after date I promise to pay J. E. Morgan Eight hundred Dollars, for value received.

Discounted at 6 %, June 1. S. J. WILLIAMS.

SOLUTION.

60 da. less 19 da. in May = 41 da., less 30 da. in June = 11 da. Therefore the note becomes due July 11/14,—11 denotes the day it is nominally due, and 14 the day it is legally due.

The term of discount is 29 da. in June plus 14 da. in July = 43 da.

The discount is the interest of $800 for 43 da. at 6 %. Therefore the discount is $\$.8 \times 6\frac{5}{6} = \$5.47$.

The proceeds = $800 − $5.47, or $794.53.

ANS.—Date of maturity, July 14; term of discount, 43 da.; discount, $5.47; proceeds, $794.53.

In finding the date of maturity, bankers generally count forward by months or by days, as the note specifies. The term of discount, when not expressed in years or months, is usually found in days.

$640. AUSTIN, TEXAS, *March* 4, 1901.

Five months from date we jointly and severally promise to pay to the order of E. R. Moore Six hundred forty Dollars, for value received, with interest at 6 %.

E. E. CORRY.

Discounted at 8 %, May 23. C. H. CHERRY.

SOLUTION.

Interest of $640 for 5 mo. 3 da., at 6 % = $16.32.
Amount of note when due, $640 + $16.32 = $656.32.
Date of maturity, March 4, + 5 mo. + 3 da. = Aug. 7.
Term of discount, from May 23 to Aug. 7, = 76 da.
Discount, interest of $656.32 for 76 da., at 8 % = $11.08.
Proceeds, $656.32 − $11.08 = $645.24.

EXERCISES (Written).

416. Find the date of maturity, the term of discount, the discount, and the proceeds of the following notes :

1. $385. New Orleans, La., *June* 19, 1901.

Sixty days from date I promise to pay to the order of B. F. Jonas Three hundred eighty-five Dollars, value received.

Discounted at 5 %, July 16. Warren Easton.

2. $1400. Demopolis, Ala., *Sep.* 2, 1900.

Four months from date I promise to pay to the order of J. M. Rice Fourteen hundred Dollars, value received.

Discounted at 6 %, Sep. 16. M. J. Rye.

3. $135.50. Little Rock, Ark., *Aug.* 20, 1902.

Three months from date, for value received, I promise to pay J. J. Miller, or order, One hundred thirty-five and $\frac{50}{100}$ Dollars. I. D. Newell.

Discounted at 6 %, Sep. 7.

4. $750. Charleston, S. C., *June* 16, 1896.

Nine months from date, for value received, I promise to pay G. W. Bolton, or bearer, Seven hundred fifty Dollars, with interest at 6 %. H. F. Lyons.

Discounted at 6 %, Oct. 14.

5. $700. Macon, Ga., *Jan.* 31, 1901.

One month after date I promise to pay to the order of J. W. Jones Seven hundred Dollars, with interest at $4\frac{1}{2}$ %, value received. S. R. Smith.

Discounted at 6 %, Feb. 8.

6. $685.30. RALEIGH, N. C., *Jan.* 7, 1896.

Sixty days after date, for value received, I promise to pay to the order of H. M. Brownfield Six hundred eighty-five and $\frac{30}{100}$ Dollars. C. K. DAVID.

Discounted Feb. 3, at 8%.

7. $500. JACKSON, MISS., *Sep.* 29, 1902.

Two months from date I promise to pay to the order of J. E. Tate Five hundred Dollars, value received.

Discounted Sep. 29, at 6%. C. W. BUCKNER.

417. ***To find the face of a note when the proceeds, time, and rate are given.***

1. Copy and complete the following note, so that T. W. Atkinson, on presenting it at the Bank of Baton Rouge, may procure $474.96, the bank discounting at 6%:

$ ...... BATON ROUGE, LA., *May* 1, 1902.

Sixty days from date I promise to pay to the order of the Bank of Baton Rouge .... Dollars, value received.

T. W. ATKINSON.

SOLUTION.

The face of the note is found thus:

The discount on $1 for 63 da. at 6% = $.0105.
The proceeds of $1 = $1 − $.0105 = $.9895.
The face required = $474.96 ÷ .9895 = $480.

RULE.—*Divide the proceeds by the proceeds of* $1 *at the given rate for* 3 *days more than the specified time.*

EXERCISES (Written).

418. Required the face of a note, that the proceeds:

1. May be $1924, for 7 mo. 15 da., at 6%.

2. May be $824, for 60 da., at 8%.

3. What sum of money, payable in 3 mo. 15 da. without interest, discounted at 8%, is equivalent to $864 cash?

4. I had a note payable in 5 mo. 3 da. without interest, which I traded to a bank, at 6% discount, and received $1240. What was the face of the note?

5 A farmer owes a merchant $925. How large a note, payable in 4 mo. 15 da., when discounted at a bank at 10%, will cancel the debt?

STOCKS AND INVESTMENTS.

419. **Stock** is the money or other property of banks, railroads, and incorporated companies.

A **share** of stock is one of the equal parts into which the stock of a company is divided; the original value of which is generally $100.

The **par value** of stock is its original value; and the **market value** of stock is the sum for which it will sell. When stock will sell for its original value, it is said to be at *par;* when it will sell for more than its original value, it is *above par*, or at a *premium;* and when for less, it is *below par*, or at a *discount.* Stock quoted at 100 is at par; at 105, is 5% above par; at 93, is 7% below par; etc.

A **dividend** is a sum paid to stockholders out of the earnings of the company, and is reckoned on the par value of the stock.

A **bond** is an instrument in writing under seal, given to secure the payment of a sum of money at a specified time. Bonds are issued for the purpose of borrowing money.

The principal bonds bought and sold as stock are government, state, city, and railroad bonds; and these are quoted according to the rate of interest which they bear. Thus, U. S. 4's are bonds issued by the United States, bearing 4% interest.

A **stock broker** is a person whose business it is to buy and sell stocks; and brokerage is the compensation he receives for his services.

420. *Brokerage is computed upon the par value of the stock.*

EXERCISES (Written).

421. *1.* Find the cost of 80 shares of railroad stock at 103½, brokerage ⅛%.

SOLUTION.—103½% + ⅛% = 103⅝%.
103⅝% of $100 = $103⅝, cost of 1 share.
80 × $103⅝ = $8290, cost of 80 shares.

2. Find the cost of 45 shares of the Southern Pacific R. R. stock at 101¼, brokerage ⅛%.

3. How much will 120 shares of the Canadian Pacific R. R. stock cost at 83⅞, brokerage ⅛%?

4. I wish to buy 24 shares of a cotton mill stock at a discount of 10%. What will they cost, brokerage ⅛%?

5. What must be paid for 160 shares of Telegraph stock at 12¼% premium, brokerage ⅛%?

6. How many shares of stock, at 101¼, can be bought for $12,165, brokerage being ⅛%?

Solution.—Since the market value of one share is $101¼, each share, including the brokerage, will cost $101⅜; hence, as many shares can be bought as $101⅜ is contained times in $12,165, which is 120.

7. How many shares of stock, at a discount of 11½%, can be bought with $10,635, brokerage ⅛%?

8. How many shares of stock, at a premium of 7¾%, can be bought with $4315, brokerage ⅛%?

9. Bought stock for 96¾ and sold it for 98½, paying ⅛% brokerage each way. How much did I make on each share?

Operation: (98½ − ⅛) − (96¾ + ⅛) = 1½; hence, $1½.

10. How many shares of stock, bought at 102½ and sold at 105¾, brokerage each way ⅛%, will yield a gain of $750?

11. What sum must be invested in U. S. 4's, at 115½, to yield an income of $480?

Solution.—The income of 1 share is $4; hence the number of shares is $480 + $4, or 120. Now 120 shares at $115½ each = $13,860.

What sum must be invested:

12. In stock at 96½, that pays an annual dividend of 6%, to yield an annual income of $1200?

13. In stock, at a premium of 3½%, that pays a semi-annual dividend of 4½%, to yield an annual income of $1512?

14. Stock is at a discount of 3¼%, but bears 3¼% interest. What sum must be invested in it to yield an annual income of $260, brokerage being ⅛%?

15. What income will be realized by investing $9760 in 4% bonds, purchased at 121⅞, brokerage ⅛% ?

Solution.—$121⅞ + $⅛ = $122, cost of share.
$9760 ÷ $122 = 80, the No. of shares bought.
1 share yields $4. 80 shares yield 80 × $4 = $320.

16. If I invest $8208 in 5 per cent bonds, at a premium of 2⅖%, paying ⅕% brokerage, what annual income will I receive ?

17. City stock is at a discount of 5%, but pays 3% interest. If I invest $6685 in it, paying ½% brokerage, what yearly income will I receive ?

18. What rate per cent will be realized from bonds bought at 125. and drawing 10% interest ?

Solution.—The cost of 1 share is $125, and the annual income of 1 share is $10 ; hence the rate per cent is $\frac{10}{125}$, or $\frac{2}{25}$, of 100% = 8%.

Find the rate % of income realized :

19. From 5% bonds, bought at 80.

20. From 6% bonds, bought at 10% discount.

21. From 3¾% bonds, bought at 12½% premium.

22. Which is the more profitable, to buy 8% bonds at 120, or 5% bonds at 80 ?

23. What per cent of income do U. S. 4's yield, if bought at a premium of 19½%, brokerage being ½% ?

24. At what price should 6% bonds be bought to yield an income of 5% ?

Solution.—Since the annual income of 1 share is $6, this must be 5% of the price ; hence the price is $6 ÷ 5% = $120, or 20% premium.

What must be paid :

25. For 7% stock to yield an income of 10% ?

26. For 8% stock to yield an income of 5% ?

27. At what price must 4% stock be bought to pay as well as 5% stock bought at 20% discount ?

28. A man bought 130 shares R. R. stock at 106¼, and afterwards sold them at 109¾. What was the gain, brokerage in each transaction being ⅛% ?

29. How much must be invested in 7% city bonds, bought at 101½, brokerage ⅛%, to yield an annual income of $840 ?

30. What annual income will a man derive from investing $10777.375 in railroad bonds paying an annual dividend of 10%, if he buys them at 98⅜, brokerage ⅛% ?

31. How many shares of the Burton Lumber Co. stock can be bought with $6874, at 61¼, brokerage ⅛% ?

32. The stock of a certain company pays a dividend of 12%. At what price must I buy it so as to realize 8% on the investment ?

33. What is the price of stock when $11,600 of it may be bought with $8729 ?

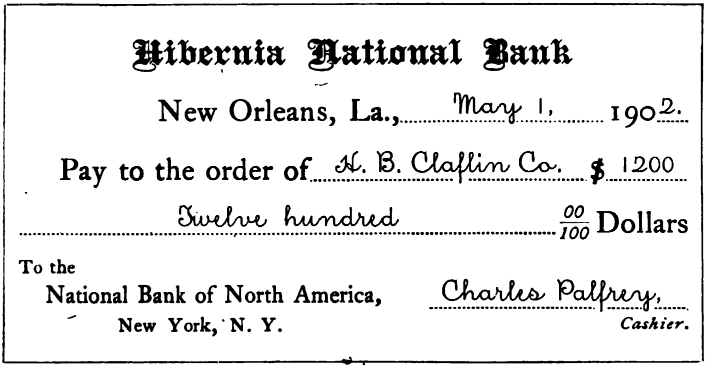
34. The trustees of a college had $48,000, and invested one-half of it in 5% railroad bonds at 95⅞, and the other half in 6% city bonds at 119⅞, paying ⅛% brokerage in each case. What income did they secure ?

EXCHANGE.

422. If a man lives in New Orleans and owes a debt in New York, can he pay it without sending the actual money? Can he pay it by a *postal money order?* Can he pay it by an *express money order?*

423. The payment of money in distant places by *drafts* is called **exchange.**

The payment of money by drafts is done through banks. Every prominent bank in this country has a correspondent in New York ; that is, another bank in which it (the first bank) has more or less money on deposit. For example, we may suppose that the National Bank of North America, located in New York, is a correspondent of the Hibernia National Bank of New Orleans. Now suppose D. H. Holmes Co., of New Orleans, owe H. B. Claflin Co., of New York, the sum of $1200. To pay the debt, an agent of D. H. Holmes Co. could apply to the Hibernia National Bank for a written order on the National Bank of North America to pay H. B. Claflin Co. $1200 out of its (the Hibernia Bank) deposits. Written orders of this kind are called drafts, and are of the form :



Hibernia National Bank

New Orleans, La., May 1, 1902.

Pay to the order of H. B. Claflin Co. $ 1200

Twelve hundred $\frac{00}{100}$ Dollars

To the
National Bank of North America,
New York, N. Y.

Charles Palfrey,
Cashier.

For this draft D. H. Holmes Co. pay the Hibernia Bank $1200 and a small sum for its services, take the draft and send it to H. B. Claflin Co., who indorse it, and have it cashed at the bank specified.

Exchange is said to be at a premium or at a discount according as a draft cost more or less than its face value.

Variations in the cost of drafts are due to fluctuations in the course of business. If a dealer in New York buys sugar in New Orleans, and settles by draft, the money remains in some bank in New York to the credit of some bank in New Orleans. If a merchant in New Orleans buys dry goods in New York, the opposite conditions prevail. Hence there may be times when the bankers of New Orleans will not have sufficient money on deposit in New York to meet the drafts they are making on that place. In this case they have to incur the expense and risk of sending money to New York, and this raises the cost of drafts in New Orleans.

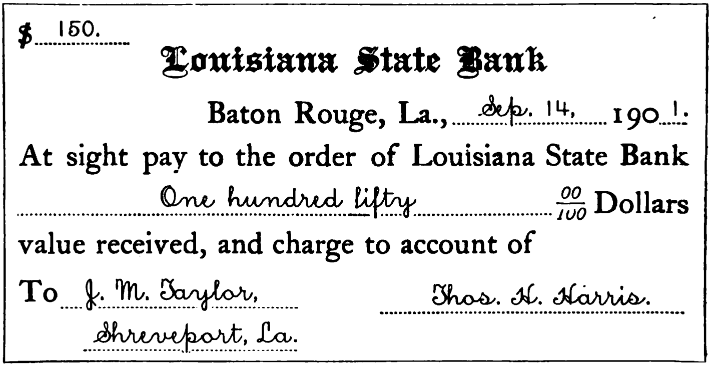
On the other hand, there may be times when the bankers of New Orleans will have large sums of money on deposit in the banks of New York on which they receive no interest. At such times they can afford to sell drafts on New York at a discount, as it avoids the expense of having the money forwarded by express.

In the Southern States, exchange on New York is usually cheap in the fall and winter months, for two reasons : (1) New York buys a large amount of cotton in the South ; and (2) money is needed in the South to "handle the cotton crop."

424. Drafts are used for collecting debts as well as for paying debts. When used for the first purpose they are called **commercial drafts.**

A commercial draft made payable on presentation is called a **sight draft,** and one payable at a specified time after presentation, or after sight, is a **time draft.**

425. The following is the common form of a sight draft:



$ 150.

Louisiana State Bank

Baton Rouge, La., Sep. 14, 1901.

At sight pay to the order of Louisiana State Bank

One hundred fifty 00/100 Dollars

value received, and charge to account of

To J. M. Taylor, Thos. H. Harris.

Shreveport, La.

In time drafts, in the place of the words "at sight" insert "—— days after sight," or "—— days after date." "After sight" means after acceptance.

In the above draft, Harris is the *drawer,* and Taylor the *drawee.*

If the drawee agrees to pay the money specified in the draft on its presentation, he writes his name under the word "Accepted," across its face. This act is called the *acceptance* of the draft.

EXERCISES (Written).

426. *1.* What is the cost of a sight draft on Mobile for \$1500, at \$1½ % premium?

Solution.—Since the rate of premium is 1½ %, \$1 of exchange will cost \$1.01½; hence the cost of \$1500 will be 1500 times \$1.01½, or \$1522.50.

Find the cost of a sight draft for :

2. $500, at 2 % premium.
3. $360, at 1⅛ % premium.
4. $1000, at 2 % discount.
5. $850, at 1⅜ % discount.
6. $480, at 1⅔ % premium.
7. $275.20, at 1¾ % discount.

8. What is the cost of a draft on Richmond for $2000, payable in 60 days after sight, at 6 % interest, exchange at 2½ % premium?

SOLUTION.—Since the premium is 2½ %, $1 of exchange is equal to $1.025, but the bank discount on $1 for 63 da. is $.0105 ; hence the cost of exchange for $1 is $1.025 – $.0105, or $1.0145. Hence the cost of $2000 is 2000 times $1.0145, or $2029.

Find the cost of the *time* drafts :

9. Face $1462, discount 1¼ %, time 30 da., interest 6 %.
10. Face $650, premium ½ %, time 60 da., interest 8 %.
11. Face $600, discount ¼ %, time 90 da., interest 9 %.

12. Find the cost of a sight draft on Augusta for $1256, at 1¾ % discount.

13. What will be the cost of a draft in New Orleans on Cincinnati for $2500, payable in 70 da., at 5 % interest, the rate of exchange being ¾ % premium?

14. How large a draft can be bought for $510, exchange being 2 % premium?

SOLUTION.—Since the rate of premium is 2 %, $1 of exchange will cost $1.02. Hence the face of a draft that will cost $510 will be as many dollars as $1.02 is contained times in $510, or $500.

Find the face of a draft which costs :

15. $412, at 3 % premium.
17. $1164, at 3 % discount.
19. $251.25, at ½ % premium.
16. $39.90, at ¼ % discount.
18. $33.77, at 2⅓ % premium.
20. $103.95, at 5½ % discount.

21. Find the face of a draft, payable 60 da. after sight, that can be bought for $5047.50, when exchange is 2 % premium and interest 6 %.

Solution.—Since the premium is 2 %, $1 of exchange is equal to $1.02, but the bank discount of $1 for 63 da. is $.0105; hence the cost of $1 exchange is $1.02 – $.0105 = $1.0095. Consequently the face of a draft that costs $5047.50 is as many dollars as $1.0095 is contained times in $5047.50, or $5000.

22. What is the face of a 30 days' draft which cost $2029, premium 2 %, interest 6 % ?

23. If exchange is 1⅜ % discount, how large a draft can be bought for $1166.80 payable in 67 da., interest 6 % ?

24. A commission merchant receives a consignment of 25 bales of cotton, averaging 500 lb. to the bale, which he sells at 8 cts. a pound, at 1½ % commission. If he remits the proceeds by a sight draft on New York, what will be its face, exchange being 1½ % premium ?

25. In the last example, what would be the face of the draft if payable in 60 da., at 6 % interest, exchange being 1¼ % discount ?

PROPORTION.

RATIO.

427. How many times does 12 yd. contain 2 yd. ? What, then, is the relation of 12 yd. to 2 yd. ?

What is the measure of 15 pints by 5 pints as a unit ? What, then, is the relation of 15 pints to 5 pints ?

What is the relation of 24 to 3 ? Of 28 bu. to 4 bu. ?

428. Ratio is the relation of one number to another of the same kind expressed by their quotient or measure.

Thus, the ratio of $15 to $3 is 5 ; of 20 to 10, 2.

The **sign of ratio** is the colon (:).

Thus, 16 : 6 is read "the ratio of 16 to 6." Therefore the value of 16 : 6 is 16 ÷ 6, or $\frac{16}{6} = 2\frac{2}{3}$. That is, an expression like 25 : 10 means that 25 is to be divided by 10.

The two numbers compared are the terms of the ratio : the first term, or dividend, is the **antecedent**; the second, or divisor, the **consequent.** The two terms together form a **couplet.**

The terms of a ratio must be like numbers. There can be no such thing as the ratio 5 acres : 6 pints, because we cannot compare acres and pints; nor can we determine the value of 5 yards : 6 feet, until the terms are reduced to the same denomination. Thus, 5 yd. = 15 ft., and 15 ft. : 6 ft. = $2\frac{1}{2}$.

Again, $\frac{1}{2} : \frac{3}{5} = \frac{5}{10} : \frac{6}{10}$ = 5 tenths : 6 tenths = $\frac{5}{6}$.

EXERCISES (Written).

429. What is the ratio (in lowest terms) :

1. Of 18 to 9 ? Of 40 to 8 ? Of 21 to 35 ?
2. Of 12 to 9 ? Of 12 to 28 ? Of 50 to 15 ?
3. Of 3 to 15 ? Of 24 to 16 ? Of 12 to 32 ?

Find the value of :

4. 21.6 : 8.1 ; 4 yd. : 8 ft. ; 5 bu. : 1 bu. 1 pk.
5. 5.1 : 1.19 ; 8 gal. : 3 qt. ; 3 qt. : 7 gal. 1 pt.
6. 1.43 : 6.5 ; 11 ft. : 5 ch. ; 3 da. : 3 hr. 9 min.

Find the value of :

7. 2 days : 6 days ; 2 ninths : 6 ninths ; $\frac{1}{2} : \frac{2}{3}$; 150 : $16\frac{2}{3}$.
8. $3\frac{1}{3}$ bu. : $2\frac{1}{2}$ bu. ; 7 tenths : 5 tenths ; $\frac{3}{4} : \frac{9}{8}$; $43\frac{1}{3} : 7\frac{2}{9}$.
9. $8\frac{1}{3}$ pk. : $2\frac{2}{9}$ pk. ; 6 fifths : 4 thirds ; $\frac{2}{5} : \frac{3}{4}$; $21\frac{7}{8} : 7\frac{1}{7}$.
10. Which is the greater, 3 : 4 or 4 : 5 ?

SOLUTION.—3 : 4 = $\frac{3}{4} = \frac{15}{20}$;
4 : 5 = $\frac{4}{5} = \frac{16}{20}$;
Therefore 4 : 5 is $\frac{1}{20}$ more than 3 : 4.

Which is the greater :

11. 3 : 4 or 12 : 16 ? *12.* 6 bu. : 10 bu. or $\$\frac{1}{2} : \$\frac{5}{6}$?
13. 7 : 10 or 15 : 20 ? *14.* 36 pk. : 27 pk. or $3\frac{1}{3} : 2\frac{1}{2}$?

How much more is :

15. \$5 : \$6 than 155 : 186 ? *16.* $\frac{3}{5}$ da. : $\frac{3}{4}$ da. than $3\frac{1}{5} : 26\frac{1}{4}$?
17. $3\frac{1}{2} : 9\frac{1}{3}$ than $14 : 43\frac{3}{4}$? *18.* 2.4 : 16 than .03 : 60 ?

PROPORTION.

430. *1.* What is the ratio of 6 to 3 ? Of 8 to 4 ?

2. Is 6 : 3 equal to 8 : 4 ?

3. What is the value of 10 : 4 ? Of 15 : 6 ? Has 10 the same relation to 4 that 15 has to 6 ?

4. What is the value of 5 : 15 ? Of 7 : 21 ? Has 5 the same relation to 15 that 7 has to 21 ?

5. Name two numbers having the relation that 3 has to 12 ; that 5 has to 30 ; that 20 has to 10.

Name two numbers having the relation of :

6. 3 to 9. 4 to 24. 6 to 30. 15 to 45.
7. 6 to 3. 12 to 4. 36 to 9. 22 to 11.
8. 2 to 8. 30 to 3. 7 to 63. 45 to 9.

9. Is 15 : 3 equal to 40 : 10 ? Would they be equal if 8 were written in the place 10 ? If 40 were changed to 50 ?

Correct the following by changing one or more of the terms :

10. $15 : 3 = 12 : 4$. $20 : 5 = 24 : 7$.
11. $8 : 24 = 5 : 16$. $10 : 2 = 3 : 15$.

12. The relation of 9 to 27 = the relation of 11 to 36.
13. The relation of 16 to 3 = the relation of 63 to 12.

insert the right number in the place of x :

14. $12 : 6 = 18 : x$. $\frac{4}{5} : \frac{2}{5} = 14 : x$.
15. $16 : 4 = x : 5$. $\frac{3}{8} : \frac{9}{8} = x : 12$.
16. $42 : x = 18 : 3$. $\frac{2}{7} : x = 3 : 30$.
17. $x : 40 = 3 : 24$. $x : \frac{3}{5} = \frac{10}{11} : \frac{2}{11}$.

431. A **proportion** is an expression of equality of ratios.

8 : 4 = 6 : 3, and 12 : 2 = 30 : 5, are proportions.

432. The **sign** of proportion is the double colon (::), which is equivalent to =.

8 : 4 :: 6 : 3 is read, 8 is to 4 as 6 is to 3, and is equivalent to 8 : 4 = 6 : 3, which is read, the ratio of 8 to 4 is equal to the ratio of 6 to 3, or 8 contains 4 as many times as 6 contains 3.

The **terms** or **proportionals** are the four numbers compared ; the **antecedents,** the first and third terms ; the **consequents,** the second and fourth terms ; the **extremes,** the first and fourth terms ; and the **means,** the second and third terms.

Thus, in 5 : 10 :: 3 : 6, 5, 10, 3, and 6 are the terms, or proportionals; 5 and 3, the antecedents; 10 and 6, the consequents; 5 and 6, the extremes; and 10 and 3, the means.

433. When the second term is equal to the third, it is said to be a **mean proportional** between the extremes.

In 4 : 6 :: 6 : 9, 6 is a mean proportional between 4 and 9.

PRINCIPLE.—*In any proportion the product of the extremes is equal to the product of the means.*

For, take any proportion, as.............. 3 : 2 :: 9 : 6,
which means............................ $\frac{3}{2} = \frac{9}{6}$.
Multiplying both sides by 2 × 6........... $3 \times 6 = 2 \times 9$.

Hence the relation of the four terms of a proportion to each other is such that, if any three of them are given, the other, or missing term, may be found thus :

To find either extreme, *Divide the product of the means by the other extreme.*

To find either mean, *Divide the product of the extremes by the other mean.*

To find the value of x in an expression like 3 : 9 :: 5 : x, is to find the number which belongs in the place of x.

Thus, $x = \frac{9 \times 5}{3} = 3 \times 5 = 15$. That is, 3 : 9 = 5 : 15.

EXERCISES (Written).

434. Find the value of x in :

1. $3 : 4 :: 6 : x$.

2. 3 da. : 10 da. $= 3\frac{2}{5} : x$.

3. $5 : 6 :: x : 7$.

4. 91 yd. : 7 yd. $= x : \$3$.

5. $8 : x :: \frac{1}{2} : \frac{3}{4}$.

6. $2.5 : 62.5 = \$6 : \$x$.

7. $x : 9 :: \frac{2}{3} : 7$.

8. $\$.25 : \$x = 2.5 : 7.5$.

9. $\frac{2}{3} : \frac{3}{4} :: \frac{7}{9} : x$.

10. $\frac{5}{6}$ pt. : $\frac{7}{8}$ pt. $= x : \$\frac{12}{17}$.

11. $x : \frac{10}{3} :: \frac{3}{4} : \frac{7}{8}$.

12. $\$6 : \$16 = 40$ ft. $: x$.

13. $\frac{5}{6} : x :: 4 : \frac{8}{15}$.

14. 34 A. : 85 A. $= \$x : \$\frac{5}{8}$.

15. $\frac{3}{4} : \frac{4}{5} :: x : \frac{16}{21}$.

16. $16\frac{2}{3} : x = 7\frac{7}{9} : 12\frac{3}{5}$.

17. $\frac{2}{9} : x :: \frac{1}{6} : \frac{3}{5}$.

18. $18\ \% : 24\ \% = \$27 : \$x$.

Applications of Proportion.

435. *1.* If 6 yards of cloth cost $18, what will 20 yards cost ?

Operation.

$$6 \text{ yd} : 20 \text{ yd.} :: \$18 : x$$

$$x = \frac{20 \times \$18}{6} = \$60.$$

Explanation.—The relation of 6 yd. to 20 yd., as implied in the problem, is the same as the cost of 6 yd. to the cost of 20 yd. Hence we have the proportion, 6 yd. is to 20 yd. as $18 is to the cost of 20 yd., from which we find, by the rule, the cost of 20 yd. = $60.

Cases may arise in which the student will not know which of the first two terms to make the antecedent. This can always be determined by considering from the problem whether or not the second consequent (or answer) is to be greater or less than the second antecedent, and then arrange the first couplet accordingly, as in the following example:

2. If 4 men can build a fence in 15 days, how long will it take 6 men to build it ?

Operation.

$$6 \text{ men} : 4 \text{ men} :: 15 \text{ da.} : x.$$

$$x = \frac{4 \times 15 \text{ da.}}{6} = 10 \text{ da.}$$

Explanation.—6 men can do the work in less than 15 da. That is, x is to be less than 15 da. ; hence the first consequent is to be less than the first antecedent. Therefore we write the proportion thus:

$$6 \text{ men} : 4 \text{ men} :: 15 \text{ da.} : x.$$

RULE.—*Make that term which has the same unit as the answer the third term.*

If the question requires the answer to be greater than the third term, make the greater of the two remaining numbers the second term; but if the answer ought to be less than the third term, make the lesser of the two numbers the second term.

Divide the product of the second and third terms by the first, and the quotient will be the fourth term.

When possible, use cancellation.

EXERCISES (Written).

1. If a man earns $60 in 9 days, how much will he earn in 6 days?

2. If a horse consumes 82 bu. of corn in 320 days, how much will he consume in 96 days?

3. If a pole 10 ft. high cast a shadow 12 ft. long, what is the height of a tree that cast a shadow 108 ft. long?

4. If 42 acres of land produce 924 bushels of corn, how many acres are required to produce 2772 bushels?

5. If 4 bushels of corn cost $$\frac{7}{8}$, what will 10 bushels cost?

6. If $4\frac{1}{3}$ yards of velvet cost $26.78, how much will $24\frac{1}{2}$ yards cost?

7. If a man goes 15 miles in 4 hours, how far will he travel in $6\frac{3}{5}$ hours at the same rate?

8. How far will a locomotive go in 8 hr. 33 mi., if it runs at the rate of 28 miles in 56 minutes?

9. If $40 draw $8 interest in 2 years, what interest will $90 draw in the same time?

10. If the interest of $225 for one year be $15.75, what would be the interest of $3625 for the same time?

11. If 8 hats can be bought for $18, how many hats can be bought for $45?

12. If 5 sheep can be bought for $20.75, how many sheep can be bought for $398.40?

13. If syrup sells at the rate of 3 quarts for 40 cents, what will be the cost of 5 quarts and 1 pint?

14. If 16 bu. of wheat be worth $19.84, what would 56 bu. 2 pk. 4 qt. be worth?

15. If $\frac{2}{3}$ of an acre is worth $22, what is $\frac{7}{11}$ of it worth?

16. If $\frac{7}{8}$ of a yard of silk is worth $2.10, what will $16\frac{1}{2}$ yards cost?

17. If $10\frac{1}{6}$ pounds of sugar is equal to $4\frac{7}{11}$ pounds of butter, how much butter is equal to $7\frac{1}{3}$ pounds of sugar?

18. The liabilities of a bankrupt merchant are $15,000, and his assets are $6562.50. How much will a creditor receive on a debt of $784?

19. A farmer sowed $7\frac{1}{2}$ bu. of grain on $18\frac{3}{4}$ acres. How many bushels, at this rate, would be needed to sow $43\frac{1}{2}$ acres?

20. B did a piece of work in 24 days, receiving $2.70 per day. C received the same amount for doing another piece of work in 18 days. How much did C receive per day?

21. Two rectangular fields contain equal areas. One of them is $19\frac{1}{2}$ rd. long and $13\frac{1}{3}$ rd. wide. What is the width of the other, its length being $18\frac{4}{7}$ rd.?

22. A piece of work can be done by 24 men in 35 days. After 18 days 8 men quit work. In how many days can the rest finish the work?

23. If 6 persons in 6 months consume 240 pounds of sugar, how much will 9 persons consume in 11 months?

Questions of this kind may be solved by proportion, thus: 6 persons in 8 months = 48 persons in 1 mo.; that is, 48 persons in 1 mo. will consume as much as 6 persons in 8 months. Similarly, 9 persons in 11 months = 99 persons in 1 mo. Hence the problem may be stated thus: If 48 persons consume 240 pounds of sugar, how much will 99 persons consume in the same time?

Hence, 6×8 persons : 9×11 persons : : 240 lb. : x lb.

$$x = \frac{\overset{\overset{5}{\cancel{40}}}{\cancel{240}} \times 9 \times 11}{\cancel{6} \times \cancel{8}} = 5 \times 9 \times 11 = 495.$$

24. If 36 men earn $324 in 18 days, how much will 42 men earn in 27 days?

25. If 12 persons consume 6 barrels of flour in 5 months, in how many months will 15 persons consume 8 barrels?

SOLUTION.—Putting x for the required number of months, remembering that 12 persons for 5 mo. $= 12 \times 5$ persons for 1 mo., and 15 persons for x mo. $= 15 \times x$ persons for 1 mo., we have

$$12 \times 5 \text{ persons} : 15 \times x \text{ persons} :: 6 \text{ bbl.} : 8 \text{ bbl.}$$

Now, since the product of the extremes equals the product of the means, we have

$$6 \times 15 \times x = 8 \times 12 \times 5$$
$$x = \frac{8 \times 12 \times 5}{6 \times 15} = 5\tfrac{1}{3}.$$

26. If 58 cows eat 29 bales of hay in 25 days, in how many days will 125 cows eat 35 bales?

27. If 8 yards of cloth, 1¼ yards wide, cost $1.25, what will 10 yards of the same quality, 1⅛ yards wide, cost?

SOLUTION.

$$8 \times 1\tfrac{1}{4} \text{ sq. yd.} : 10 \times 1\tfrac{1}{8} \text{ sq. yd.} :: \$1.25 : \$x.$$
$$x = \frac{1.25 \times 10 \times 1\frac{1}{8}}{8 \times 1\frac{1}{4}} = \frac{1.25 \times 10 \times 9 \times 4}{8 \times 5 \times 8} = 1.40\tfrac{5}{8}.$$

$1\frac{1}{8} = \frac{9}{8}$. Write the 9 above and the 8 below; this is the same as multiplying both numerator and denominator by 8. Again, $1\frac{1}{4} = \frac{5}{4}$; write the 5 below and the 4 above, which is multiplying both terms by 4.

28. If 12 men in 35 days build a wall 140 ft. long, 6 ft. high, and 3 ft. thick, how many men can in 40 days build a wall 144 ft. long, 5 ft. high, and 2 ft. thick?

SOLUTION.

$$12 \times 35 \text{ men} : 40 \times x \text{ men} :: 140 \times 6 \times 3 \text{ cu. ft.} : 144 \times 5 \times 2 \text{ cu. ft.}$$
$$140 \times 6 \times 3 \times 40 \times x = 144 \times 5 \times 2 \times 35 \times 12$$
$$x = \frac{144 \times 5 \times 2 \times 35 \times 12}{140 \times 6 \times 3 \times 40} = 6.$$

29. If 15 persons consume 6 barrels of flour in 9 months, how long will it take 20 persons to consume 10⅔ barrels?

30. If a pasture of 36 acres supplies 60 sheep 30 days, how long will one of 48 acres supply 120 sheep ?

31. If 16 horses eat 96 bu. of oats in 42 da., in how many days will 7 horses eat 66 bushels ?

32. If 12 men can perform a work in 30 days, how many men are required to do 7 times as much in $\frac{1}{4}$ of the time ?

Denote one work by 1 and the other by 7; then we have

$$12 \times 30 \text{ men} : \tfrac{1}{4} \times 30 \times x \text{ men} :: 1 : 7.$$

33. If a pile of wood 18 ft. long, 4 ft. wide, 6 ft. high, cost $30.24, what will be the cost of a pile 80 ft. long, 8 ft. wide, and 4 ft. high ?

34. If a marble slab 8 ft. long, 5 ft. wide, and $2\frac{1}{3}$ ft. thick, weighs 7280 pounds, what will a slab 9 ft. long, 6 ft. wide, and $2\frac{1}{2}$ ft. thick weigh ?

35. If a load of hay 10 ft. by $3\frac{1}{2}$ ft. by $2\frac{1}{4}$ ft. is worth \$$12\frac{1}{2}$, what is the worth of a load $11\frac{3}{11}$ ft. by $4\frac{2}{5}$ ft. by $1\frac{3}{4}$ ft. ?

36. If 6 men in 24 days, working 10 hours a day, can build a wall 48 yards long, what length of a similar wall can 15 men build in 18 days, working 8 hours a day ?

37. If 6 men in 24 days, working 10 hours a day, can build a wall 48 ft. by 10 ft. by $2\frac{2}{5}$ ft., how many men can build a wall 72 ft. by 9 ft. by $2\frac{2}{3}$ ft. in 18 days, working 8 hours a day ?

38. If 32 men can dig a ditch 40 rd. long, 6 ft. wide, and 3 ft. deep in 9 days, working 8 hours a day, to what depth can 6 men dig a ditch 15 rd. long and $4\frac{1}{2}$ ft. wide in 12 days, working 6 hours a day ?

39. If a lot of land 18 rd. long and 16 rd. wide is worth $35, what is the length of a lot 24 rd. wide, that is worth $200 ?

40. A piece of gold $2\frac{2}{3}$ in. long, $1\frac{5}{7}$ in. wide, and $1\frac{1}{2}$ in. thick weighs $4\frac{7}{9}$ pounds. What is the width of a piece $4\frac{4}{5}$ in. long and $\frac{3}{4}$ in. thick, that weighs $7\frac{21}{40}$ pounds?

41. If 27 men can perform a piece of work in 19 days, how many men can do another work $\frac{7}{9}$ as great in $\frac{3}{5}$ of the time ?

All these problems may be solved like examples 3, 4, and 5 in Miscellaneous Problems, page 297.

PROPORTIONAL PARTS.

436. *1.* Divide 60 into three parts proportional to 3, 4, and 5.

SOLUTION.—3, 4, and 5 are the parts of 12, and the corresponding proportional parts of 60, or 5 × 12, are 5 × 3, 5 × 4, and 5 × 5, or 15, 20, and 25.

Or, we may say, $\frac{3}{12}$ of 60 = 15, the first part; $\frac{4}{12}$ of 60 = 20, the second part; and $\frac{5}{12}$ of 60, or 25, the third part.

2. Divide $93 into parts proportional to $\frac{1}{2}$, $1\frac{1}{3}$, $\frac{3}{4}$.

SOLUTION.—$\frac{1}{2}$, $1\frac{1}{3}$, $\frac{3}{4}$ = respectively, 6 twelfths, 16 twelfths, 9 twelfths, and these are proportional to 6, 16, 9. Therefore the required parts are $\frac{6}{31}$ of $93, $\frac{16}{31}$ of $93, $\frac{9}{31}$ of $93, or $18, $48, $27.

EXERCISES (Oral and Written).

1. Divide $45 into parts proportional to 1, 3, 5.
2. Divide $423 into parts proportional to $\frac{3}{5}$, $1\frac{1}{2}$, $\frac{1}{4}$.
3. Divide 120 bu. into parts proportional to 2, 5, 6, 7.
4. Divide 973 da. into parts proportional to 1, $2\frac{1}{3}$, $\frac{3}{5}$, $\frac{7}{10}$.
5. Divide 81 marbles into parts proportional to $\frac{3}{5}$, $\frac{3}{4}$.
6. Divide 6853 lb. into parts proportional to $1\frac{3}{4}$, $2\frac{4}{5}$, $5\frac{5}{6}$.

7. A father divides 42 apples between Ella and Henry, giving Ella 4 as often as he gives Henry 3. How many apples does each receive?

8. In counting 988 pieces of money, I observed that in every 76 pieces there were 18 dollar pieces, 21 half-dollar, and 37 nickel. How many pieces of each in all?

9. Plumbers' solder contains 2 parts lead and 1 part tin. How much of each in 75 lb. solder?

10. A gold chain weighs 3 oz. 12 pwt. and is 14 carats fine. How much pure gold is in the chain?

11. What is the weight of the pure gold in 500 lb. of gold dollars?

12. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. How much of each is there in a room 18 ft. by 16 ft. by 15 ft.?

13. Type-metal is composed of 39 parts of lead and 11 parts of antimony. How many pounds of each in 400 lb. of type?

14. Bell-metal contains 78 parts of copper to 22 parts of tin. How much of each in a bell weighing 245 lb.?

15. Divide $72 among James, John, and Henry so that John shall have 3 times as much as James, and Henry as much as James and John together.

Solution.—Let 1 represent James's part, then
3 represents John's part, and
4 represents Henry's part.

Hence James has $\frac{1}{8}$ of $72, or $9; John, $\frac{3}{8}$ of $72, or $27; and Henry, $\frac{4}{8}$ of $72, or $36.

16. In a load of 272 pumpkins, melons, and gourds there are twice as many melons as pumpkins, and $4\frac{1}{3}$ times as many gourds as melons and pumpkins together. How many of each?

17. It is 728 feet from A to E. The distance from B to C is five times the distance from A to B; from C to D is $2\frac{2}{3}$ times the distance from A to C; and from D to E is $1\frac{3}{7}$ times the distance from B to D. How far is it from A to B; B to C? etc.

PARTNERSHIP.

437. A **partnership** is an association of two or more persons for the transaction of business.

A partnership association is called a **company, firm,** or **house**; the members of it, **partners**; and the property invested in the business by the partners, **capital, joint stock,** or **stock in trade.**

When the capital or property of the partners is invested an *equal* time, the partnership is called **simple**: when their capital is invested an *unequal* time, the partnership is called **compound.**

1. A, B, and C entered into partnership for 1 yr.; A put in \$450, B \$540, and C \$720, and their net profits were \$380. What was each partner's share?

SOLUTION BY PROPORTION.

\$450, A's capital.	\$1710 : \$450 : : \$380 :	\$100, A's share of gain.
\$540, B's capital.	\$1710 : \$540 : : \$380 :	\$120, B's " " "
\$720, C's capital.	\$1710 : \$720 : : \$380 :	\$160, C's " " "
\$1710, Total capital.		\$380, total gain.

SOLUTION BY PROPORTIONAL PARTS.

$\frac{450}{1710} = \frac{5}{19}$, A's part of capital ; $\frac{5}{19}$ of \$380 = \$100, A's gain.
$\frac{540}{1710} = \frac{6}{19}$, B's part of capital ; $\frac{6}{19}$ of \$380 = \$120, B's gain.
$\frac{720}{1710} = \frac{8}{19}$, C's part of capital ; $\frac{8}{19}$ of \$380 = \$160, C's gain.

PRINCIPLE.—*In simple partnership the gain or loss is shared by the partners in proportion to the amounts of their capital.*

EXERCISES (Written).

438. *1.* B and C were partners in business for 4 years; B put in \$3600 and C \$4200, and their profits were \$910. What was each partner's share of the profits?

2. James caught 25 fishes, William 35, and Thomas 15, and they sold them all for 60 cts. How much of the money should each receive?

3. A, B, and C formed a partnership in business; A put in \$7000, B \$5600, and C \$4284, and their loss was \$536. What was each partner's share of the loss?

4. The capital of two partners is proportional to 5 and 4, and their profits are \$990. What is the share of each?

5. The capital of four partners is proportional to $2\frac{1}{3}$, $2\frac{3}{4}$, $3\frac{1}{2}$, and $3\frac{5}{6}$, and their profits are \$1000. What is each partner's share?

6. Frank and Moses sold 2 baskets of apples at a profit of 20 cts. each. Frank owned $\frac{4}{5}$ of the apples in the first basket, and Moses $\frac{5}{6}$ of those in the second. To how much of the gain is each entitled?

7. A and B traded in partnership 2 years, making an annual profit of $3240; during the first year A owned $\frac{3}{7}$ of the stock, and during the second year B owned $\frac{5}{9}$ of it. What is each partner's share of the total profits?

8. Three partners, A, B, and C, furnish capital as follows: A, $300 for 5 mo.; B, $450 for 4 mo.; C, $350 for 6 mo. They gain $270. What is each partner's share?

SOLUTION.

$300 × 5 = $1500. $\frac{1500}{5400} = \frac{5}{18}$. $\frac{5}{18}$ of $270 = $75, A's share.
$450 × 4 = $1800. $\frac{1800}{5400} = \frac{1}{3}$. $\frac{1}{3}$ of $270 = $90, B's share.
$350 × 6 = $2100. $\frac{2100}{5400} = \frac{7}{18}$. $\frac{7}{18}$ of $270 = $105, C's share.
$5400.

EXPLANATION.—The use of $300 for 5 mo. is the same as the use of $1500 for 1 mo.; the use of $450 for 4 mo. is the same as the use of $1800 for 1 mo.; and the use of $350 for 6 mo. is the same as the use of $2100 for 1 mo. We now divide the gain, $270, into parts proportional to $*1500*, $1800, and $2100, as these represent the amounts invested by the partners for *equal* times.

PRINCIPLE.—*In compound partnership, the gain or loss is shared by the partners in proportion to the products of the amounts of their capital by the time of investment.*

9. A and B enter into partnership. A puts in $650 for 8 mo.; B, $775 for 12 mo., and their profits are $377. Find the share of each.

10. The amounts invested by two partners are proportional to 6 and 8; the capital of the first is in trade $5\frac{2}{3}$ mo., and that of the latter $6\frac{3}{4}$ mo. If they gain $1111, what will be the share of each?

11. In a partnership for 1 yr., A at first put in $5000, and 4 mo. after $2000 more; B at first put in $6000, and 6 mo. after withdrew $1500; and C at first put in $3000, and 9 mo. after put in $4000 more. The whole gain was $4500. What was the share of each?

12. A, B, and C entered into partnership Jan. 1, with a joint capital of $7200, of which A owned $\frac{3}{8}$, B $\frac{2}{5}$, and C the remainder. June 1, A decreased his capital by $\frac{2}{3}$, Aug. 1, B increased his capital by $\frac{3}{4}$, and Oct. 1, C increased his capital by $\frac{7}{9}$. The net profits for the year were $2370. Find the share of each.

MIXTURES.

439. ***To find the average price of a mixture of different articles, when the quantity of each article and its price are given.***

1. A grocer mixes 20 lb. of sugar, worth 15 cts. per lb., with 24 lb. worth 10 cts., and 28 lb. worth 9 cts. What is a pound of the mixture worth ?

SOLUTION.

20 lb.	of sugar	@ 15 cts	=	$3.00
24 lb.	" "	@ 10 cts.	=	$2.40
28 lb.	" "	@ 9 cts.	=	$2.52
72 lb.	" "		=	$7.92

Hence 1 lb. of the mixture = $7.92 ÷ 72, or 11 cts.

RULE.—*Find the value of each article, and divide the total value by the number of articles.*

EXERCISES (Written).

440. *1.* If a mixture be made of 8 lb. of candy, worth 15 cts. a pound, 12 lb. worth 20 cts., and 20 lb. worth 25 cts., what will one pound of it be worth ?

2. If 20 lb. of tea, worth 40 cts. a pound, 30 lb. worth 60 cts., 40 lb. worth $32\frac{1}{2}$ cts., and 60 lb. worth 38 cts., be mixed together, what will the mixture be worth per pound ?

3. A man bought 30 acres of land, @ $5 ; 15 acres, @ $7 ; and 35 acres, @ $6.40. What was the average price per acre ?

4. If 18 gallons of whisky worth $2.50 per gallon, and 28 gallons worth $3, are mixed with 14 gallons of water, which cost nothing, what is a gallon of the mixture worth ?

5. A thermometer stood, from 6 to 9 A.M., at 65° ; from 9 to 11 A.M., at 74°; from 11 A.M. to 2 P.M., at 80°; and from 2 to 6 P.M., at 78°. What was the average temperature of the day?

441. ***To find what proportions of several articles of different values are required to form a compound of a particular value.***

1. A grocer has sugar worth 6 cts., 7 cts., 12 cts., and 13 cts. a pound, which he wishes to mix together so as to form a mixture worth 9 cts. a pound. What proportions of the different sugars must he take ?

OPERATION.

		(1)	(2)	(3)	(4)	(5)
9	6	$\frac{1}{3}$		3		4
	7		$\frac{1}{2}$		3	3
	12		$\frac{1}{3}$		2	2
	13	$\frac{1}{4}$		4		3

EXPLANATION.—In selling the sugars which are worth less than 9 cts. for 9 cts., there will be a gain ; and in selling those which are worth more than 9 cts. for 9 cts., there will be a loss. Now, we desire to take such amounts of each that the *gain* will balance the *loss*. On 1 lb. of the 6-ct. sugar there will be a gain of 3 cts. ; hence, on $\frac{1}{3}$ of a lb. there will be a gain of 1 ct. On 1 lb. of the 13-ct. sugar there will be a loss of 4 cts. ; hence, on $\frac{1}{4}$ of a lb. there will he a loss of **1** ct. That is, if we take $\frac{1}{3}$ of a lb. of the 6-ct. sugar as many times as we take $\frac{1}{4}$ of a lb. of the 13-ct. sugar, the gain and loss will be exactly equal. Hence we write $\frac{1}{3}$ and $\frac{1}{4}$ in the 1st column for the proportional amounts to be taken of the sugars whose prices they stand opposite.

In the same manner we find the proportional amounts $\frac{1}{2}$ and $\frac{1}{3}$ for the 7-ct. and 12-ct. sugars, which we write in the 2d column.

Now, since we may take $\frac{1}{3}$ any number of times, provided we also take $\frac{1}{4}$ the same number of times, we may multiply them by 12, the L. C. M. of their denominators, and thus reduce them to 4 and 3, which we write in the 3d column. Similarly, multiplying $\frac{1}{2}$ and $\frac{1}{3}$ by 6, we obtain 3 and 2, which we write in the 4th column. Hence the required proportional amounts are 4 lb. of the 6-ct. sugar, 3 lb. of the 7-ct., 2 lb. of the 12-ct., and 3 lb. of the 13-ct., which proportional numbers we write in the 5th column.

It is evident that problems of this kind may have an unlimited number of sets of answers ; for any multiple of the proportional numbers found also expresses the proportional quantities to be taken of each.

RULE.—*Write the several prices or qualities in a column, and the average price or quality at the left.*

Form couplets by comparing any price or quality less, with one that is greater than the average rate, and place the reciprocal of the difference between the price and average, opposite the price.

Multiply the fractional terms of each of the couplets by the L. C. M. of their denominators, and the products will be the proportional numbers required.

EXERCISES (Written).

442. *1.* A farmer has hogs worth $2, $3, $6, and $10 a head. What number of each should he sell to obtain an average price of $5 a head?

2. A dry goods merchant has ribbon worth, respectively, 10 cts., 14 cts., 17 cts., 20 cts., 24 cts., and 30 cts. a yard. How many yards of each must he sell so as to realize an average price of 18 cts. per yard?

3. A grocer has soda worth, respectively, 10 cts., 15 cts., and 18 cts. a pound. How much of each must he take so as to form a mixture worth 12 cts. a pound?

As *two* of the values are *above* the average, and *one below*, we compare the latter with each of the former.

OPERATION.

12	10	$\frac{1}{2}$		3		3	
	10		$\frac{1}{2}$		3	3	6
	15		$\frac{1}{3}$		2	2	2
	18	$\frac{1}{6}$		1		1	1

4. If three qualities of syrup are worth, respectively, 42 cts., 54 cts., and 70 cts. a gallon, what quantity of each is required to form a mixture worth 60 cts. per gallon?

5. A merchant has four kinds of starch, worth 3, 5, 6, and 9 cts. per pound. How much of each kind must be taken to form a mixture worth 7 cts. a pound?

6. What proportion of water at *no* value, and whiskies worth $2, $2½, and $3 per gallon, must be used to form a mixture worth $1½ a gallon?

7. A farmer has 15 sheep worth $3 a head. He also has sheep worth $5, $8, and $10 a head. How many of each of the latter must he put with all the former, in order that he may sell the whole at an average price of $7 ?

OPERATION.

$$7\left\{\begin{array}{r|l} 3 & 3 \times 5 = 15 \\ 5 & 1 \times 5 = 5 \\ 8 & 2 \times 5 = 10 \\ 10 & 4 \times 5 = 20 \end{array}\right.$$

We find the proportional numbers 3, 1, 2, and 4, as before. Now, since any multiple of these is also a set of answers, we multiply each by such a number (5) as will reduce the first one (3) to 15, as we desire 15 of the first to be taken. Hence the answer is 15 of the $3 sheep, 5 of the $5, 10 of the $8, and 20 of the $10 sheep.

8. A grocer has syrups worth 35, 55, 70, and 85 cts. per gallon. How many gallons of each must he mix with 18 gallons of the 70-ct. syrup to form a mixture worth 65 cts. per gallon ?

9. A man has traveled 5 hr. at the rate of 3 mi. per hour. How many hours must he now travel, at the rate of 4, 7, and 9 mi. per hour, in order that his average rate will be 6 mi. per hour ?

10. Three fourths of a mixture of 35 gallons is alcohol. What proportion of mixtures containing 95 and 100 per cent of alcohol must be mixed with the former in order that $\frac{4}{5}$ of the mixture shall be alcohol ?

11. A farmer has hogs worth $2, $4, $9, and $13 a head. How many of each kind must he take so as to have 64 in all, worth $7 per head ?

OPERATION.

$$7\left\{\begin{array}{r|l} 2 & 6 \times 4 = 24 \\ 4 & 2 \times 4 = 8 \\ 9 & 3 \times 4 = 12 \\ 13 & 5 \times 4 = 20 \\ \hline & 16)64(4 \end{array}\right.$$

We find the proportional numbers 6, 2, 3, and 5, as before. Now, since any multiple of these numbers is a set of answers, we multiply each by such a number (64 ÷ 16, or 4) as will reduce their sum (16) to 64, as we desire 64 in all. Hence the answer is 24 of the $2 hogs, 8 of the $4, 12 of the $9, and 20 of the $13 hogs.

If the total number required is not divisible by the sum of the proportional numbers, the latter may be varied by taking different multiples of the couplets. Thus, in the last example, if 74 hogs were required in all, we may multiply the first couplet 6 and 5 by 2, and the second, 2 and 3, by 3, which gives the proportional numbers 12, 6, 9, and 10, whose sum is 37. Now, 74 ÷ 37 = 2; hence the required numbers would be 24, 12, 18, and 20.

12. A merchant has nuts worth 24, 29, 35, and 46 cts. a pound. How much of each must be taken to form a mixture of 51 pounds, worth 30 cts. a pound?

13. A farmer sold 63 cows for $1260. For one kind he received $15 a head; for another, $25; and for another, $40. How many of each kind did he sell?

The average price in this case is $1260 ÷ 63, or $20.

14. A paid $100 for 100 head of cows, sheep, and chickens. For the cows he paid $10 each, for the sheep $3, and for the chickens $$\frac{1}{2}$. How many of each did he buy?

AVERAGE OF PAYMENTS.

443. Average of payments is the process of finding the time for paying, all together, several debts due at different times, without loss to debtor or creditor.

The **term of credit** is the time that must elapse before a debt matures, the **average term of credit** is the time to elapse before several debts, due at different times, may all be paid at once, and the **average time** is the date on which the average term of credit expires.

444. ***To find the average term of credit and the equated time.***

1. Aug. 10, A owes B $140; of which $30 is due at once, $50 in 2 months, and $60 in 4 months. When may the $140 be equitably discharged by a single payment?

OPERATION.

$$\begin{array}{rl} 30 \times 0 = & 0 \\ 50 \times 2 = & 100 \\ \underline{60} \times 4 = & \underline{240} \\ 140 &)340(2\tfrac{3}{7} \end{array}$$

$2\frac{3}{7}$ mo. = 2 mo. $12\frac{6}{7}$ da.
= 2 mo. 13 da.
Aug. 10 + 2 mo. 18 da. = Oct. 23.

EXPLANATION.—A is entitled to the use of:

$30 for 0 months = use of $1 for 0 months;
$50 for 2 months = use of $1 for 100 months;
$60 for 4 months = use of $1 for 240 months.

That is, A is entitled to the use of $1 for 340 months, and therefore to the use of $140 for $\frac{1}{140}$ of 340 months, or $2\frac{3}{7}$ mo.

The average term of credit is then $2\frac{3}{7}$ mo., or 2 mo. 13 da., a fraction of a day being considered a day; and the average time is 2 mo. 13 da. from and after Aug. 10, which is Oct. 23.

RULE.—*Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit. This added to the date from which the credits were reckoned will give the average time of payments.*

EXERCISES (Written).

445. *1.* Find the average term of credit for the payment of $300 in 2 mo., $340 in 5 mo., and $360 in $6\frac{1}{3}$ mo.

2. A owes B $1500, of which $600 is due in 4 months, $400 in 5 months, $300 in 8 months, and the remainder in 12 months. What is the average term of credit?

3. Five hundred dollars due in 10 mo., $800 due in 12 mo., and $700 due in 15 mo., is equivalent to the *whole* due in what time?

4. I owe $375 due to-day, $350 in 4 mo., and $475 due in 8 mo. Find the average term of credit.

5. A person bought a bill of goods Jan. 1, 1902, amounting to $744, of which $\frac{1}{3}$ was due in 20 days, $\frac{1}{6}$ in 35 days, and the remainder in 50 days. What was the equated time of payment?

6. Bought goods as follows: June 1, 1902, $200 on 4 mo. credit; July 20, $300 on 5 mo.; Aug. 10, $400 on 3 mo. What is the average term of credit and the equated time of payment?

SOLUTION.

200 × 0 da. = 0 da. (0 da. from Oct. 1 to Oct. 1.)
300 × 80 da. = 24000 da. (80 da. from Oct. 1 to Dec. 20.)
400 × 40 da. = 16000 da. (40 da. from Oct. 1 to Nov. 10.)
900)40000 da. (45 da. from Oct. 1 to Nov. 15.)
$44\frac{4}{9}$ da.

The 1st item is due June 1, plus 4 mo., or Oct. 1; the 2d, July 20, plus 5 mo., or Dec. 20; the 3d, Aug. 10, plus 3 mo., or Nov. 10. The terms of credit, reckoned from Oct. 1, the earliest date at which any of the sums become due, are 0 da., 80 da., and 40 da. The average term from that date is 45 da., and the equated time is therefore Oct. 1, plus 45 da., or Nov. 15.

7. Find the equated time for the payment of three notes, as follows: $450, dated May 12, 1901, for 60 da.; $540, dated July 15, 1901, for 90 da.; and $900, dated April 27, 1901, for 100 da.

8. Bought goods as follows: Aug. 15, 1900, on 3 mo., $600; Sep. 10, 1900, on 4 mo., $750; Nov. 5, 1900, on 6 mo., $900. Find the equated time of payment.

9. A merchant owes the sums named in the margin for goods bought at the time specified, and he gives his note for the entire amount, payable in one sum, at the equated time. When does this note mature?

$800, Mar. 18, credit of 40 da.
$900, June 2, " " 55 da.
$250, May 11, " " 70 da.
$300, July 17, " " 28 da.

10. A stationer bought of F. F. Hansell & Bro. the following bills of stationery:

May 1, a bill of $300, on 4 mo. credit.
June 12, " " " $450, " 5 " "
" 27, " " " $200, " 3 " "
July 15, " " " $320, " 4 " "
Aug. 2, " " " $730, " 6 " "

What is the equated time?

POWERS AND ROOTS.

INVOLUTION.

446. *1.* How much is 5^2 or 5×5? $(\frac{2}{3})^2$?

2. How much is 7^2? 9^2? 3^2? 10^2? 12^2?

3. How much is $(\frac{1}{3})^2$? $(\frac{2}{5})^2$? $(\frac{3}{2})^2$? $(\frac{7}{10})^2$? $(\frac{5}{12})^2$?

4. How much is 5^3 or $5 \times 5 \times 5$? $(\frac{1}{2})^3$ or $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$?

5. How much is 7^3? 9^3? 3^3? 10^3? 12^3?

6. How much is $(\frac{1}{3})^3$? $(\frac{2}{5})^3$? $(\frac{3}{2})^3$? $(\frac{7}{10})^3$? $(\frac{5}{12})^3$?

447. The **power** of a number is the result of multiplying that number by itself a certain number of times.

The power of a number is named from the number of times it is taken as a factor.

Thus: 16, which is 4×4 or 4^2, is the second power of 4;
64, which is $4 \times 4 \times 4$ or 4^3, is the third power of 4;
256, which is $4 \times 4 \times 4 \times 4$ or 4^4, is the fourth power of 4.

The second power is usually called the *square*, and the third power the *cube*, because the area of a square is the product of two equal factors, and the volume of a cube is the product of three equal factors.

A power is indicated by an exponent (Art. 103).

448. Involution is the process of finding the powers of numbers.

EXERCISES (Written).

449. *1.* Find the fourth power of 6 or 6^4.

SOLUTION.

$$6 \times 6 \times 6 \times 6 = 36 \times 6 \times 6 = 216 \times 6 = 1296.$$

2. Find the second power or square of 12; 15; 42.

3. Find the third power or cube of 5; 12; 15; 51.

4. Find the square of 2.5; 7.42; .05.

5. Find the cube of 3.5; 1.05; .025.

Find the value of:

6. 24^2. *7.* 15.04^2. *8.* $(\frac{3}{4})^2$. *9.* $(2\frac{1}{2})^4$.
10. 30^3. *11.* $.0018^3$. *12.* $(\frac{2}{5})^3$. *13.* $(3\frac{1}{3})^3$.
14. 15^4. *15.* 2.031^2. *16.* $(\frac{5}{6})^2$. *17.* $(9\frac{1}{4})^2$.

EVOLUTION.

450. A **root** of a number is one of the equal factors that produce the number.

451. Evolution is the process of finding roots, and is the reverse of involution.

Thus, since 25 is the *square* of 5, 5 is the *square root* of 25; and since 125 is the *cube* of 5, 5 is the cube root of 125.

The **sign of evolution** is $\sqrt{\ }$, which is called the radical sign.

The **index** of the root is the figure placed over the radical sign, to denote what root is to be taken. When no index is written, the index 2 is understood.

Thus, $\sqrt{9} = 3$ is read, the square root of 9 is 3; and $\sqrt[3]{64} = 4$, the cube root of 64 is 4.

A **perfect power** is a number whose exact root can be found; and an **imperfect power** is a number whose exact root can not be found.

Square Root.

452. *1.* What is the square root of 36 ? Why ?

2. What is the square root of .49 ? Why ?

3. What is the square root of $\frac{16}{25}$? Why ?

Find the value of:

4. $\sqrt{4}$; $\sqrt{16}$; $\sqrt{64}$; $\sqrt{100}$; $\sqrt{1.69}$.
5. $\sqrt{9}$; $\sqrt{25}$; $\sqrt{81}$; $\sqrt{121}$; $\sqrt{\frac{81}{100}}$.
6. $\sqrt{\frac{4}{9}}$; $\sqrt{49}$; $\sqrt{.16}$; $\sqrt{144}$; $\sqrt{13\frac{4}{9}}$.
7. $\sqrt{\frac{1}{4}}$; $\sqrt{2\frac{1}{4}}$; $\sqrt{.04}$; $\sqrt{11\frac{1}{9}}$; $\sqrt{5\frac{19}{25}}$.

8. Name the perfect integral squares between 1 and 100.

9. What is the square or second power of 31 ? How was the 9 in the product obtained ? The 6 ? The 1 ?

$$\begin{array}{r} 31 \\ 31 \\ \hline 31 \\ 93 \\ \hline 961 \end{array}$$

10. What is the square root of 961 ? May the 3 of the root be found by taking the square root of the 9 ? Having found the 3, may the 1 in the root be found by dividing the 6 by 2 × the 3 ?

Find the square root of :

11. 441 or 4 hunds. 4 tens 1 one.

$\sqrt{4} = ?$ $4 + (2 \times 2) = ?$ Therefore, $\sqrt{441} = ?$ Prove it.

12. 1024 or 9 hunds. 12 tens 4 ones.

13. 2025 or 16 hunds. 40 tens 25 ones.

453. Extraction of the square root is the process of finding one of the two equal factors that produce a number.

A **perfect square** is a number which can be resolved into two equal factors. The perfect integral squares between 1 and 100 are $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$.

454. Extracting the square root of integers depends on three principles, which are now to be explained.

Since $36 = 30 + 6 =$ tens + ones, we may square 36 thus :

$$\begin{array}{rcrcl}
36 & = & 30+6 & = & \text{tens} + \text{ones}. \\
\underline{36} & = & \underline{30+6} & = & \underline{\text{tens} + \text{ones}.} \\
216 & = & (30\times 6) + 6^2 & = & (\text{tens}\times\text{ones}) + \text{ones}^2. \\
\underline{108\ \ } & = & \underline{30^2 + (30\times 6)\ \ } & = & \underline{\text{tens}^2 + (\text{tens}\times\text{ones})} \\
1296 & = & 30^2 + 2\,(30\times 6) + 6^2 & = & \text{tens}^2 + 2\,\text{tens} \times \text{ones} + \text{ones}^2.
\end{array}$$

That is, the square of any number composed of tens and ones equals

I. *The square of the tens, plus two times the tens times the ones, plus the square of the ones, or* $t^2 + 2\,t \times o + o^2$.

If any integer be squared,
the square of the units will lie between 0 and 100 ;
the square of the tens will lie between 100 and 10000 ;
the square of the hunds. will lie between 10000 and 1000000 ;
and so on.

Hence, if an integer which is a perfect square be separated into periods of two figures each, beginning at the right the square of the units' figure of the root will lie wholly in the first period ; the square of the tens' figure of the root will lie wholly in the second period ; the square of the hundreds' figure of the root will lie wholly in the third period, etc., etc.

Therefore, II. *There are as many figures in the square root of an integral perfect square as there are 2-figure periods in that square.*

III. *The greatest square contained in the last or left hand period is the square of the left hand figure of the root.*

The above principles apply to decimals if they are pointed off into 2-figure periods by beginning at tenths and passing to the right, annexing a cipher when necessary to an exact number of periods.

455. ***To extract the square root of a number.***

1. Find the square root of 3249.

Solution.—Since there are two periods in 32,49, there will be two figures in its square root, tens and ones, and by I we have

$$\text{tens}^2 + 2 \text{ times the tens} \times \text{ones} + \text{ones}^2 = 3249.$$

By III the greatest square in 32, viz., 25 is the square of the tens' figure of the root; therefore the tens' figure is 5, and the tens 50. It only remains now to find the unknown units' figure.

Putting 50 in the place of tens, we have

$$2500 + 2 \text{ times } 50 \times \text{units} + \text{units}^2 = 3249;$$

or $$2 \text{ times } 50 \times \text{units} + \text{units}^2 = 3249 - 2500 = 749;$$

or $$(2 \times 50 + \text{units' figure}) \times \text{units' figure} = 749.$$

That is, 749 is the product of the unknown figure by (2 × 50 + the unknown figure).

Hence, if we use 2 × 50 or 100 as a trial divisor, we obtain a quotient, namely 7, which is either equal to or greater than the required figure. Put this quotient for the unknown figure, and we have 2 × 50 + 7, or 107, for a true or complete divisor. Now, dividing 749 by 107, we find that 7 is the correct figure for the units' place.

Therefore the square root of 3249 is 57.

The work may now be intelligently and more conveniently performed as follows:

		32,49	57
		25	
Trial divisor, 2 × 50	= 100	749	
Trial quo. of 749 by 100 =	7		
Complete divisor,	107	749	

The greatest square in the left hand period is 25, which we subtract from 32, and to the remainder 7 annex the next period, taking 5, the square root of 25, for the tens' figure of the root.

Our trial divisor is obtained by taking 2 times the tens already found, namely 50. 749 ÷ 100 gives 7 for a trial quotient, which added to 2 × 50 gives the complete divisor 107. Multiplying this complete divisor by 7, we obtain 749, which is equal to the dividend.

In obtaining any figure of the root except the first, the figure or figures of the root already found are regarded as tens, and the figure sought as units; that is, each succeeding figure of the root is found in the same manner as the second figure of a root expressed by two figures.

2. Extract the square root of 132496.

FULL PROCESS.

$$\begin{array}{r|l}
 & 13{,}24{,}96\ \underline{|364} \\
 & 9 \\
2\times 30 = 60 & 424 \\
60+6 = 66 & \underline{396} \\
2\times 360 = 720 & 2896 \\
720+4 = 724 & \underline{2896}
\end{array}$$

ABBREVIATED METHOD.

$$\begin{array}{r|l}
 & 13{,}24{,}96\ \underline{|364} \\
 & 9 \\
66 & 424 \\
 & \underline{396} \\
724 & 2896 \\
 & 2896
\end{array}$$

3. Find the square root of 10 to six places of decimals.

Having obtained four figures of the answer, the other three may be obtained by dividing the last remainder, 1756, with three 0's annexed, by 2 × the answer already found (regarded as an integer).

Therefore the required root is 3.162277.

OPERATION.

$$\begin{array}{r|l}
 & 10.00{,}00{,}00\ \underline{|3.162} \\
 & 9 \\
61 & 100 \\
 & \underline{61} \\
626 & 3900 \\
 & \underline{3756} \\
6322 & 14400 \\
 & \underline{12644} \\
\end{array}$$

$$6324)1756000(277+$$

The rule in such cases is, that one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found regarded as an integer.

RULE.—*Separate the number into 2-figure periods, beginning with ones in an integer, and with tenths in a decimal.*

Find the greatest square in the left hand period, and place its root on the right for the first root-figure. Subtract its square from the left hand period, and to the remainder annex the next period for a dividend.

Double the part of the root already found, and place it on the left for a trial divisor; divide the dividend, exclusive of its right hand figure, by the trial divisor, and take the quotient for the next figure of the root, and also annex it to the trial divisor, to form the complete divisor.

Multiply the complete divisor by the second figure of the root; subtract the product from the dividend, and to the remainder annex the next period for a dividend.

Proceed with the second, and with each succeeding dividend, in the same manner as with the first.

NOTES.—1. If any dividend is less than the divisor, write a 0 in the root, and also on the right of the divisor, and annex the next period to the dividend for a new dividend.

2. If there is a remainder after all the periods have been used, annex periods of decimal 0's, and extend the work to any desired degree of exactness, writing + after the root.

3. To extract the square root of a mixed number, first reduce it to a mixed decimal number, or to an improper fraction.

4. To find the square root of a fraction whose terms are perfect squares, extract the square root of the numerator and of the denominator for the terms of the required root; when the terms are not perfect squares, reduce the fraction to a decimal, and then extract the square root.

EXERCISES (Written).

456. Find the square root of:

1. 1156. *2.* 15625. *3.* 522729. *4.* 5764801.
5. 5625. *6.* 59049. *7.* 297025. *8.* .000729.
9. 2401. *10.* 46656. *11.* 117649. *12.* 169.9616.
13. 4096. *14.* 201.64. *15.* 390625. *16.* 7728400.
17. 9025. *18.* 52441. *19.* .537289. *20.* .00021025.

Find the value of:

21. $\sqrt{1442\frac{1}{4}}$. *22.* $\sqrt{85069\frac{4}{9}}$. *23.* $\sqrt{.180625}$.

Find the value to three decimal places of:

24. $\sqrt{2}$. *25.* $\sqrt{18}$. *26.* $\sqrt{\frac{7}{8}}$. *27.* $\sqrt{1.47}$.

28. $\sqrt{3}$. *29.* $\sqrt{75}$. *30.* $\sqrt{\frac{3}{8}}$. *31.* $\sqrt{12.6\frac{7}{8}}$.

32. $\sqrt{5}$. *33.* $\sqrt{50}$. *34.* $\sqrt{\frac{2}{5}}$. *35.* $\sqrt{25.0\frac{3}{4}}$.

Applications of Square Root.

457. *1.* Find the side of a square field which contains 302500 sq. yards.

The side of a square is found by extracting the square root of its area.

2. What is the length of a square garden that contains as much as one 50 yd. long and 32 yd. wide?

3. How many rods in the side of a square field that contains 1440 acres?

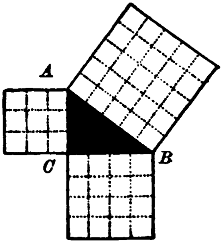
4. If fencing cost 18 cts. per yd., how much more would it cost to fence a field 378 yd. long and 168 yd. wide than to fence an equivalent square field?

5. How many yards in the side of a square which contains one acre of land?

6. There are 15129 peaches in an orchard. How many trees are there if the number of trees is equal to the number of peaches on each tree?

7. A number of gentlemen built a church for $7225, each contributing as many dollars as there were gentlemen. How many gentlemen were there?

458. *In a right triangle* (*A C B*), *the square on the hypotenuse* (*A B*) *is equal to the sum of the squares on the two legs* (*other sides*).



Hence the formulas:

Hypotenuse

$$= \sqrt{(\textit{one leg})^2 + (\textit{other leg})^2};$$

Either leg

$$= \sqrt{(\textit{hypotenuse})^2 - (\textit{other leg})^2}.$$

8. The two legs of a right triangle are 27 and 36 ft. Required the hypotenuse.

SOLUTION.—Hypotenuse = $\sqrt{27^2 + 36^2} = \sqrt{2025} = 45$.

9. The hypotenuse of a right triangle is 20 and one of the legs is 12. Find the other leg.

SOLUTION.—Leg = $\sqrt{20^2 - 12^2} = \sqrt{256} = 16$.

Find the hypotenuse of a right triangle :

10. Whose legs are 20 and 48 yd., respectively.

11. Whose legs are 33 and 56 ch., respectively.

12. One of the legs of a right triangle is 24 yd. and the hypotenuse is 74 yd. Find the other leg.

13. A room is 42 ft. long and 30 ft. wide. How far is it from one corner to the opposite corner ?

14. Two vessels sail from the same port, one sailing directly north 6 miles an hour, the other west 8 miles an hour. How far apart will they be in 13 hr. ?

15. A ladder 37 ft. long leans against a house while its foot is 12 ft. from the building. How high does the ladder reach ?

16. A pole was broken 52 ft. from the bottom, and the top, clinging to the stump, fell so that its end struck the ground 39 ft. from the foot. Find the length of the pole.

17. A rope 130 ft. long, with one end fastened to a pin in the street, will reach on one side to a window 78 ft. high, and on the other to a window 50 ft. high. What is the width of the street ?

Extraction of the Cube Root.

459. Extraction of the cube root is the process of finding one of the three equal factors that produce a number.

A **perfect cube** is a number which can be resolved into three equal factors. The perfect cubes between 1 and 1000 are $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$. When a number is not a perfect cube, it contains a perfect cube and something over. Thus the greatest perfect cube in 35 is 27; in 90, 64 ; in 400, 343; in 625, 512.

460. The extraction of the cube root depends on three principles, which we will now notice.

Since $36 = 30 + 6 =$ tens + ones, we may square 36 thus:

$$\begin{array}{rcr}
36 & = & 30 + 6 \\
36 & = & 30 + 6 \\
\hline
216 & = & (30 \times 6) + 6^2 \\
108 & = & 30^2 + (30 \times 6) \qquad \\
\hline
1296 & = & 30^2 + 2\,(30 \times 6) + 6^2 \\
36 & = & 30 + 6 \\
\hline
7776 & = & (30^2 \times 6) + 2\,(30 \times 6^2) + 6^3 \\
3888 & = & 30^3 + 2\,(30^2 \times 6) + (30 \times 6^2) \qquad \\
\hline
46656 & = & 30^3 + 3\,(30^2 \times 6) + 3\,(30 \times 6^2) + 6^3
\end{array}$$

That is, the cube of any number composed of tens and ones equals

I. *The cube of the tens, plus three times the product of the square of the tens and the ones, plus three times the product of the tens and the square of the ones, plus the cube of the ones.*

This statement may be represented by the following formula, in which the letter t stands for tens and the letter o for ones:

$$(t + o)^3 = t^3 + 3 \times t^2 \times o + 3 \times t \times o^2 + o^3.$$

If any integer be cubed,

the cube of the units will lie between 0 and 1 thousand;
the cube of the tens will lie between 1 thous. and 1 million;
the cube of the hunds. will lie between 1 mill. and 1 billion;
and so on.

Hence if an integer which is a perfect cube be separated into periods of three figures each, beginning at the right, the cube of the units of the root will lie wholly in the first period; the cube of the tens' figure of the root will lie wholly in the second period; the cube of the hundreds' figure of the root will lie wholly in the third period; etc., etc.

Therefore, II. *There are as many figures in the cube root of any integral perfect cube as there are 3-figure periods in that cube.*

III. *The greatest cube contained in the left hand period is the cube of the left hand figure of the root.*

The above principles apply to decimals, if they are pointed off into 3-figure periods, by beginning at tenths and passing to the right, annexing one or two ciphers if necessary to an exact number of periods.

461. ***To extract the cube root of a number.***

1. Find the cube root of 157464.

SOLUTION.

Since there are two periods in 157,464, there will be two figures in the cube root, tens and ones, and by I we have

$$t^3 + 3\,t^2 \times o + 3\,t \times o^2 + o^3 = 157{,}464.$$

By III the greatest cube in 157, viz. 125, is the cube of the tens' figure of the root; therefore the tens' figure is 5, and the tens 50. It only remains now to find the unknown ones' figure.

Putting 50 in the place of t, or tens, we have

$$50^3 + 3 \times 50^2 \times o + 3 \times 50 \times o^2 + o^3 = 157{,}464;$$
$$\text{or} \quad 3 \times 50^2 \times o + 3 \times 50 \times o^2 + o^3 = 157{,}464 - 50^3;$$
$$\text{or} \quad (3 \times 50^2 + 3 \times 50 \times o + o^2) \times o = 32464.$$

That is, 32464 is the product of the unknown figure by [3 × 50^2 + 3 × 50 × (unknown figure) + (unknown figure)^2].

Hence, if we use 3 × 50^2, or 7500, as a trial divisor, we obtain a quotient, viz. 4, which is either equal to or greater than the required figure. Put this quotient for the unknown figure, and we have $3 \times 50^2 + 3 \times 50 \times 4 + 4^2 = 8116$ for a true or complete divisor. Now dividing 32464 by 8116, we find that 4 is the correct figure for the units' place.

Therefore the cube root of 157464 is 54.

The work is more conveniently performed as follows :

OPERATION.

$$\begin{array}{lcr|l}
 & & \multicolumn{2}{l}{157{,}464\ \underline{\big|\,54}} \\
 & & \multicolumn{2}{l}{125} \\ \cline{4-4}
\text{Trial divisor, } 3\times50^2 & = & 7500 & 32464 \\
3\times50\times4 & = & 600 & \\
4^2 & = & \underline{16} & \\
\text{Complete divisor,} \quad . \quad . \quad . & & 8116 & \underline{32464}
\end{array}$$

The greatest cube in the left hand period is 125, which we subtract from 157, and to the remainder 32 annex the next period, taking 5, the cube root of 125, for the tens' figure of the root.

Having found the tens, 50, we now take 32464 for a dividend and 3×50^2, or 7500, for a trial divisor, and find the trial quotient 4. With this quotient we now complete the divisor by adding to 7500 the sum of $3 \times 50 \times 4$ and 4^2, which gives 8116. This we multiply by 4 and obtain 32464, which is the same as the dividend.

In extracting the cube root, only 2 periods of figures are considered in connection. Hence,

Any figure of the root after the second is found in the same manner as the second figure of a root expressed by two figures.

This principle is fully illustrated in the solution of the following example.

2. Find the cube root of 244140625.

FULL PROCESS.

$$\begin{array}{lcr|l}
 & & \multicolumn{2}{l}{244{,}140{,}625\ \underline{\big|\,625}} \\
 & & \multicolumn{2}{l}{216} \\ \cline{4-4}
\text{1st. Trial divisor, } 3\times60^2 & = & 10800 & 28140 \\
3\times60\times2 & = & 360 & \\
2^2 & = & \underline{4} & \\
\text{1st. Complete divisor,} \quad . \quad . \quad . \quad . & & 11164 & \underline{22328} \\
\text{2d. Trial divisor, } 3\times620^2 & = & 1153200 & 5812625 \\
3\times620\times5 & = & 9300 & \\
5^2 & = & \underline{25} & \\
\text{2d. Complete divisor,} \quad . \quad . \quad . & & 1162525 & \underline{5812625}
\end{array}$$

ABBREVIATED METHOD.

		244,140,625	625
		216	
1st. Trial divisor, .	10800	28140	
1st. Complete divisor,	11164	22328	
2d. Trial divisor, .	1153200	5812625	
2d. Complete divisor,	1162525	5812625	

3. Extract the cube root of 4 to five decimal places.

Having obtained four figures of the answer, the other two may be obtained by dividing the last remainder, with 0's annexed, 307299700, by 3 times the square of the answer (1587) already found.

Hence the required root is 1.58740.

	4.000,000,000	1.587
	1	
300	3000	
475	2375	
67500	625000	
71164	569312	
7489200	55688000	
7522429	52615003	

7555658)307299700(40 +

The rule in such cases is, that two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.

RULE.—*Separate the number into 3-figure periods, beginning with ones in an integer, and with tenths in a decimal.*

Find the greatest cube in the left hand period, and place its root on the right for the first root-figure. Subtract its cube from the left hand period, and to the remainder annex the next period for a dividend.

Square the part of the root already found, multiply the result by 3, annex two ciphers, and place it on the left as a trial divisor; divide the dividend by the trial divisor, and take the quotient as the second figure of the root. To

obtain the true divisor, add to the trial divisor 3 times the product of the last root-figure by the root previously found, annexing one cipher, and also add the square of the last root-figure.

Multiply the true divisor by the last root-figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Proceed with the second, and each succeeding dividend, in the same manner as with the first, until all the periods are used.

NOTE.—The notes under Art. 455 are applicable to cube root with the change of "square" to "cube."

EXERCISES (Written).

462. Find the cube root of:

1. 13824. *2.* 421875. *3.* 48228.544.
4. 42875. *5.* .019683. *6.* 244140625.
7. 74088. *8.* 175.616. *9.* 187149248.
10. 54872. *11.* .405224. *12.* 34169931125.

Find the value of:

13. $\sqrt[3]{1728}$. *14.* $\sqrt[3]{2571353}$. *15.* $\sqrt[3]{14348907}$.

Find the value to four decimal places:

16. $\sqrt[3]{2}$. *17.* $\sqrt[3]{3}$. *18.* $\sqrt[3]{5}$. *19.* $\sqrt[3]{6}$.
20. $\sqrt[3]{\frac{2}{3}}$. *21.* $\sqrt[3]{\frac{3}{4}}$. *22.* $\sqrt[3]{\frac{1}{9}}$. *23.* $\sqrt[3]{.4}$.

Applications of Cube Root.

463. *1.* What is the length of the edge of a cube whose volume is 2197 cu. feet?

The edge of a cube is equal to the cube root of the volume.

2. A cubical block contains 474552 cu. in. What is the length of all its edges together?

3. What is the entire surface of a cube whose volume is 91125 cu. ft.?

4. I wish to construct a cubical cistern to hold 729 gallons of water. What must be one of its equal dimensions?

5. A merchant bought a bale of cloth containing as many bolts as there were yards in each bolt, for $911¼, and paid as many cents for each yard as there were bolts. How many yards were there?

6. A man wishes to make a cubical box to hold 1 bushel. What must be the length of its edge?

7. What is one dimension of a cubical bin that will hold the same number of bushels of wheat as a bin that is 92$\frac{4}{7}$ in. long, 56 in. wide, and 6 feet deep?

8. What will it cost to plaster the bottom and sides of a cubical reservoir which will hold 53,361 gallons of water, at 4 cents a sq. foot?

PROGRESSIONS.

464. A **progression** is a series of numbers which increases or decreases regularly or according to some fixed law.

Thus, (1) 5, 7, 9, 11, 13, or 17, 14, 11, 8, 5,
or (2) 2, 4, 8, 16, 32, or 9, 3, 1, ⅓, ⅑,
are progressions.

It will be noticed that in the first two progressions the series are made by successive additions or subtractions, while in the last two the series are made by successive multiplications or divisions.

Series of the first kind (1) are called *arithmetical progressions.*

Series of the second kind (2) are called *geometrical progressions.*

ARITHMETICAL PROGRESSION.

465. In an arithmetical progression there are five things to be considered :

the first term,	for short,	called	*first ;*
the last term,	"	"	*last ;*
the number of terms,	"	"	*number ;*
the common difference,	"	"	*difference ;*
the sum of the terms,	"	"	*sum.*

Thus, in the series 4, 9, 14, 19, 24, 29,
first = 4, *last* = 29, *number* = 6, *difference* = 5 (= 9 − 4, or 14 − 9, etc.), *sum* = 99 (= 4 + 9 + 14 + 19 + 24 + 29).

466. ***To find last when first, number, and difference are given.***

1. If the series 3, 5, 7, 9, etc., were continued until there were 40 terms in all, what would the last term be ?

EXPLANATION.	OPERATION.
The 2d term $= 3 + 1 \times 2$;	$40 - 1 = 39$;
" 3d " $= 3 + 2 \times 2$;	$39 \times 2 = 78$;
" 4th " $= 3 + 3 \times 2$;	$3 + 78 = 81$.—Ans.
....	
" 40th " $= 3 + 39 \times 2$.	

FORMULA : *last* = *first* + (*number* −1) × *difference.*

EXERCISES (Written).

467. Find the :

1. 10th term of 5, 9, 13, 17, etc.
2. 20th term of 2, 5, 8, 11, etc.
3. 50th term of 7, 9, 11, 13, etc.
4. 16th term of 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, etc.

Find the last term of an increasing series in which :

5. First = 4, number = 15, difference = 3.
6. First = 6, number = 22, difference = $\frac{2}{3}$.
7. First = $5\frac{1}{2}$, number = 35, difference = $\frac{3}{4}$.

8. A man bought 100 chickens, paying 5 cts. for the first, 10 cts. for the second, 15 cts. for the third, and so on. What did he pay for the last chicken ?

9. The youngest of 17 men is 22 years old, and the common difference between their ages is $1\frac{1}{2}$ years. How old is the oldest man ?

468. ***To find sum when first, last, and number are given.***

1. Find the sum of 3, 5, 7, 9, 11, 13.

EXPLANATION.

$$\begin{aligned}
\text{Sum} &= 3 + 5 + 7 + 9 + 11 + 13.\\
\text{Sum} &= 13 + 11 + 9 + 7 + 5 + 3.\\
\hline
2 \times \text{sum} &= 16 + 16 + 16 + 16 + 16 + 16.\\
2 \times \text{sum} &= 16 \times 6 = (3 + 13) \times 6.\\
\text{Sum} &= \frac{(3 + 13) \times 6.}{2}
\end{aligned}$$

OPERATION.

$3 + 13 = 16;$

$16 \div 2 = 8;$

$8 \times 6 = 48.$

FORMULA : $$sum = \left(\frac{first + last}{2}\right) \times number.$$

EXERCISES (Written).

469. Find the sum of :

1. 6, 10, 14, 18, 22, 26, 30, 34.
2. 3, 10, 17, 24, 31, 38, 45, 52.
3. $\frac{1}{2}$, 2, $3\frac{1}{2}$, 5, $6\frac{1}{2}$, 8, $9\frac{1}{2}$, 11.

Find the sum of :

4. 20 terms of the series 3, 11, 19, etc.
5. 30 terms of the series 7, 10, 13, etc.
6. 16 terms of the series 2, $2\frac{2}{3}$, $3\frac{1}{3}$, etc.
7. 24 terms of the series $\frac{2}{3}$, $\frac{11}{12}$, $1\frac{1}{6}$, etc.

8. A man traveled 7 miles the first day, 11 miles the second, 15 miles the third, and so on. How far did he go in 15 days ?

9. How many strokes does a clock strike in 12 hours ?

10. How much will it cost a man to have his horse shod if he pays 5 cts. for the first nail, 8 cts. for the second, 11 cts. for the third, and so on, there being 8 nails in each of the four shoes ?

11. A stone when dropped from a height falls 16.1 ft. in the first second, 48.3 ft. in the second, 80.5 ft. in the third, and so on in arithmetical progression. How far will it fall in the 10th second? How far will it fall in 10 seconds?

12. A man bought 22 cows, at $28 each. He sold the first cow for $2, the second for $5, the third for $8, and so on, in an arithmetical series. How much did he gain on all?

13. One hundred stones lie in a straight line, 2 yd. apart. A lad starts at the first stone, and brings each of the others separately to the point of starting. How far does he travel?

GEOMETRICAL PROGRESSION.

470. In a geometrical progression there are five things to be considered:

the first term,	for short,	called	*first;*
the last term,	"	"	*last;*
the number of terms,	"	"	*number;*
the common ratio,	"	"	*ratio;*
the sum of the terms,	"	"	*sum.*

Thus, in the series 5, 10, 20, 40, 80,
first = 5, *last* = 80, *number* = 5, *ratio* = 2 (= 10 ÷ 5, or 20 ÷ 10, etc.),
sum = 155 (= 5 + 10 + 20 + 40 + 80).

471. ***To find last when first, number, and ratio are given.***

1. If the series 3, 6, 12, 24, etc., were continued until there were 10 terms, what would the last term be?

EXPLANATION.

The 2d term $= 3 \times 2^1$;
" 3d term $= 3 \times 2^2$;
" 4th term $= 3 \times 2^3$;
. . . .
" 10th term $= 3 \times 2^9$.

OPERATION.

$10 - 1 = 9$;
$2^9 = 512$;
$3 \times 512 = 1536$.

FORMULA: *last* = *first* × $(ratio)^{number-1}$

EXERCISES (Written).

472. Find the:

1. 10th term of the series 7, 14, 28, etc.
2. 6th term of the series 5, 20, 80, etc.
3. 5th term of the series 2, $2\frac{2}{3}$, $3\frac{5}{9}$, etc.
4. 7th term of the series 2, 1, $\frac{1}{2}$, etc.
5. 9th term of the series $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{32}$, etc.

6. If a boy gathers 5 chestnuts on Monday, 15 chestnuts on Tuesday, 45 chestnuts on Wednesday, and so on, how many will he gather the following Saturday?

7. A merchant doubles his capital every 3 years. If he begins with $4000, how much will he have at the end of 18 years?

8. If the first term is $100 and the ratio 1.06, what is the 5th term? Or, what is the amount of $100 at compound interest for 4 years at 6%?

9. What is the amount of $450 for 6 years, at 5% compound interest?

473. ***To find sum when first, last, and number are given.***

1. Find the sum of 4, 12, 36, 108, 324.

EXPLANATION.

$$\begin{array}{rl}
3 \times \text{sum} = & 972 + 324 + 108 + 36 + 12. \\
\underline{\text{Sum} =} & \underline{} \quad \underline{324} + \underline{108} + \underline{36} + \underline{12} + \underline{4}. \\
2 \times \text{sum} = & 972 + \ \ 0 + \ \ 0 + \ 0 + \ 0 - 4. \\
2 \times \text{sum} = & 972 - 4 = 324 \times 3 - 4. \\
\text{Sum} = & \frac{324 \times 3 - 4}{2} = \frac{324 \times 3 - 4}{3 - 1}.
\end{array}$$

OPERATION.

$324 \times 3 = 972;$

$972 - 4 = 968.$

$3 - 1 = 2;$

$968 \div 2 = 484.$

$$\text{FORMULA: } sum = \frac{last \times ratio - first}{ratio - 1}$$

EXERCISES (Written).

474. Find the sum of:

1. 5, 10, 20, 40, 80, 160, 320, 640.
2. 5, 15, 45, 135, 405, 1215, 3645.
3. $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8, 16, 32, 64.

Find the sum of :

4. 7 terms of the series 4, 6, 9, $13\frac{1}{2}$, etc.
5. 10 terms of the series 32, 16, 8, 4, etc.
6. 6 terms of the series $\frac{9}{16}$, $\frac{3}{4}$, 1, $\frac{4}{3}$, etc.
7. 9 terms of the series 35, 7, 1.4, etc.

8. If a man were to buy 11 mules, paying 10 cents for the first, 30 cts. for the second, 90 cts. for the third, and so on, what would he pay for the whole ?

9. How large a debt may be discharged in a year by paying $3 the first month, $6 the second month, $12 the third month, and so on ?

10. If a mother should give her daughter 1 cent at birth, and double it on each succeeding birthday, how much would the daughter be worth when she became 21 years of age, provided she spent none of it ?

MENSURATION.

475. Mensuration treats of the measurements of lines, surfaces, solids, and angles.

What is a line ? A surface ? A solid ? An angle ?

By what are lines measured ? Surfaces ? Solids ? Angles ?

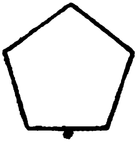
476. Parallel lines are lines which are everywhere the same distance apart.

When is one line perpendicular to another ?

Draw, as nearly as you can, (1) two lines parallel to each other. (2) Two lines perpendicular to each other.

What is a plane ? A plane figure ?

477. A **polygon** is a plane figure bounded by straight lines.



The **perimeter** of a polygon is the distance around it, or the sum of all its sides.

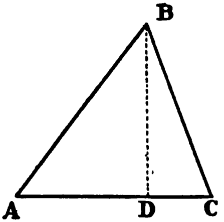
The **base** of a polygon is the side on which it is supposed to stand. A **diagonal** is a straight line joining two vertices not adjacent. A polygon is **equilateral** when its sides are all equal to one another.

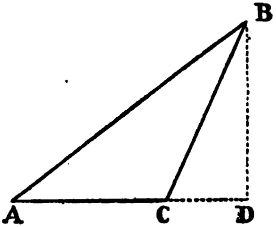
A polygon of three sides is called a **triangle**; of four sides, a **quadrilateral**; of five sides, a **pentagon**; of six sides, a **hexagon**; etc.

Draw a triangle; a quadrilateral; a pentagon; a hexagon.

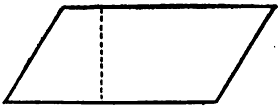
What is a circle? What is the diameter of a circle? What is the radius?

478. The **vertical angle** of a triangle, as B, is the angle opposite the base; and the **altitude**, as BD, is the line drawn from the vertical angle perpendicular to the base, or the base produced.





479. A **parallelogram** is a quadrilateral whose opposite sides are parallel. The opposite sides are also equal.

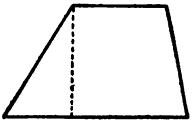


When a parallelogram is right-angled, it is a **rectangle**; and when the four sides of a rectangle are equal, it is a **square.**

The **altitude** of a parallelogram is the perpendicular distance between the base and the opposite side.

480. A **trapezoid** is a quadrilateral having two of its sides parallel.

The **altitude** of a trapezoid is the perpendicular distance between its parallel sides. The parallel sides are called **bases.**



EXERCISES (In Construction).

481. Draw (if without instruments) as nearly as you can, the following:

1. An equilateral triangle. A triangle whose altitude is equal to (1) the base; (2) twice the base; (3) half of the base.

2. A parallelogram whose altitude is equal to (1) the base; (2) ⅓ of the base; (3) 2 times the base; (4) 1½ times the base.

3. A trapezoid whose upper base is equal to (1) ½ the lower base; (2) ⅔ of the lower base. A trapezoid whose altitude is equal to (1) the lower base; (2) the upper base; (3) ⅓ of the lower base.

4. Draw a rectangle whose length is equal to (1) 2 times the breadth; (2) 1⅓ times the breadth; (3) 3 times the breadth.

5. An equilateral pentagon; an equilateral hexagon.

6. A circle (1) whose diameter is 3 inches; (2) whose radius is 2 inches.

Perimeters of Plane Figures.

EXERCISES (Oral and Written).

482. Find the perimeter of:

1. An equilateral triangle, whose base is 4⅓ inches.

2. An equilateral quadrilateral, whose base is 2½ inches.

3. An equilateral pentagon, whose base is 3.4 feet.

4. An equilateral hexagon, whose base is 1⅔ yards.

5. A parallelogram, whose adjacent sides are 9 and 6 feet.

6. A rectangle, whose length is 36 ft. and whose diagonal is 39 ft.

7. A trapezoid, whose bases are 21 and 7 inches, altitude 12 in., and one of its diagonals 20 inches.

The perimeter of a circle is called **circumference.**

483. PRINCIPLE.—*The circumference of a circle is equal to a straight line which is nearly 3.1416 times the diameter.*

Hence, *circumference* = *diameter* × 3.1416;
diameter = *circumference* ÷ 3.1416.

8. Find the circumference of a circle whose diameter is 12 feet.

9. What is the circumference of a wheel whose diameter is 34 ft.?

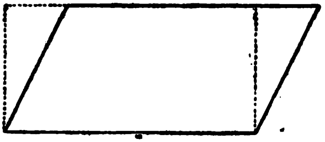
10. What is the diameter of a wheel whose circumference is 36 ft.?

11. If it is 421 yd. around a circular garden, how far is it through it?

12. A wheel, whose diameter is 4 ft. 6 in., is rolled over once on the ground. What is the length of its track?

Area of Surfaces.

EXERCISES (Written).

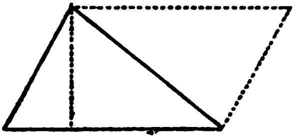


484. PRINCIPLE.—*The surface of a parallelogram is equal to that of a rectangle having the same base and altitude.*

Hence, *area of a parallelogram = base × altitude.*

1. Find the area of a parallelogram whose base is 26 ft. and altitude 15 in.

2. How many acres in a piece of land in the form of a parallelogram, the base being 24.5 rd. and the altitude 18 rd.?



485. PRINCIPLE.—*The surface of a triangle is equal to one half of that of a parallelogram having the same base and altitude.*

Hence, *area of a triangle* = $\frac{1}{2}$ (*base × altitude*).

3. How many acres in a triangular field whose base is 128 rd. and whose altitude is 75 rd.?

4. Find the area of a triangular garden whose base is 7 ch. and whose altitude is 5 ch.

5. How many square yards in the gable end of a house 36 ft. wide and 13 ft. 6 in. high?

6. What is the area of a triangle whose sides are 10, 8, and 6 ft.?

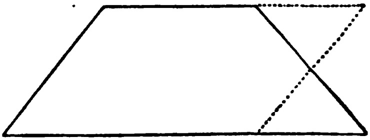
SOLUTION.—$(10 + 8 + 6) \div 2 = 12 = \frac{1}{2}$ of the sum of the sides.
$12 \times (12 - 10) \times (12 - 8) \times (12 - 6) = 12 \times 2 \times 4 \times 6 = 576$.
$\sqrt{576} = 24$; hence the area is 24 sq. ft.

RULE.—*From the half sum of the three sides, subtract each side separately; then multiply the half sum and the three remainders together, and extract the square root of the product. The result will be the area.*

7. What is the area of a triangle whose sides are 3, 4, and 5 inches?

What is the area of a triangle:

8. Whose sides are 10, 24, and 26 rods?
9. Whose sides are 25, 36, and 49 chains?
10. Whose sides are 16, 34, and 30 feet?
11. Whose sides are each 60 rods?



486. PRINCIPLE.— *The surface of a trapezoid is equal to that of a parallelogram having the same altitude and a base equal to the average of the two bases of the trapezoid.*

Hence, $$\textit{area of trapezoid} = \frac{\textit{sum of bases}}{2} \times \textit{altitude}.$$

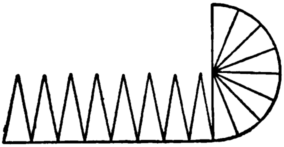
12. What is the area of a trapezoid whose bases are 16 ft. and 12 ft., and altitude 10 ft.?

13. What is the area of a trapezoid whose bases are 64 ft. and 48 ft., and altitude 55 ft.?

14. How many acres in a field in the form of a trapezoid, whose parallel sides are 60 rd. and 52 rd., and the distance between them 48 rd.?

15. What is the value of a piece of land in the shape of a trapezoid whose bases are 34 ch. and 32 ch., and altitude 25 ch., at $12.50 per acre?

487. PRINCIPLE.—*The surface of a circle is equal to that of a rectangle whose breadth is the radius and whose length is ½ of the circumference.*



Hence, *area of circle = radius × ½ of circ.*

Since diameter = 2 × radius,
and circ. = diameter × 3.1416,
$area\ circle = (radius)^2 \times 3.1416;$
$and\ radius = \sqrt{area \div 3.1416}.$

16. Find the area of a circle whose radius is 6 ft.

17. Find the area of a circle whose diameter is 7 ft.

18. Find the area of a circle whose circumference is 12 ft.

19. Find the radius of a circle whose area is 64 sq. yd.

20. What is the value of a circular garden whose diameter is 15 chains, at $12.50 per acre?

21. How long a rope will it take to fasten a horse to a post so that he may graze over 2 acres?

488. A **prism** is a solid, two faces of which are equal and parallel polygons, and the other faces parallelograms.

The **bases** of a prism are the equal and parallel polygons; the **lateral faces** are all the faces except the bases; the **lateral surface** is the sum of the lateral faces; the **lateral edges** are the straight lines in which the lateral faces meet; and the **altitude** is the perpendicular distance between the bases.

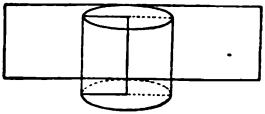
A **prism** is triangular, quadrangular, pentagonal, hexagonal, etc., according as its bases are triangles, quadrilaterals, pentagons, etc.

A **right prism** is a prism whose lateral faces are rectangles; or whose lateral edges are perpendicular to the bases.

489. A cylinder is a solid bounded by a uniformly curved surface, with two bases which are equal and parallel circles.

The **altitude** of a cylinder is the perpendicular distance between its bases, and the **lateral surface** is the entire surface except the bases.

490. PRINCIPLE.—*The lateral surface of a right prism or cylinder is equal to that of a rectangle having the same altitude and a base equal to the perimeter of the base of the solid.*



Hence, *lateral surface of a right prism or cylinder = altitude × perimeter of base.*

22. Find the lateral surface of a prism whose base is a pentagon, each side of which is 6 ft., and its altitude 9 ft.

23. What is the lateral surface of a right prism whose altitude is 15 ft., and the base a triangle, each side of which is 4 ft. 8 in ?

24. Find the lateral surface of a cylinder whose altitude is 12 in., and the diameter of whose base is 8 in.

SOLUTION.—8 in. × 3.1416 = 25.1328 in., circumference of base; 25.1328 × 12 = 301.5936 ; hence 301.5936 sq. in.—Ans.

25. The altitude of a cylinder is 6 ft., and the diameter of its base 2 ft. 6 in. What is the lateral surface ?

The entire surface of a prism or cylinder is found by adding the area of the bases to the lateral surface.

Find the entire surface :

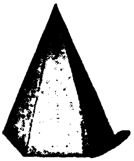
26. Of a cube whose edge is 5 ft.

27. Of a cylinder whose altitude is 7 ft., and the radius of whose base is 2 ft.

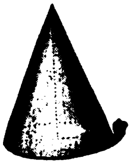
28. Of a triangular prism whose altitude is 6 ft., and the sides of whose bases are 3, 4, and 5 ft.

491. A **pyramid** is a solid, one face of which is a polygon, and the other faces are triangles, which meet at a common point, called the vertex.

The **altitude** of a pyramid is the perpendicular distance from the vertex to the base.



492. A **cone** is a solid having one base, which is a circle, and a curved surface terminating in a point, called the vertex.



The **altitude** of a cone is the same as that of a pyramid.

The **slant height** of a regular pyramid is the straight line drawn from its vertex perpendicularly to one side of the base.

The **slant height** of a cone is a straight line from the vertex to the circumference of the base.

The **lateral surface** of a pyramid or cone is all the surface except the base.

493. PRINCIPLE.—*The lateral surface of a pyramid or cone is equal to that of a triangle whose altitude is the slant height, and whose base is the perimeter of the base of the solid.*

Hence, *lateral surface of pyramid or cone*
$= \frac{1}{2}$(*slant height*) × *perimeter of base.*

29. What is the lateral surface of a triangular pyramid whose slant height is 12 ft., and each of the sides of whose base is 2 ft.?

SOLUTION.—2 ft. × 3 = 6 ft., perimeter of the base. 6 × $\frac{1}{2}$ of 12 = 36; 36 sq. ft., lateral surface.

30. What is the lateral surface of a cone, the slant height of which is 20 ft., and the radius of the base 3 ft.?

SOLUTION.—3 ft. × 2 × 3.1416 = 18.8496 ft. circum. of base. 18.8496 × $\frac{20}{2}$ = 188.496; hence 188.496 sq. ft. is the lat. surface.

The entire surface of a pyramid or cone may be found by adding the area of the base to the area of the lateral surface.

Find the entire surface:

31. Of a pyramid, the base of which is 15 ft. square, and the slant height 42 ft.

32. Of a cone, the slant height of which is 60 ft., and the radius of the base 9 ft.

33. Of a pyramid, the slant height of which is 40 ft., and the base a triangle, each of whose sides is 5 ft.

34. Of a cone, the slant height of which is 6 ft., and the diameter of the base 2 ft. 8 in.

494. A **sphere** is a solid bounded by a curved surface all points of which are equally distant from a point within called the center.

The **diameter** of a sphere is a straight line drawn through the center and terminated both ways by the surface. The **radius** is a straight line drawn from the center to the surface.

495. PRINCIPLE.—*The surface of a sphere is equal to that of four circles each having a radius equal to that of the sphere.*

Hence, *surface of sphere* = 4 (*radius*)^2 × 3.1416.

35. Find the surface of a globe 3 ft. in diameter.

36. Find the surface of a ball whose radius is 7½ in.

37. The earth is 7912 miles in diameter. How many square miles in its surface?

Volume of Solids.

EXERCISES (Written).

496. PRINCIPLE.—*The volume of a prism or cylinder is equal to that of a rectangular solid having the same altitude and an equivalent base.*

Hence, *volume of prism or cylinder*
= area of base × altitude.

1. Find the volume of a square prism, the altitude of which is 10 ft., and each side of the base 4 ft.

SOLUTION.—4 × 4 = 16 ; 16 sq. ft. = the area of base. 16 × 10 = 160 ; hence 160 cu. ft. is the volume.

2. Find the volume of a cylinder whose altitude is 6 ft., and the radius of whose base is 5 ft.

SOLUTION.—$5^2 \times 3.1416 = 78.54$; 78.54 sq. ft. = area of base. 78.54 × 6 = 471.24 ; hence 471.24 cu. ft. is the volume.

3. What is the volume of a pentagonal prism whose altitude is 16 in., and the area of whose base is 18.4 sq. in.?

4. What is the volume of a cylinder whose altitude is 8 ft., and the diameter of whose base is 1 ft. 6 in.?

5. What is the value of a piece of timber, in the shape of a triangular prism, whose length is 15 ft., and the sides of whose bases are 6, 8, and 10 in., at 12 cts. per cu. ft.?

6. How many gallons of water will a cylindrical cistern hold, the altitude of which is 11 ft., and the diameter of the bases 8 ft.?

497. PRINCIPLE.—*The volume of a pyramid or cone is equal to ⅓ of the volume of a prism or cylinder, respectively, having the same base and altitude.*

Hence, *volume of pyramid or cone*
= area of base × ⅓ altitude.

7. What is the volume of a pyramid, the altitude of which is 15 ft., and the base 6 ft. square?

SOLUTION.—$6 \times 6 \times \frac{15}{3} = 180$; 180 cu. ft. is the volume.

8. Find the volume of a cone, the altitude of which is 9 ft., and the radius of the base 5 ft.

SOLUTION.—$5^2 \times 3.1416 = 78.54$; 78.54 sq. ft. = area of base.
$78.54 \times \frac{9}{3} = 235.62$; 235.62 cu. ft. is the volume.

Find the volume:

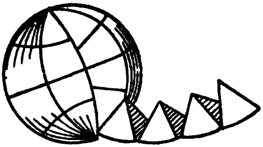
9. Of a pyramid whose altitude is 60 in., and the base a triangle 10 in. on each side.

10. Of a cone whose altitude is 36 ft., and the circumference of the base 314.16 ft.

11. What is the value of a piece of timber in the form of a pyramid, the altitude of which is 18 ft., and the base a triangle whose sides are 9 in., 12 in., and 15 in., at 15 cts. per cu. ft.?

12. How many bushels will a conical vessel hold whose altitude is 6 ft. and radius of base 2 ft.?

498. PRINCIPLE.—*The volume of a sphere is equal to that of a pyramid whose altitude is the radius of the sphere and whose base is equal to the surface of the sphere.*



Hence, *volume of sphere*

$$= \tfrac{4}{3}\,(radius)^3 \times 3.1416.$$
$$= (diameter)^3 \times .5236.$$

13. Find the volume of a sphere whose diameter is 5 inches.

SOLUTION.—$5^3 \times .5236 = 65.45$; hence 65.45 cu. in.—Ans.
The decimal $.5236 = \frac{1}{6}$ of 3.1416.

14. Find the volume of a globe whose diameter is 20 inches.

15. Find the volume of a sphere whose radius is 3 ft.

16. How many cubic inches of lead in a ball 3 in. in diameter?

17. A cu. ft. of iron weighs about 447 pounds. What is the weight of a cannon ball 8 in. in diameter?

GOVERNMENT LANDS.

499. Under the administration of Thomas Jefferson the public lands were laid off according to the following system:

A north and south line, called the *principal meridian* (**P. M.**), was run, its course being marked by stones, posts, etc.

Next, lines were run 6 miles apart, and parallel to the **P. M.**, thus dividing the country into *strips* called **ranges** (**R.**), which are numbered in order, both east and west, from the **P. M.**

Thus, **R.** 3 E. is the 3d strip or range east of the **P. M.**, and **R.** 4 W. is the 4th range west of the **P. M.**

Next, a line was run east and west, called the **base line** (**B. L.**).

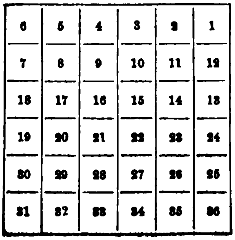
Next, lines were run 6 miles apart, and parallel to the **B. L.**, thus dividing the ranges into squares of 36 sq. mi. each, called **townships**, which are numbered in order, both north and south, from the **B. L.**

Thus, **Tp.** 18 N. **R.** 5 W. is the 18th township north of the **B. L.** and in the 5th range west of the **P. M.**

A township is divided into 36 square miles, called **sections** (Sec.).

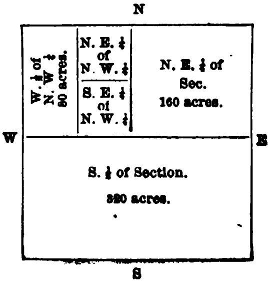
The U. S. Land Office recognizes the following divisions of a section:

Half Section = 1 mi. by $\frac{1}{2}$ mi. = $\frac{1}{2}$ sq. mi. = 320 acres.
Quarter Section = $\frac{1}{2}$ mi. by $\frac{1}{2}$ mi. = $\frac{1}{4}$ sq. mi. = 160 acres.
Half-Quarter Section = $\frac{1}{2}$ mi. by $\frac{1}{4}$ mi. = $\frac{1}{8}$ sq. mi. = 80 acres.
Quarter-Quarter Section = $\frac{1}{4}$ mi. by $\frac{1}{4}$ mi. = $\frac{1}{16}$ sq. mi. = 40 acres.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

SECTIONS OF A TOWNSHIP.


N
W. ½ of N. W. ¼ 80 acres.
N. E. ¼ of N. W. ¼
S. E. ¼ of N. W. ¼
N. E. ¼ of Sec. 160 acres.
W
E
S. ½ of Section. 320 acres.
S

A SECTION ENLARGED.

EXERCISES (Written).

500. Let the student draw a square representing a section, divide it into halves, quarters, etc., and point out the land designated in the following.

How many acres in the:

1. W. ½ and N. ½ of N. E. ¼ of Sec. 20?
2. N. W. ¼ and W. ½ of N. E. ¼ of Sec. 21?
3. S. E. ¼ and E. ½ of S. W. ¼ of Sec. 17?
4. W. ½ of S. W. ¼ and S. W. ¼ of N. W. ¼ of Sec. 5?
5. S. W. ¼ of N. E. ¼ and S. E. ¼ of N. W. ¼ of Sec. 11?
6. N. ½ of S. W. ¼ and N. W. ¼ of S. E. ¼ of Sec. 31?
7. E. ½, S. W. ¼, and E. ½ of N. W. ¼ of Sec. 3?
8. N. W. ¼ of S. E. ¼, N. E. ¼ of S. W. ¼, S. E. ¼ of N.W. ¼ and W. ½ of N. E. ¼ of Sec. 10?

Parallel Problems.

501. *1.* How many acres in a triangle whose base is 15 chains and altitude 12 chains?

2. What is the value of a triangular piece of land whose base is 528 yards and altitude 165 yards, at $12½ per acre?

3. The three sides of a triangular board are 6, 5, and 5 inches. Find the area of the board.

4. The three sides of a triangular garden are 12, 39, and 45 chains. How many acres in the garden?

5. The diagonal of a rectangle is 13 inches, which is 1 inch more than the length of the rectangle. Find the width of the rectangle.

6. A ladder 52 ft. long stands up against a wall. How far must it be drawn out at the bottom that the top may be lowered 4 ft.?

7. A box is 2 ft. long, 2 ft. wide, and 1 ft. deep. How long a cord will it take to reach from a lower to the opposite upper corner?

8. A room is 21 ft. long, 16 ft. wide, and 12 ft. high. How far is it from an upper corner to the opposite lower one?

9. Find the diagonal of a square whose side is 1.

10. Find the diagonal of a cube whose side is 1.

11. How many acres in a parallelogrammic field whose base and altitude are, respectively, 4½ and 3⅓ ch.?

12. A man has a trapezoidal field whose parallel sides are 480 and 736 yd., and whose altitude is 176 yd. What is the value of the field at \$6¾ an acre?

13. The altitude of a cone is 40 in. and the diameter of its base 18 in. What is the slant height?

14. A pole 90 ft. high is supported by three guys attached to the top, and reaching the ground at the distances of 120 ft., 216 ft., and 400 ft. from the foot of the pole. Find the length of the guys.

15. How long, omitting the fraction, is the length of the tire of a wheel whose diameter is 8 ft.?

16. If the length of a man's step is 2 ft. 6 in., how many *whole* steps will he make in walking around a circular field which is one mile in diameter?

17. How many acres, omitting the fraction, are there in a circular garden whose diameter is 6 chains?

18. If a tether permits a horse to graze anywhere between 10 and 100 yards from the tether-stick, on how many acres can he graze?

19. If the greatest square log that can be cut from a round log is 5 in. by 5 in., what is the diameter of the round log, omitting the fractions?

20. What is the diameter of a round log if a cord which will just reach around it is 13 ft. 5 in. long?

21. What is the entire surface of a right prism whose altitude is 6 in. and whose base is 5 in. square?

22. What is the entire surface of a right cylinder whose altitude is 6 inches and radius of base 5 inches?

23. The slant height of a right pyramid is 11 in. and the base is 4 in. square. Find the number of square inches in the entire surface.

24. Find the entire surface of a right cone whose altitude is 12 inches and the diameter of whose base is 10 inches.

25. How many square inches of map surface on a 10-inch school globe, omitting the fraction ?

26. What would it cost to paint a spherical cannon-ball 15 inches in diameter at $½ per sq. foot ?

27. How many board feet in a piece of timber 20 in. long, 6 in. wide, and 5 in. thick ?

28. A cylindrical tank is 8 ft. in diameter and 7 ft. high. How many gallons of water will it hold ?

29. What is the weight of a marble pyramid whose base is 6 in. square and whose altitude is 5 in., if a cu. inch of marble weighs $\frac{1}{10}$ of a pound ?

30. A cu. foot of silver weighs 654⅝ lb. What is the weight of a silver cone 20 in. in diameter and 18 in. in altitude ?

31. The diameter of a sphere is 6 feet. How many cu. ft. does it contain, omitting the fraction ?

32. A cubic foot of cast-iron weighs 450½ lb. What is the weight of a cannon-ball whose diameter is 12 inches ?

33. Draw a diagram and find the area of a man's land which is described as follows : The E. ½ of the W. ½, the S. ½ of N. E. ¼, the N. E. ¼ of S. E. ¼, and the W. ½ of S. E. ¼, of section 9, township 7 N., R. 4 E.

MISCELLANEOUS PROBLEMS.

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502. A few important problems are here solved by the analytic method, and the student may use that method in the solution of such of the miscellaneous problems as the teacher may desire.

1. If a man can do $\frac{5}{8}$ of a certain work in a day, how long will it take him to do $\frac{2}{3}$ of it?

SOLUTION.—If he can do $\frac{5}{8}$ of it in 1 day,
he can do $\frac{1}{8}$ of it in $\frac{1}{5}$ day,
and $\frac{8}{8}$, or all of it, in $\frac{8}{5}$ days,
and $\frac{2}{3}$ of it in $\frac{2}{3}$ of $\frac{8}{5} = 1\frac{1}{15}$ days.

2. If $3\frac{3}{14}$ acres of land cost $\$37\frac{1}{2}$, how many acres can be bought for $70?

ANALYSIS.—If $3\frac{3}{14}$ A. cost $\$37\frac{1}{2}$, 1 A. will cost as many dollars as $3\frac{3}{14}$ is contained times in $\$37\frac{1}{2}$, which is $11\frac{2}{3}$; if 1 A. cost $\$11\frac{2}{3}$, $70 will buy as many A. as $11\frac{2}{3}$ is contained times in 70, which is 6.

OPERATION.

$3\frac{3}{14}$ A. $= \$37\frac{1}{2}$
1 A. $= \$11\frac{2}{3}$
$\$70 \div \$11\frac{2}{3} = 6$

3. If 4 men in 3 days earn $60, how much will 7 men earn in 5 days?

ANALYSIS.—If 4 men in 3 da. earn $60, 1 man in 3 da. will earn $\frac{1}{4}$ of $60, or $15; if 1 man in 3 da. earn $15, 1 man in 1 day will earn $\frac{1}{3}$ of $15, or $5; if 1 man in 1 da. earn $5, 1 man in 5 da. will earn 5 times $5, or $25; and if 1 man in 5 da. earn $25, 7 men in 5 da. will earn 7 times $25, or $175.

OPERATION.

Men.	Da.	Dol.
4	3	60
1	3	15
1	1	5
1	5	25
7	5	175

SECOND SOLUTION.

7 men in 3 da. will earn $\frac{7}{4}$ of what 4 men will in 3 da.
7 men in 5 da. " " $\frac{5}{3}$ of $\frac{7}{4}$ " " " "
$\frac{5}{3}$ of $\frac{7}{4}$ of $\$60 = \$175$.

4. **If 5 men in 6 days of 8 hours each can mow 16 acres, how many days of 6 hours each must 10 men work to mow 24 acres ?**

SOLUTION.—6 hr. per day will require $\frac{8}{6}$ as many days as 8 hr. per day; 10 men will require $\frac{5}{10}$ as many days as 5 men; 24 acres will require $\frac{24}{16}$ as many days as 16 acres.

Since the first group requires 6 days, the second will require

$$\tfrac{8}{6} \text{ of } \tfrac{5}{10} \text{ of } \tfrac{24}{16} \text{ of 6 da.} = \text{6 da.}$$

The following is a convenient method of solving all problems of this character:

(1) Draw a vertical line. (2) Write the terms of the supposition (the terms preceded by *if*) as follows: on the left write the living or moving term, under it write the elements of time or distance, and on the right at the bottom the work, the remaining terms of supposition. (3) Write the terms of demand (those preceded by *how much* or *how many*) on the opposite sides of the terms of supposition having the same name. (4) Proceed as in cancellation, remembering that the terms on the side where the blank occurs are the factors of the divisor.

(men) 5	10 (men)
(days) 6	———
(hours) 8	6 (hours)
(acres) 24	16 (acres)

5. If 16 men can excavate a cellar 40 ft. long, 36 ft. wide, and 8 ft. deep in 12 days of 8 hours each, in how many days of 10 hours each can 8 men excavate a cellar 30 ft. long, 27 ft. wide, and 6 ft. deep ?

STATEMENT.

(men) 16	8 (men)
(days) 12	———
(hours) 8	10 (hours)
(ft. long) 30	40 (ft. long)
(ft. wide) 27	36 (ft. wide)
(ft. deep) 6	8 (ft. deep)

6. One third of the trees in an orchard is peach, $\frac{1}{4}$ is apple, $\frac{1}{5}$ is plum, and the remainder, which is 39 trees, is cherry. How many trees in all ?

ANALYSIS.—Since $\frac{1}{3}$ of *all* is peach, $\frac{1}{4}$ apple, $\frac{1}{5}$ plum, the peach, apple, and plum together are equal to $\frac{47}{60}$ of the *whole*, which includes all except $\frac{13}{60}$; hence the cherry is $\frac{13}{60}$ of all. That is, $\frac{13}{60}$ of all is equal to 39 trees; hence $\frac{1}{60}$ is 3 trees, and $\frac{60}{60}$ is 60 times 3 trees, or 180 trees.

OPERATION.

$$\tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} = \tfrac{47}{60}$$
$$\tfrac{60}{60} - \tfrac{47}{60} = \tfrac{13}{60}$$
$$39 \div \tfrac{13}{60} = 180$$

7. A farmer spent $\frac{1}{5}$ of his money for sheep, and $\frac{3}{4}$ of the remainder for hogs, $\frac{2}{3}$ of the last remainder for chickens, and had \$12 left. How much had he at first?

Analysis.—Having spent $\frac{1}{5}$, he had $\frac{4}{5}$ left; and as he spent $\frac{3}{4}$ of this, he had $\frac{1}{4}$ of it, or $\frac{1}{4}$ of $\frac{4}{5} = \frac{1}{5}$ left; of this he spent $\frac{2}{3}$, hence he had $\frac{1}{3}$ of it, or $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ left. But as he had \$12 left, $\frac{1}{15}$ of the whole is equal to \$12; hence the whole is 15 times \$12, or \$180.

Operation.

$$\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{1}{15}$$
$$\$12 \div \frac{1}{15} = \$180$$

8. By selling oranges at 3 cts. each, I shall lose 27 cts.; by selling them at 7 cts. each, I shall gain 33 cts. How many oranges have I?

Solution.—A difference of 7 cts. — 3 cts. (= 4 cts.) in the price per orange produces a difference of 27 cts. + 33 cts. (= 60 cts.) in the sale of all. Therefore I have as many oranges as 4 cts. is contained times in 60 cts.; that is, 15.

9. If I mix 20% of water with 8 gal. of wine, how many gallons of water will there be in the mixture?

Solution.—100% of the mixture = the mixture.
20% " = the water.
80% " = the wine = 8 gal.
1% " = $\frac{1}{80}$ of 8 gal. = .1 gal.
100% " = 100 × .1 gal. = 10 gal.
Therefore the water is 10 gal. − 8 gal. = 2 gal.

10. A miller grinds for $\frac{1}{8}$ toll. How much grain did a farmer carry to the mill if he took away 35 bu. of meal?

Solution.—$\frac{8}{8}$ of the am't carried = the corn.
$\frac{1}{8}$ " " = the toll.
$\frac{7}{8}$ " " = the meal, 35 bu.
Therefore the am't carried = 35 bu. ÷ $\frac{7}{8}$ = 40 bu.

11. If 4 oxen or 5 horses consume 40 bu. of corn in a a certain time, how much will 16 oxen and 7 horses consume in the same time?

Solution.—Since 4 oxen or 5 horses can consume 40 bu., etc., we infer that 4 oxen consume as much as 5 horses; hence 1 ox will consume $\frac{5}{4}$ as much as 1 horse, and 16 oxen 16 times $\frac{5}{4}$, or 20 times as much as 1 horse, or as much as 20 horses. Hence the problem may be stated thus· If 5 horses consume 40 bu. in a certain time, how much will 20 + 7, or 27 horses, consume? Since 5 horses consume 40 bu., 1 horse will consume 8 bu. and 27 horses will consume 27 times 8 bu., or 216 bu.

12. A man agreed to work 20 days for $3 a day and his board, and to pay $1 a day for board when idle; at the end of the time he received $44. How many days was he idle?

Solution.—On each idle day he lost $3 that he might have earned, and $1 for board, or $4 in all.

If he had worked the whole time, he would have received $60; but as he received only $44, he lost through idleness $16.

Since he lost $4 for each idle day and $16 altogether, he must have been idle as many days as $4 is contained times in $16; that is, 4 days.

13. A tank is supplied by a spring. If 13 men can bail out the water in 5 hours, and 9 men can do it in 7 hours, in what time can 6 men do the work?

Solution.—Let the amount of water flowing from the spring into the tank in 1 hour be called the *flow*, and the amount bailed out by 1 man in 1 hr., the *unit of work*. Then

13 men in 5 hr. do 65 units of work,
9 men in 7 hr. do 63 units of work.

Hence the flow in, 7 − 5, or 2 hr., is equivalent to 65 − 63, or 2 units of work. Therefore the flow = 1 unit of work, or the work of 1 man.

During the 5 hr. that the 13 men worked the flow was equivalent to 5 × 1, or 5 units of work; hence the water in the tank when full represents 65 − 5, or 60 units of work.

Of the 6 men, one will discharge the flow, and the number of hours required for the remaining 5 to bail out the tank will be the number of times that 5 units of work is contained in 60 units of work; that is, 12 hr.

14. The divisor is 18, the quotient 12, and the remainder 13. What is the dividend?

15. The multiplicand is the sum of 136 and 154, and the multiplier the difference between 172 and 64. What is the product?

16. The minuend is the product of 64 by 25, and the remainder the quotient of 4625 by 37. What is the subtrahend?

17. Required the G. C. D. of 2145 and 3471, and the L. C. M. of 407 and 888.

18. Add $\frac{3}{4}$, $7\frac{1}{2}$, $\frac{3}{5}$ of $4\frac{1}{6}$, and $\frac{2}{3}$ of $7\frac{1}{2}$.

19. Subtract the difference between $5\frac{1}{2}$ and $2\frac{2}{3}$ from the sum of $3\frac{1}{5}$ and $1\frac{5}{6}$.

20. James paid \$$8\frac{1}{2}$ for pants, \$$3\frac{3}{5}$ for a vest, and had \$$\frac{5}{8}$ left. How much had he at first ?

21. How many yards of cloth can be bought for \$$7\frac{1}{2}$ at \$$\frac{5}{6}$ a yard ?

22. A man bought $16\frac{1}{2}$ acres of land at \$$5\frac{1}{3}$ per acre. What did the land cost ?

23. Divide $\frac{2}{7}$ of $8\frac{3}{5}$ by $\frac{3}{10}$ of $8\frac{8}{9}$.

24. Multiply the sum of $7\frac{1}{2}$ and $5\frac{1}{3}$ by their difference.

25. What number added to $\frac{8}{9}$ of $5\frac{2}{5}$ will produce $7\frac{1}{2}$ times $\frac{4}{5}$?

26. What is the cost of 5670 lb. of hay at \$11.50 a ton ?

27. If flour is worth \$$7\frac{1}{2}$ a barrel, what part of a barrel can be bought for \$$\frac{3}{5}$?

28. If $\frac{7}{8}$ of a number is 35, how much is $\frac{3}{5}$ of it ?

29. A bale of cotton weighs 450 lb. If $\frac{3}{5}$ of it sells at $7\frac{1}{2}$ cts. a lb., and the remainder at $8\frac{2}{3}$ cts., what does the bale bring ?

30. If $\frac{3}{4}$ of a lot is worth \$30, how much is $\frac{7}{8}$ of it worth ?

31. A lad spent $\frac{3}{4}$ of his money for candy. If he had spent $\frac{2}{5}$ of it, he would not have spent as much as he did by 14 cents. How much money had he ?

32. A barrel is half full of water ; if 4 gal. more are poured in, it will be $\frac{3}{5}$ full. How much water will the barrel hold ?

33. A man sold $\frac{5}{12}$ of his hogs at \$$4\frac{1}{2}$ each, and received \$135. How many hogs had he ?

34. Three boys caught 169 fishes, which they shared in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. How many did each boy receive ?

35. What number added to 37.25 will produce 64 and 375 millionths ?

36. What is the number whose parts are 42.3, 6.083, and 62 and 325 ten thousandths?

37. A box whose capacity is 101.34 gal. contains 74.65873 gal. of water. How much more will it hold?

38. Divide the sum of $4\frac{3}{8}$ and 3.2435 by their difference.

39. If a pole $7\frac{1}{2}$ ft. high cast a shadow $12\frac{1}{2}$ ft. long, what is the height of a tree that casts a shadow $163\frac{3}{8}$ ft?

40. A train travels 22 miles in .88 of an hour. What is its rate per hour?

41. A train runs from New York to Philadelphia, a distance of 90 miles, in 2 hr. 18 min. What is its rate per hour?

42. Three-fourths of John's weight is equal to $\frac{12}{13}$ of Henry's. If Henry weighs 52 lbs., what does John weigh?

43. If telegraph poles are 88 yd. apart, and a passenger, on a train, counts 23 poles every 3 min., at what rate is the train traveling per hour?

44. What is the value of a field 605 yd. long and $40\frac{1}{2}$ rd. wide, at $\$8\frac{1}{2}$ per acre?

45. Change $\frac{2}{3}$ of $1\frac{1}{8}$ to a decimal.

46. Change .15625 to a common fraction.

47. A merchant owns $\frac{2}{3}$ of a ship. If $\frac{7}{10}$ of his share is valued at $15995, what is the value of the ship?

48. If 560 is subtracted from .458 of a number, .234 of the number will be left. What is the number?

49. Reduce .625 of a pound troy to lower integers.

50. How many steps, 2 ft. 3 in. each, will a person make in walking 3 miles?

51. What is the value of a pile of bricks 12 ft. long, 5 ft. wide, and 6 ft. high, at $8 per thousand bricks, the dimensions of each brick being 8, 4, and 2 inches?

52. A rectangular field is 120 rods long and 75 rods wide. How many boards, each 12 ft. long, will inclose it with a fence 5 boards high?

53. If fencing cost $2\frac{1}{2}$ cents a foot, how much more will it cost to fence a rectangular field 120 rd. long and 30 rd. wide than an equivalent square field ?

54. A man sold the N. ½ of S. E. ¼, and the S. E. ¼ of N. E. ¼ of section 16, at $12½ per acre. What did he receive ?

55. What will it cost to dig a ditch 120 yd. long, 3 ft. wide, and 2½ ft. deep, at 40 cts. per cu. yard ?

56. What per cent of 8⅓ is 1¼ ?

57. Three sevenths of a number is what per cent of $\frac{5}{9}$ of it ?

58. What per cent of pure gold is there in a gold dollar ?

59. What per cent of the Avoir. pound is the Troy pound ?

60. Sold a horse for $72 and lost 20 %. What did the horse cost ?

61. What per cent was made on a farm which cost $6250 and sold for $7000 ?

62. A owned 30 % of a vessel and sold 15 % of it ; B owned 45 % of the vessel and sold 12 % of it ; A and B sold at the same rate, and together received $1287. What was the value of the vessel at that rate ?

63. Sold a cow for $57, and gained the same per cent that I would have lost had I sold her for $29. What per cent did I gain ?

64. Sold an ox for $41.25, at a gain of 25 %. What % would have been lost had he been sold for $27 ?

65. Sold 20 % of a quantity of flour at a loss of 25 %, and received $1.80. What was the cost of all ?

66. Bought 100 cigars for $2.50. What should I price them each so that I may deduct 33⅓ % of the price and still make a profit of 20 % ?

67. By selling eggs at 3½ cts. each I shall make 40 cents, and by selling them at 2¾ cts. each I shall lose 11 cts. How many eggs have I, and what did they cost me ?

68. A man bought stock at a discount of 19½ %, and sold it at a premium of 7⅛, paying ½ % brokerage each transaction. What % did he gain ?

69. Three fourths of a number lacks 12 of being as much as 5/6 of 36. What is the number ?

How far is it, in chains, from the center :

70. Of S. W. 1/4 of S. E. 1/4 of sec. 11 to the center of N. E. 1/4 of N. W. 1/4 of sec. 11 ?

71. Of N. E. 1/4 of sec. 32 to the N. E. corner of S. W. 1/4 of sec. 11 of the same Tp. ?

72. Of Tp. 4 N. R. 3 E. to the N. E. corner of S. E. 1/4 of sec. 10, Tp. 2 S. R. 5 W. ?

73. A room is 3/5 as wide as it is long, and contains 135 sq. ft. What are its dimensions ?

74. What is the length of one side of a square field whose area is 22½ acres ?

75. What is one dimension of a cubical tank that will hold the same amount of water as one that is 18¾ ft. long, 15 ft. wide, and 12 ft. high ?

76. A box, whose length is 2 times its width and whose width is 3 times its height, contains 6174 cu. in. What are its dimensions ?

77. How much leather will it take to cover the largest ball that can be cut from a ten-inch cube ?

78. A farmer had a number of sheep ; at one time he sold 10 %, at another time 16⅔ % of the remainder, and at another time 20 % of what he then had, and then had 432 sheep left. How many had he at first ?

79. A commission merchant receives $1611.60 with instructions to invest it in flour at $5 per barrel. How many barrels does he buy, his commission being 2 % ?

80. A gentleman owns a house which he values at $2450. For what sum must he have it insured so that if the property is destroyed he will lose nothing, rate of insurance being 2 % ?

81. If 8 men or 14 boys can dig a ditch 120 ft. long in 5 days, how many boys are needed to help 3 men in order that they may dig a ditch 405 ft. long in 9 days ?

82. If a dividend of 9½ % is declared on the stock of a railroad, how many shares are owned by a stockholder who receives $608?

83. In what time will $972 yield $211.41 interest, at 6 %?

84. What must be the face of a note, payable in 8 mo. 9 da., so that when discounted at a bank, at 9 %, the proceeds may be $1275?

85. A, B, and C rented a pasture for $55.20; A put in 25 cows for 6 weeks; B, 30 cows for 7 weeks; and C, 48 cows for 5 weeks. What part of the rent ought each to pay?

86. How many pounds of tea worth 36, 42, and 45 cts. a pound must be mixed together so that the mixture may be worth 40 cts. a pound?

87. B agreed to work for 30 days at $2½ a day and his board, and to pay $¾ a day for board when idle; at the end of the time he received $36. How many days did he work?

88. A borrows of B $400 for 6 months; and B afterward borrows $200 of A. How long should A allow him to keep it to balance the favor?

89. A block of wood is 21 in. long, 18⅔ in. wide, and 7 in. high. How much more cloth will it take to cover it than an equivalent cubical block?

90. A owes B $600, of which $\frac{5}{12}$ is due in 6 mo., $\frac{3}{10}$ of the remainder in 8 mo., and the balance in 12 mo. What is the average term for the whole sum?

91. John is flying his kite with a cord 225 ft. long. If it is 112 ft. from where John stands to a point directly under the kite, how high is the kite, allowing 13 ft. for the sag of the cord?

92. B owes $1000 due in 6 mo., $800 due in 5 mo., and $1200 due in 3 mo. If he pays ⅖ of the whole in 2 mo., when ought the remainder to be paid?

93. A and B formed a partnership for 1 year, A putting in $1200 and B $400. How much more must B put in at the end of 8 mo. to receive one-half of the profits?

94. What is the area of a triangle whose sides are 50, 48, and 14 ft.?

95. What is the square root of .14356521?

96. What is the cube root of .001906624?

97. What is the area of the lateral surface of a cubical box which contains 110592 cu. in.?

98. If A leaves Baltimore for Washington at the same time that B leaves Washiugton for Baltimore, how long before they will meet, if A can travel the whole distance in 12½ hours, and B in 16⅔ hours?

99. A can do a piece of work in 3 days, B in 4 days, and C in 8 days. How long will it take all to do the work?

100. A can do a work in 10 days, B in 12 days, and C in 18 days. How many days must A and B work together, so that, with the assistance of C for the remainder of the time, the work may be finished in 5 days?

101. At what time between 3 and 4 o'clock is the minute hand 18 minute spaces in advance of the hour hand?

102. What change is produced in the income by selling $5000 of 6% stocks at 10% discount, and investing the proceeds in 10% stocks at a premium of 25%?

103. If I sell 3½% stock at 88½, at what price must I buy 4½% stock, that there may be no loss of income?

104. What sum must I invest in 4% stock, at 98, to realize an income of $200 a year?

105. A merchant has candies worth 30, 40, and 50 cts. a pound, of which he forms a mixture by taking 3 times as much of the second as of the first, and 2 times as much of the third as of the second, and sells the mixture at 50 cts. a pound. What % does he gain?

106. The diameter of a wheel is 2 ft. How many times will it revolve in rolling a rod?

107. Find the area of a circular ring of which the inner and outer diameters are 8 and 13.

108. Find the volume of a spherical shell whose inner and outer radii are 5 and 8.

109. The surface of a sphere is equal to the lateral surface of a cylinder. If the diameter of each is 12 ft., what is the height of the cylinder?

110. A note for $4000, dated May 1, 1899, bearing interest at 6%, has the following indorsements upon it: May 21, 1900, $800; June 10, 1901, $1200; Aug. 10, 1902, $1500. What was due May 1, 1903?

111. I rent a house for $535.60 payable in 6 mo. without interest. If I settle the amount now, what ought I to pay, allowing 6½% discount?

112. How much ready money is equal to $2100 payable in 3 mo., $2100 payable in 6 mo., and $2100 payable in 9 mo., money being worth 8%?

113. How many men are required to build a wall 500 ft. long, 2⅔ ft. thick, and 24 ft. high, in 64 da. of 10 hr. each, if 20 men can build a wall 33⅓ ft. long, 3 ft. thick, and 14 ft. high, in 9 da. of 9⅓ hr. each?

114. A vessel leaves New York for Liverpool on Monday at 10 o'clock A.M., and sails at the rate of 13½ miles per hour. A second vessel leaves the same place for Liverpool on Tuesday at 2 o'clock P.M., and sails at the rate of 18 miles per hour. When will the latter overtake the former?

115. A can do a certain work in 8 days, B the same in 12 days, and C in 16 days. They all work together and receive $18.20. How should it be divided among them?

116. How many three-inch cubical blocks can be packed in a box 2 ft. long, 1 ft. 3 in. wide, and 9 in. deep, inside measurement?

117. The diameter of the earth is about 8000 mi. and that of the sun about 880000 mi. How many times larger is the sun than the earth?

118. If a manufacturer sells to a jobber at 10% profit, and the jobber sells to the retailer at 20% profit, and the retailer sells to the planter at 25% profit, what part of the price paid by the planter is consumed in profits?

119. A laborer agreed to work for $12 a month, payable at the end of each month. After working 12 months, and receiving no pay, how much is he entitled to, at 6% simple interest?

120. A man bought 80 shares of bank stock at 98; he held it for 2 yr. 6 mo., receiving a semi-annual dividend of 5% on the same, and then sold it at 104. Money being worth 8% simple interest, what was his gain?

121. When cotton is quoted in New Orleans at 9 cts. a pound, what should it be worth in Liverpool, £1 being quoted at $4.86, and freight 1½ cts. a pound?

122. How many hektoliters of wheat will a bin contain that is 5 m. square, and 2.5 m. deep?

123. How many gallons of water will a cylindrical vessel hold whose diameter is 10 ft., and height 8 ft.?

124. A, B, and C plant 1080 acres of corn, of which A's interest is 2 times B's, and B's 3 times C's. They sell the entire crop, amounting to 28½ bu. to the acre, at 36 cts. per bu. What is each man's share of the profit?

125. A traveler on arriving at New Orleans, found his watch, which indicated the correct time of the place from which he started, 1 hr. 12 min. fast. How many degrees of longitude had he passed over, and in what direction?

126. A cistern has 3 pipes; the first can fill it in 40 min., the second in 50 min., and the third can empty it in 1 hr. If these pipes are all opened at 11 o'clock A.M., the cistern being empty, at what time will it be filled?

127. In still water A can row a boat 7½ mi. in 1½ hr., and B, 38½ mi. in 4⅙ hr. At what rate per hour can they together row a boat up stream in a river where the current is 2 mi. per hr.?

128. The area of the bottom of a rectangular solid is 375 sq. in.; of one side, 225 sq. in.; and of one end, 135 sq. in.

What is one of the dimensions of a cube whose volume is equal to that of the solid ?

129. How many men and women will earn \$375⅔ in 30⅔ days, if 4 men or 5 women earn \$53⅔ in 7⅔ days ?

130. A contracted to do a piece of work in 9 days for \$28.40 ; after working 4 days he employed B to assist him, and at the close of the 7th day employed C, and together they finished the work at the end of the 9th day. How ought the money to be divided among them, B being a ⅓ better man than C, and A a ¼ better man than B ?

131. If 60 cows can eat off the grass of a pasture in 8 weeks, and 90 cows can eat it in 5 weeks, how long would it take 50 cows to eat it ?

See solution of ex. 18.

ANSWERS.

N. B.—When a problem has two or more answers, with few exceptions only the last answer is given. When an answer is a sum of money, the mills are generally not regarded if less than 5; and when 5, or more than 5, are considered 1 cent. It is not deemed necessary or advisable to give the answers to all the problems in the book; and therefore the answers to many of the simpler ones are omitted. To determine whether or not any result is correct, students should be required to check the work.

Page **4.**—*3,* Twenty-four, 24. *6,* Seventy-five, 75. *9,* Eight hundred five, 805. *13,* Nine hundred ninety-nine, 999. *16,* 5 tens 1 one, 51. *18,* 4 hunds. 5 tens 5 ones, 455. *20,* 4 hunds. 0 tens 1 one, 401.

Page **5.**—*22,* 5 tens 8 ones, fifty-eight. *27,* 1 hund. 0 tens 1 one, 101. *28,* 8 hunds. 8 tens 8 ones, eight hundred eighty-eight.

Page **6.**—*2,* 75 bills. 326 mills. 424 thous. 3 hundreds, seventy-five billion three hundred twenty-six million four hundred twenty-four thousand three hundred. *5,* 80 bills. 0 mills. 160 thous. 3 units, eighty billion one hundred sixty thousand three.

Page **7.**—*2,* 6 thous. 46 units, 6,046. *7,* 43 thous. 8 units, 43,008. *15,* 13 bills. 13 mills. 13 thous. 13 units, 13,013,013,013.

Page **8.**—*3,* 421 dollars and 8 cents. *7,* 64 dollars and 5 mills. *8,* 300 dollars 5 cents and 4 mills. *13,* 5 mills. *16,* 125 dollars 48 cents 3 mills.

Page **9.**—*2,* $100.02. *4,* $.974. *7,* $43,071,009. *10,* $721.043.

Page **15.**—***3,*** 362. ***5,*** 362. ***7,*** 336. ***9,*** 7863. ***11,*** 2515. ***12,*** 2710420.

Page **16.**—***14,*** 404. ***16,*** 4,320. ***18,*** 5,164. ***20,*** 54,545. ***22,*** 50,248. ***24,*** 67,786. ***25,*** 4,578 lb. ***26,*** $11,775. ***27,*** 181.

Page **17.**—***29,*** 465,146. ***32,*** 4,766 lb. ***34,*** 9187. ***36,*** $3,075. ***37,*** $274.92. ***39,*** $10,408.86. ***41,*** $21,279.50. ***43,*** 4,874,618.

Page **18.**—***44,*** 9,030,349. ***45,*** 55,820.—***2,*** $3,955.15. ***4,*** 2,176. ***6,*** 703 mi.

Page **19.**—***8,*** $9,463.40. ***10,*** 3,812 mi. ***12,*** 96,162. ***14,*** 1,570. ***16,*** 1,570.

Page **20.**—***18,*** $295,038.48. ***20,*** $39,191.62. ***22,*** $12,572.

Page **26.**—***1,*** 18. ***4,*** 51. ***5,*** 147. ***7,*** 1,472. ***9,*** 77,607.

Page **27.**—***11,*** $3.35. ***13,*** 4552 cents. ***15,*** 90,626. ***17,*** 104,134. ***19,*** 13,516. ***21,*** 9,999. ***23,*** 482,995. ***25,*** 5,001,105. ***27,*** 43,209,907. ***29,*** 82,577. ***31,*** 88,357. ***33,*** 9,326,982. ***35,*** 9,889,899. ***36,*** 96,797. ***37,*** 67,219. ***38,*** 239,502. ***39,*** 3,776,385. ***41,*** $73.35.

Page **28.**—***44,*** 588 lb. ***46,*** 46 yr. ***48,*** 801,024. ***49,*** 28,787.

Page **29.**—***52,*** $251.29. ***53,*** $367.40. ***54,*** 1,237,311 sq. mi. ***56,*** 115,815.—***2,*** $2,373.25. ***4,*** 791 ft. ***6,*** $90.35.

Page **30.**—***8,*** $3,594.85. ***10,*** $451.45. ***12,*** $1,942.25. ***14,*** $221.15. ***16,*** 50 yr. ***18,*** $490.26. ***20,*** 58,219 sq. mi.

Page **36.**—***1,*** 975. ***2,*** 3,882. ***3,*** 3,025. ***4,*** 2,922. ***8,*** 7,884. ***11,*** 3,269. ***13,*** 18,304. ***15,*** 60,688. ***18,*** 58,728.

Page **37.**—***19,*** 9,268 lb. ***21,*** 4,480. ***23,*** 42,240. ***24,*** $131,179. ***26,*** 6,246. ***27,*** 18,648. ***29,*** 28,350. ***31,*** 92,016.

Page **38.**—***35,*** 176,220,000. ***37,*** 1,635,500,000. ***39,*** 62,280,000,000. ***41,*** 216,000 lb.

Page **39.**—***1,*** 1,536. ***2,*** 9,500. ***3,*** 45,528. ***5,*** 2,890. ***6,*** 56,520. ***9,*** 5,382. ***13,*** 187,318. ***14,*** 4,749,972. ***15,*** 7,846,272. ***17,*** 1,956,564. ***19,*** 8,974,702. ***22,*** 156,466,720. ***23,*** 6,351,105,618. ***25,*** 11,234,275. ***26,*** 2,361,055,599. ***28,*** 2,026,441,428. ***30,*** 3,841,167,050. ***32,*** 1,199,528,823. ***34,*** 1,833,590,250.

36, 1,533,115,750. ***38,*** 1,694,006,080. ***40,*** 94,752. ***42,*** 47,430 lb. ***43,*** 101,888.

Page **40.**—***46,*** 9,711. ***48,*** 15,696. ***50,*** 19,012 lb. ***52,*** 48,272 lb. ***54,*** 43,200 lb. ***56,*** 21,900 lb.—***2,*** $3,216. ***4,*** 11,655 da. ***6,*** $5,122. ***8,*** 63,360.

Page **41.**—***10,*** 481 mi. ***12,*** $1,569.44. ***14,*** $368.75. ***16,*** $797.60. ***18,*** $327.70. ***20,*** 6,180. ***22,*** 253,485.

Page **42.**—***24,*** 8,763. ***26,*** $27,744. ***28,*** 94,185. ***30,*** 2,160. ***32,*** 70,215,270,400.

Page **48.**—***2,*** 2,377. ***3,*** 486. ***4,*** 2,113. ***5,*** 2,701. ***8,*** 9,526, 6 rem. ***11,*** 8,125, 3 rem. ***12,*** 4,167, 1 rem. ***14,*** 7,146, 2 rem. ***16,*** 68,598, 4 rem. ***18,*** 6,885, 5 rem. ***20,*** 2,627, 4 rem. ***23,*** 74,022, 3 rem. ***26,*** 68,079. ***28,*** 2,714, 2 rem.

Page **49.**—***30,*** 6,216. ***32,*** 244. ***34,*** 626. ***36,*** 5,087.

Page **50.**—***4,*** 5, 71 rem. ***5,*** 7, 17 rem. ***6,*** 7, 48 rem. ***7,*** $8\frac{4}{13}$. ***8,*** $3\frac{54313}{97353}$. ***9,*** $4\frac{2235}{63854}$.

Page **51.**—***1,*** 33. ***2,*** 132. ***3,*** 225. ***4,*** 417. ***5,*** 450. ***6,*** $114\frac{8}{19}$. ***7,*** $47\frac{50}{76}$. ***9,*** 762. ***11,*** 888. ***13,*** 345. ***15,*** 672. ***17,*** 2454. ***19,*** 7426. ***21,*** 1782. ***23,*** 272 rem. ***25,*** 469.

Pago **52.**—***26,*** $93\frac{525}{626}$. ***27,*** $168\frac{13}{587}$. ***29,*** $13\frac{5390}{5421}$. ***30,*** 134. ***31,*** 225. ***33,*** 7,328. ***35,*** 1276. ***37,*** 180071. ***38,*** 25. ***40,*** 186. ***41,*** 873. ***43,*** 1178. ***45,*** 113. ***47,*** $\$136\frac{360}{365}$.

Page **53.**—***49,*** 207. ***51,*** 3750 hr. ***53,*** 125. ***54,*** 225.

Page **54.**—***1,*** 3, 31 rem. ***2,*** 10, 35 rem. ***3,*** 6, 3 rem.

Page **55.**—***4,*** 237 rem. ***6,*** 13, 5 rem. ***8,*** 18 quo. 560 rem. ***10,*** $12. ***12,*** 230 cts. ***14,*** 260. ***16,*** 5 and 22,180 sq. mi. over ***17,*** about 28.

Page **57.**—***1,*** $2070. ***2,*** $323.

Page **58.**—***3,*** $1088. ***5,*** $1038. ***7,*** 1548 in. ***10,*** 43 A. ***11,*** 65 da. ***13,*** $6120. ***14,*** $4305.—***2,*** 125. ***4,*** $85. ***6,*** 1802. ***8,*** 24 bu.

Page **59.**—***10,*** 9. ***12,*** 12. ***14,*** 96 ft. ***16,*** 451 lb. ***18,*** 540. ***20,*** 58 bar. ***22,*** $177.12. ***24,*** 30 da. ***26,*** $51.

Page **60.**—***28,*** $510. ***30,*** 550. ***32,*** $5850.

Page **63.**—***1,*** 2^2, 3^2, 7. ***3,*** 3^4, 5^2. ***4,*** 2^{10}, 3, 7. ***6,*** 2, 3^2, 7,

31. *8,* 2, 5, 7^2, 11, 13. *10,* 2^3, 3, 5^2, 13. *12,* 2^7, 503. *14,* 2, 3, 5, 7, 11. *16,* 2^3, 3, 7^2, 43.

Page **64.**—*2,* 32. *4,* 36. *6,* 54. *8,* 53. *9,* 31. *11,* 7. *13,* 13.

Page **65.**—*16,* 1021. *17,* 1117. *18,* $37: 20, 27, 31. *20,* 54 bu., 36 loads. *22,* 11 ft., 256. *24,* 33.

Page **67.**—*1,* 240. *2,* 6300. *4,* 51051. *6,* 5160. *8,* 6300. *10,* 9702. *12,* 5940. *14,* 2400. *15,* 1260. *17,* 3600 yd. *19,* $2100. *20,* 2100 min., 105, 84, 70, 50.

Page **69.**—*1,* 2. *3,* 3. *5,* 4. *7,* 5. *9,* 5. *11,* 336. *13,* 32. *15,* 400 hr. *16,* 30. *18,* 7 cts.

Page **69.**—*1,* $\frac{5}{6}$, $\frac{7}{9}$, $\frac{3}{7}$. *2,* $\frac{5}{6}$, $\frac{5}{7}$. *3,* $\frac{5}{11}$, $\frac{2}{3}$, $\frac{22}{39}$. *4,* $\frac{1}{2}$. *5,* $\frac{19}{31}$. *6,* $\frac{3}{20}$. *7,* $\frac{7}{10}$. *8,* $\frac{63}{149}$. *9,* $\frac{19}{25}$. *10,* $\frac{15}{37}$. *11,* $\frac{10}{31}$. *12,* $\frac{17}{19}$.

Page **80.**—*1,* $\frac{43}{3}$. *3,* $\frac{840}{13}$. *5,* $\frac{63}{5}$. *7,* $\frac{1132}{15}$. *9,* $\frac{187}{10}$. *11,* $\frac{1723}{20}$. *13,* $\frac{221}{9}$. *15,* $\frac{5056}{51}$. *16,* $\frac{151101}{300}$.

Page **81.**—*1,* 20, $18\frac{3}{5}$. *2,* $16\frac{1}{4}$, $10\frac{7}{15}$. *3,* $37\frac{1}{3}$, $41\frac{3}{4}$. *4,* 15. *5,* $10\frac{18}{23}$. *6,* $77\frac{2}{21}$. *7,* $12\frac{1}{2}$. *8,* $8\frac{1}{3}$. *9,* $23\frac{69}{100}$. *10,* $8\frac{7}{24}$. *11,* $1\frac{13}{20}$. *12,* $63\frac{3}{4}$.

Page **83.**—*1,* $\frac{120}{144}$, $\frac{52}{144}$, $\frac{51}{144}$. *2,* $\frac{15}{60}$, $\frac{50}{60}$, $\frac{18}{60}$, $\frac{35}{60}$, $\frac{16}{60}$. *3,* $\frac{20}{84}$, $\frac{27}{84}$, $\frac{48}{84}$. *4,* $\frac{48}{80}$, $\frac{50}{80}$, $\frac{35}{80}$, $\frac{44}{80}$, $\frac{46}{80}$. *6,* $\frac{50}{75}$, $\frac{45}{75}$, $\frac{57}{75}$, $\frac{40}{75}$, $\frac{175}{75}$. *8,* $\frac{54}{72}$, $\frac{60}{72}$, $\frac{63}{72}$, $\frac{64}{72}$, $\frac{42}{72}$, $\frac{44}{72}$.

Page **85.**—*1,* $2\frac{1}{24}$. *2,* $2\frac{13}{25}$. *3,* $61\frac{1}{6}$. *5,* $1\frac{7}{10}$. *6,* $26\frac{11}{24}$. *8,* $2\frac{3}{5}$. *9,* $86\frac{14}{15}$. *11,* $2\frac{39}{80}$. *12,* $72\frac{23}{24}$. *15,* $236\frac{3}{8}$. *18,* $22\frac{11}{12}$. *19,* $15\frac{5}{12}$. *21,* $343\frac{1}{3}$. *23,* $76\frac{1}{15}$. *25,* $8\frac{71}{210}$. *27,* $$133\frac{58}{75}$.

Page **86.**—*28,* $146\frac{27}{40}$. *29,* $43\frac{1}{8}$. *30,* $$42\frac{37}{40}$. *32,* $281\frac{13}{24}$ pk. *34,* $$166\frac{11}{20}$. *36,* $15\frac{29}{40}$ yd., $$49\frac{13}{40}$. *37,* $103\frac{17}{40}$ mi., $$39\frac{11}{12}$. *41,* $31.

Page **88.**—*1,* $\frac{19}{110}$. *2,* $\frac{79}{224}$. *4,* $\frac{37}{189}$. *6,* $\frac{11}{40}$. *8,* $\frac{19}{80}$. *10,* $1\frac{11}{12}$. *12,* $4\frac{19}{28}$. *14,* $3\frac{25}{36}$. *16,* $10\frac{1}{2}$. *18,* $39\frac{31}{63}$. *20,* $86\frac{81}{82}$. *22,* $5\frac{3}{10}$. *24,* $1\frac{23}{24}$. *26,* $17\frac{5}{12}$. *28,* $80\frac{59}{60}$. *30,* $\frac{26}{55}$. *31,* $8\frac{1}{375}$.

Page **99.**—*32,* $\frac{7}{12}$. *33,* $\frac{3}{5}$. *34,* $1\frac{5}{12}$. *35,* $\frac{3}{4}$. *37,* $22\frac{43}{60}$ yd. *39,* $7\frac{1}{3}$ lb. *41,* $19\frac{1}{12}$. *42,* $77\frac{5}{24}$ yd. *43,* $$2\frac{1}{8}$.

Page **93.**—*1,* $\frac{2}{5}$. *2,* $\frac{4}{7}$. *3,* $\frac{5}{14}$. *4,* $\frac{2}{3}$. *6,* 1. *8,* $\frac{9}{28}$. *10,* 4. *12,* $2\frac{1}{2}$. *14,* $14\frac{2}{3}$. *16,* 12. *18,* $190\frac{2}{3}$. *20,* $2. *22,* $$7\frac{1}{2}$. *24,* $570\frac{5}{16}$ cts. *26,* 72 cts. *28,* $2185. *30,* 369. *32,* $836\frac{1}{4}$. *34,* $3461\frac{1}{2}$. *36,* $1091\frac{4}{5}$. *38,* $1020\frac{4}{9}$. *40,* $$11.61\frac{1}{3}$.

Page **94.**—*43,* $\$19.27$. *44,* $\$335\frac{5}{12}$, $\$172\frac{1}{2}$, $\$67\frac{1}{12}$. *46,* $\$34\frac{1}{6}$. *47,* $\$22$. *48,* $\$46\frac{1}{2}$. *50,* $34\frac{4}{5}$ yr., $14\frac{32}{35}$ yr. *52,* $\$1667\frac{1}{4}$. *54,* $\frac{7}{44}$. *56,* $\$256\frac{5}{14}$. *57,* $\$2700$.

Page **96.**—*1,* $\frac{5}{7}$. *2,* $\frac{2}{7}$. *3,* $\frac{3}{2}$. *5,* $5\frac{3}{5}$. *6,* $\frac{41}{63}$. *8,* $2\frac{1}{7}$. *11,* $2\frac{1}{4}$. *12,* $1\frac{5}{16}$. *15,* $1\frac{5}{7}$. *18,* $1\frac{36}{55}$. *21,* $3\frac{3}{20}$. *24,* $1\frac{121}{338}$. *27,* $15\frac{1}{6}$. *28,* $2\frac{1}{40}$. *29,* $\frac{1}{4}$. *31,* $\frac{2}{5}$. *32,* $\frac{665}{748}$. *33,* $\frac{10}{49}$. *34,* $2\frac{767}{1944}$. *35,* $\frac{4216}{30375}$. *38,* $\frac{21}{128}$. *40,* $4\frac{1}{3}$. *42,* 48.

Page **97.**—*44,* $3\frac{1}{3}$. *46,* $\frac{35}{122}$. *47,* $1\frac{31}{39}$. *50,* 1. *51,* $4\frac{40}{71}$. *53,* $\frac{9}{16}$. *54,* $\frac{3}{8}$. *56,* $\frac{1}{6}$. *58,* $1\frac{1}{5}$. *60,* $1\frac{1}{95}$. *62,* $4\frac{2}{5}$. *64,* $7\frac{9}{14}$. *66,* $5\frac{1}{15}$. *68,* $33\frac{9}{10}$. *70,* $109\frac{1}{3}$. *71,* $\$24\frac{2}{3}$.

Page **98.**—*72,* $\$155\frac{3}{7}$. *74,* $12\frac{2}{3}$ ft. *75,* $25\frac{3}{5}$ mi. *78,* $4\frac{29}{49}$. *80,* $16\frac{5}{22}$. *82,* $21\frac{3}{5}$. *85,* $3\frac{1}{11}$. *87,* $4\frac{6}{13}$. *89,* $3\frac{49}{52}$. *91,* 5.

Page **99.**—*92,* $56\frac{5}{8}$ lb. *94,* $\$2\frac{6}{35}$. *96,* $\$\frac{76}{77}$. *98,* $12\frac{148}{153}$. *100,* $21\frac{7}{10}$ bu. *102,* 35. *104,* $\$17\frac{1}{3}$. *105,* 48. *106,* $7\frac{13}{16}$. *107,* 545.

Page **100.**—*1,* $\frac{8}{9}$. *2,* $\frac{12}{23}$. *3,* $\frac{7}{40}$. *4,* $\frac{1}{7}$. *6,* $\frac{50}{287}$. *8,* $\frac{3}{4}$. *10,* $\frac{1}{8}$. *12,* $\frac{21}{20}$. *14,* $\frac{16}{9}$. *16,* $\frac{2}{25}$. *17,* $\frac{4}{21}$. *18,* $\frac{1}{6}$.

Page **101.**—*2,* $19\frac{1}{2}$. *3,* 6. *4,* 216. *6,* $53\frac{2}{3}$. *8,* 852. *10,* $\$387.50$. *12,* $\$16.50$. *14,* $\$17,600$.

Page **102.**—*16,* $\$10,044$. *18,* 6.9 cts.

Page **103.**—*4,* $2:10\frac{10}{11}$, $2:43\frac{7}{11}$. *5,* $4:5\frac{5}{11}$, $4:38\frac{2}{11}$. *6,* $5:10\frac{10}{11}$, $5:43\frac{7}{11}$. *8,* 288. *9,* 42. *10,* $10:54\frac{6}{11}$.

Page **104.**—*11,* $8:10\frac{10}{11}$. *12,* (1) 2 mi., (2) 53 mi. *13,* (1) $1:21\frac{9}{11}$, (2) $7:21\frac{9}{11}$, $7:54\frac{6}{11}$, (3) $11:43\frac{7}{11}$. *14,* $4:8\frac{8}{11}$, $4:34\frac{10}{11}$.—*2,* 6. *3,* 30. *4,* A 120, B 60, C 40. *5,* 30.

Page **105.**—*7,* 24 mi. *8,* 45 mi. *9,* $5\frac{5}{11}$. *10,* $\frac{4}{5}$ hr. *11,* 96. *12,* 40 min. *13,* 33 da. *14,* All 10, A 20, B 30, C 60.

Page **106.**—*2,* $26\frac{1}{24}$. *4,* $12\frac{17}{88}$. *6,* $1794\frac{19}{24}$ lb. *8,* $\$64\frac{47}{60}$. *10,* $34\frac{1}{2}$. *12,* 10. *14,* $204\frac{1}{4}$ mi. *16,* $8\frac{1}{3}$. *18,* $3\frac{19}{24}$. *20,* $\$146\frac{59}{120}$.

Page **107.**—*22,* 15 yr. *24,* $20\frac{4}{7}$. *26,* $15\frac{3}{16}$. *27,* $8\frac{17}{25}$. *29,* $\frac{4}{35}$ smaller. *31,* $\$30$. *33,* $28\frac{7}{8}$ ft. *35,* 120. *37,* $2\frac{5}{6}$.

Page **108.**—*39,* $13\frac{1}{3}$, $33\frac{1}{3}$, $13\frac{1}{3}$. *41,* $\frac{1}{4}$, $\frac{5}{16}$, $\frac{7}{16}$: 160 A, 200 A, 280 A. *43,* $2\frac{6}{17}$ hr. *45,* $\$30\frac{25}{26}$. *47,* 9. *49,* $\$5\frac{1}{4}$.

Page **109.**—*51,* $21\frac{1}{3}$ lb. *53,* $2\frac{1}{2}$. *55,* $\frac{4}{5}$, $\frac{5}{6}$. *57,* 600. *59,* $\$2240$. *61,* 36.

Page **112.**—*1,* Nine *and* 8 hundredths. *2,* Four *and* 62 thousandths. *3,* Nine hundred ninety *and* 99 thousand 99 hundred-thousandths. *4,* Seventeen thousand *and* 17 ten-thousandths. *5,* Eight ten-millionths.

Page **113.**—*1,* .17. *2,* .305. *3,* .00568. *4,* 14.1. *5,* 500.021. *6,* .000000013. *7,* 16.0000000016.

Page **114.**—*2,* .0500, .0370, .6041, 5.3000. *3,* 7.000, .035, .060, .140, .100. *8,* 2.10000, 16.00000, .37400, .00036.

Page **115.**—*1,* $\frac{1}{4}$. *3,* $\frac{16}{25}$. *5,* $\frac{29}{40}$. *7,* $\frac{1}{16}$. *9,* $\frac{32}{125}$. *11,* $\frac{41}{80}$. *13,* $\frac{9}{200}$. *15,* $\frac{439}{625}$. *18,* $\frac{5}{8}$. *19,* $\frac{1}{12}$. *20,* $\frac{1}{8}$. *21.* $\frac{5}{32}$. *23,* $\frac{3}{500}$. *25,* $\frac{3}{5}$. *27,* $\frac{5}{176}$. *29,* $\frac{3}{224}$. *31,* $8\frac{3}{5}$. *33,* $11\frac{1}{18}$. *35,* $3\frac{5}{12}$. *37,* $71\frac{7}{12}$. *39,* $12\frac{3}{80}$. *41,* $9\frac{27}{2200}$.

Page **116.**—*1,* .8. *2,* .25. *3,* .375. *5,* .15. *7,* .16. *9,* .425. *11,* .8125. *13,* .3409+. *15,* .024. *17,* .0555+. *19,* .0016+. *21,* 18.75. *23,* 9.125. *25,* 19.266+. *27,* 4.7375. *29,* .03625. —*1,* 35.595. *2,* 72.985. *3,* 3.345. *5,* 210.08595. *7,* 594.8407. *9,* 5455.0318. *11,* 1068.1425. *13,* 42.325.

Page **117.**—*15,* 341.274. *17,* \$737.1925. *19,* \$16.334. *21,* 50.3975 yr.

Page **118.**—*1,* 15.517. *2,* 3.536. *3,* 21.5256. *5,* 19.746725. *7,* 376.67412. *9,* 6.6535. *11,* 24.2061. *13,* 7.375. *14,* 3.17. *15,* 8.0525. *16,* 14.52. *17,* .3903 ft. *20,* 756.75 lb. *22,* .5927. *23,* 266.6205 mi. *24,* \$310.93. *25,* \$3.9975.

Page **119.**—*1,* 28.8. *2,* 22.656. *3,* 5132. *5,* .000305. *7,* 1081.92. *9,* 8.7188. *11,* 1.016688. *13,* .00108. *15,* .000147. *17,* .013824. *19,* 1.42462. *21,* 33.0225. *23,* .075924.

Page **120.**—*27,* 45760. *29,* 73.01. *31,* 407000. *33,* 22609. *35,* 2162.46. *37,* 72918. *39,* 1619800. *41,* 54.223. *43,* .32125. *45,* \$10484.375. *47,* 1203.625 lb. *49,* \$16.03. *50,* \$512.49. *51,* .04503125. *53,* \$99.0825.

Page **121.**—*54,* \$158.26.—*1,* .25. *2,* 7.4. *3,* 7300. *5,* .00009.

Page **122.**—*7,* 11100. *9,* .4261. *11,* .019. *13,* .408. *15,* 225. *17,* .48. *19,* 21.543+. *21,* 356.111+. *23,* 42.8. *27,*

.063659. *29,* .0003465. *31,* .00043. *33,* .47. *35,* 4.123. *37,* .016254. *39,* .00141+. *41,* 1.34. *43,* $9.35. *44,* 248.

Page **123.**—*45,* 90.9 yd. +. *46,* 119.3 +. *47,* 273. *49,* 24.75. *50,* 41.25. *51,* 45.66.

Page **124.**—*2,* $11⅓. *4,* $9.50. *6,* $163. *9,* $232.50.

Page **125.**—*2,* 36. *4,* 9⅘. *5,* 28. *7,* 294⅖, 110⅖, 73⅗.

Page **126.**—*1,* $26.95. *2,* $.86. *3,* $155.91. *4,* $77.40. *5,* $37.76. *6,* $218.26. *7,* $257.98.—*1,* $23. *2,* $30.41. *3,* $11.50. *4,* $2.23. *5,* $218.70.

Page **127.**—*1,* $4.07.

Page **128.**—*2,* $102.72. *3,* $54.19. *4,* $100.30. *5,* $35.23.

Page **129,** Art. 216.—*2,* $8.09. *4,* $.78+. *6,* $793.98. *8,* 7½ hr. *10,* $3537.69.

Page **130.**—*12,* $1.54 gain. *14,* 124. *16,* $5.26. *18,* $82.71. *23,* $\frac{9}{16}$. *25,* $\$\frac{503}{800}$, $62875. *26,* 186.49375. *27,* 1.56825+. *28,* .546875.

Page **131.**—*30,* 1200. *32,* $3,962.49. *34,* $2000. *36,* $102,356¼. *37,* $14.9755. *38,* $3.70. *39,* 601.60 bu.+. *40,* 69⅛, $293.78. *41,* $904.04.

Page **135.**—*1,* 35 yd. *2,* 45 yd. *3,* 28½ yd. *4,* 3774 yd. *6,* 5,838 yd. *7,* 74 ft. *8,* 120 ft. *10,* 233 ft. *11,* 10574 ft. *13,* 199 in. *14,* 561 in. *15,* 2009 in. *16,* 1935 in. *17,* 1822 in. *18,* 204132 in. *19,* 760926 in. *20,* 633767 in. *21,* 462 l. *22,* 30095 l. *23,* 44575 l. *24,* 80040 l.

Page **136.**—*26,* 2 ft. 3 in. *27,* 2 ft. 4 in. *28,* 87 rd. 1 yd. 1 ft. 6 in. *29,* 2 rd. 10 l. *30,* 2 yd. 1 ft. $\frac{5}{7}$ in. *31,* 177 rd. 4 yd. 10 in. *32,* 18¾ l. *33,* 186 rd. 3 yd. 2 ft. *35,* 1 ft. 1.5 in. *36,* 2 ft. 2.1 in. *37,* 5 yd. 1 ft. 2.04 in. *38,* 1 yd. 1 ft. 6.45 in. *39,* 2 rd. 12.5 l. *40,* 70 ch. *41,* 1 ft. 11.22 in. *42,* 257 rd. 2 yd. 1 ft. 3.12 in.

Page **137.**—*1,* 4 yd. 3 in. *2,* 9 yd. 8 in. *3,* 242 rd. 5 yd. 2 ft. 1 in. *4,* 2 ch. 3 rd. *5,* 20 rd. 2 yd. 1 ft. *6,* 3 mi. 240 rd. 1 yd. 1 ft. *7,* 102 rd. 3 yd. *8,* 5 mi. 43 ch. *9,* 1 mi. 34 rd. 5 yd. 1 ft. 3 in. *10,* 3 mi. 201 rd. 2½ yd. *11,* 12 ch. 2 rd. 14 l. *12,* 10 mi. 40 rd. 1 yd. 2 in. *14,* $\frac{1}{48}$ yd. *15,* $\frac{1}{44}$ rd. *16,*

$\frac{1}{496}$ rd. *17,* $\frac{10}{231}$ rd. *18,* $\frac{1}{7200}$ mi. *19,* $\frac{3}{6400}$ mi. *20,* $\frac{3}{640}$ mi. *21,* $\frac{19}{1920}$ mi. *23,* .5303 rd. +. *24,* .6363 rd. +. *25,* .1138 mi. +. *26,* 3193 mi. +. *27,* .3759 mi. +.

Page **139.**—*1,* 360 sq. rd. *2,* 30879 sq. yd. *4,* 118416 sq. rd. *5,* 96 sq. ft. *8,* 152550 sq. ft. *10,* 14009 sq. ch. *11,* 4 sq. yd. 5 sq. ft. 136 sq. in. *13,* 2 sq. rd. 22 sq. yd. 7½ sq. ft. *15,* 38 sq. rd. 2 sq. yd. 1½ sq. ft. *16,* 23 A. 63 sq. rd. 14¼ sq. yd. *19,* 2 Tp. 12 sq. mi. 272 A.

Page **140.**—*1,* 20836 cu. in. *2,* 174618 cu. in. *5,* 1167200 cu. in. *6,* 22160617 cu. in. *7,* 22 cu. yd. 6 cu. ft. *8,* 3 cu. yd. 10 cu. ft. 1094 cu. in. *10,* 26 cd. 72 cu. ft. 1343 cu. in. *12,* 20 cu. ft. 432 cu. in. *13,* 71 cu. ft. 192 cu. in. *14,* 1166.4 cu. in. *16,* 79 cu. ft. 898.56 cu. in. *17,* $\frac{1}{39}$ cu. yd. *19,* .046875 cu. yd. *20,* .725 cd.

Page **143.**—*1,* 31 gi., .96875 gal. *2,* 21 gi., .65625 gal. *5,* 59 pt., .921875 bu. *7,* 31 pt., .484375 bu. *9,* 3 gal. 2 gi. *11,* 3 qt. 1 pt. *13,* 4 qt. 1⅜ pt. *15,* 15.875 gal. *16,* 24.65625 bu.

Page **144.**—*17,* 61 gal. 3 qt. 1 pt. *18,* 154 gal. 2 qt. *19,* 72 gal. 1 gi. *23,* 26 bu. 1 pk. *24,* 32 bu. 3 qt. *25,* 36 bu. 3 pk. 1 pt. *29,* ⅛ gal.

Pago **146.**—*1,* 160268 oz. *2,* 238 oz. *5,* 4572 gr., .79375 lb. *6,* 3984 gr., .6916 lb. +. *9,* 3228 gr., .5604 lb. +. *10,* 506 gr., .08784 lb. +. *14,* 8 lb. 5 oz. 5 pwt. *15,* 1 lb. 7 oz. 8 pwt. 15 gr. *16,* 61 T. 13 cwt. *18,* 582 lb. 15 oz. *20,* 4 lb. 4 ℥ 5 ʒ 2 ℈. *21,* 1 lb. 7 ℥ 3 ʒ 1 ℈ 7 gr.

Page **148.**—*1,* 15,612 sec. *2,* 38 da. *4,* 509 hr. *5,* 2,305,465 min. *7,* 37 min. 30 sec. *8,* 17 hr. 52 min. 48 sec. *9,* 6 hr. 15 min. *11,* 6 da. 12 hr. 46 min. 15 sec. *13,* 20 da. 20 hr. *15,* $\frac{1}{3840}$ da. *16,* .2609375 da. *18,* 3 da. 7 hr. 13 min. 21 sec. *19,* (2) 10 hr. 2 min.

Page **151.**—*3,* 36,030″. *4,* 66,418′. *6,* .0536 cir. +. *8,* 2° 46′ 40″. *9,* 11° 17′ 42″. *11,* 42° 27′. *12,* 165° 37′ 30″. *13,* 9 cir. 280°.

Page **153.**—*1,* 512 far. *4,* 6157 far. *5,* 4811 far. *7,*

£.3875. *9,* 5s. 6d. $2\frac{2}{3}$ far. *10,* £8 9s. 3 far. *11,* £1 6s. 9d. *12,* £6 17s. *13,* £26 8s. 5d.

Page **154.**—*18,* $50.61. *19,* $1.52. *20,* $100.33. *21,* $.11. *24,* $3.89. *25,* $2.87. *28,* £25 18s. 6d. *29,* £31 6s. 5d. nearly. *31,* £205 9s. 8d. 3 far. *33,* £126 11s. 1 far. nearly. *34,* 3235.75 fr., 2623.95 mk. *36,* 5181.35 fr., 4201.68 mk.

Page **155.**—*1,* 71 rd. 5 yd. 6 in. *2,* £2 10s. 3d. 2 far. *3,* 48 lb. 8 ℥ 4 ʒ. *4,* 86 mi. 88 rd. 2 yd. 1 ft. *6,* £18 13s. 1d. 1 far. *8,* 28 bu. 1 qt. *9,* 16 T. 9 cwt. 31 lb.

Page **156.**—*11,* 17s. 6d. *12,* 2 rd. 4 yd. 2 ft. $1\frac{1}{2}$ in. *13,* 3 T. 12 cwt. 30 lb. 12 oz. *15,* 6 da. 17 hr. 45 min. *17,* (C. is for cir. and S. for sign) 7 S. 18° 5′. *19,* 6 mi. 214 rd. *20,* $24.70. *21,* nearly 229.

Page **160.**—*1,* £4 8s. 6d. *2,* 3 cwt. 10 lb. 9 oz. *4,* 1 bu. 3 pk. 7 qt. 1 pt. *6,* 43 rd. 14 ft. $1\frac{3}{4}$ in. *8,* 7 T. 12 cwt. 10 lb. 8 oz. *10,* 2 bu. 2 pk. 3 qt. $1\frac{1}{5}$ pt. *12,* 13 mi. 319 rd. 2 yd. 1 ft. *14,* 3. *15,* 11. *16,* 8. *17,* 9. *19,* 11.

Page **161.**—*2,* $2.61. *4,* $.62. *6,* $204.34. *8,* 314,496 sec. *10,* 1782 mi. *12,* 248 rd. 4 yd. 2 ft. 8 in. *14,* 7 lb. 8 oz. 7 pwt. 6 gr. *16,* 66. *19,* 314.9. *20,* 240.

Page **162.**—*22,* $55.65. *24,* $62.09. *26,* 22,325+. *28,* 1289 bu.+. *30,* 113 bu. *32,* 34 bu. 3 pk. 4 qt. *34,* 29 lb. 10 oz. 2 dr. 2 scr. *35,* 17 da. 16 hr. 57 min. 36 sec. *36,* 3 lb. 10 oz. 11 pwt. 9 gr. *37,* 3 lb. 1 oz. 1 pwt. 7 gr. *38,* 6 wk. 2 da. 4 hr. 21 m.

Page **163.**—*39,* Lost $10.79. *41,* $82. *43,* $102. *45,* $6,849\frac{27}{37}$. *47,* $210. *49,* $40. *51,* $1.38. *53,* $.08. *55,* 40. *57,* $5.13.

Page **164.**—*59,* 40. *61,* 39,528,000. *62,* 40,320. *64,* 21,932. *65,* 222 da. $5\frac{1}{3}$ hr. *67,* 876,576 hr. *68,* 7 da. 12 hr.

Page **166.**—*7,* 160 sq. in. *8,* 36 sq. ft. *10,* 1,558 sq. yd. *11,* $2\frac{1}{4}$ A. *12,* 6 A. *14,* $96. *15,* $255. *16,* $469.50. *18,* 3,600. *19,* $265.32.

Page **167.**—*20,* 22,320.—*1,* $122\frac{4}{9}$ sq. yd. *2,* $137\frac{1}{9}$ sq. yd. *3,* $173\frac{1}{18}$ sq. yd. *4,* $38.37. *5,* $12.85. *6,* $23.03. *7,* $10.46.

Page **168.**—*1,* 28$\frac{1}{3}$ yd., 30 yd. *2,* 45$\frac{1}{2}$ yd. *3,* $15.56 *4,* $2.40.

Page **170.**—*8,* $64. *9,* $82.50. *10,* $55.41. *11,* 504 bu. *12,* 562$\frac{1}{2}$ lb. *13,* 976$\frac{9}{16}$ *15,* 15.

Page **171.**—*16,* 5\frac{1}{2}$.—*1,* 90.—*2,* 33$\frac{3}{4}$. *4,* 224$\frac{32}{75}$. *5,* 72 da. *6,* 1822$\frac{1}{2}$.

Page **172.**—*2,* 24. *4,* 40.17+. *5,* 57.6. *7,* $75.20. Art. 315. *2,* 15.

Page **173.**—*4,* 810. *5,* $8.55. *6,* $12.84.

Page **174.**—*1,* 144. *3,* 76. *5,* 74.

Page **175.**—*1,* $392, $266.40. *2,* 43,947. *3,* $235.20. *4,* $1048.40.—*1,* 93$\frac{9}{11}$. *3,* 64.

Page **176.**—*5,* 29$\frac{31}{36}$ yd. *7,* $18.75. *8,* $40.62. *10,* $90.30. *12,* 27$\frac{3}{11}$ hr. *14,* $142.80 (the walls are supposed to be 1 ft. thick). *17,* 99.9 da.+.

Page **177.**—*19,* 4,320 lb. *21,* $182.91.

Page **181.**—*1,* 180 m. *3,* 1500 g. *5,* .425 l. *10,* .0006584 Kg. *11,* .0024 sq. Hm. *13,* .03937079 in. *15,* 1.5432 gr.

Page **182.**—*1,* $51.44. *2,* $1.97. *3,* 98.84 A. *4,* 10.0339 Ha. *5,* 88.9056 Kg. *6,* $22.51. *7,* 29 hr. 43 min.+. *9,* 3456 l. *11,* 139.482 Hg. *13,* 17.526 dm.

Pa ge **183.**—*15,* $5.64. *16,* $1059.48.

Page **185.**—*1,* 8° 15′. *2,* 10′ 30″. *3,* 1° 3′. *5,* 31° 48′.
7, 10° 30′ E. *8,* 31° 30′ W. *10,* 45° 8′ W. *11,* 80° 16′ 45″ E.
13, 97° 0′ 36″ W. *14,* 22° 33′ 39″ W.—*1,* 10 min. 17$\frac{14}{15}$ sec.
2, 29 min. 56$\frac{3}{5}$ sec.

Page **186.**—*3,* 5 hr. 5 min.
sec. *6,* 7 hr. 57 min. 59$\frac{1}{5}$ sec. A. 23$\frac{1}{15}$ sec. *5,* 2 hr. 9 min. 31$\frac{11}{15}$
sec. A. M. *10,* 52 min. 12$\frac{13}{15}$ sec. P. M. *8,* 9 hr. 52 min. 20$\frac{2}{15}$

Page **187.**—*2,* .07. *5,* 1.25. *14,* .06.
2.105. *19,* $\frac{1}{25}$. *22,* $\frac{5}{4}$. *25,* $\frac{3}{4}$. *30,* $\frac{14}{5}$. *31,* .25. *16,* .066. *17,*
$\frac{23}{8}$. *35,* $\frac{1}{200}$. *38,* $\frac{1}{68}$. *40,* 50 %. *42,* 25 %. *4* $\frac{1}{6}$. *33,* $\frac{1}{6}$. *34,*
120 %. *51,* 60 %. *45,* 66$\frac{2}{3}$ %. *49,*

Page **190.**—*1,* $440.40. *3,* $21.87. *4,* $38.641.
7, 4.788. *9,* 1836. *10,* $11,016. *11,* $222. *12,* *5,* $39.60.
$845.52.

14, \$434. *16;* \$30.80, \$24.20. *17,* \$117. *19,* 429 bu. oats. *20,* \$13,656$\frac{2}{3}$.

Page **192.**—*1,* $16\frac{2}{3}\%$, 5%. *2,* $2\frac{2}{29}\%$, $7\frac{1}{2}\%$. *3,* $37\frac{1}{2}\%$, $4\frac{1}{2}\%$. *5,* $1\frac{2}{3}\%$, $\frac{175}{256}\%$. *7,* 96%. *8,* $28\frac{8}{9}\%$. *10,* 10%. *11,* $16\frac{2}{3}\%$, $83\frac{1}{3}\%$.

Page **193.**—*13,* $53\frac{1}{3}$. *14,* 12%. *15,* $33\frac{1}{3}\%$. *16,* $7\frac{11}{12}\%$. *18,* $21\frac{1}{3}\%$.

Page **194.**—*2,* 5440. *3,* 744. *4,* 1488. *5,* $14222\frac{2}{9}$. *6,* $4385\frac{5}{7}$. *9,* \$2100. *11,* 1400 ft. *12,* $714\frac{2}{7}$. *14,* \$3125. *15,* $41\frac{11}{29}$ yr.

Page **195.**—*16,* 185 ft. *17,* 4500 bu. *18,* 130. *19,* \$86,800. *20,* \$8,400. *24,* \$20.

Page **196.**—*1,* 160. *2,* \$180.69+. *3,* 240. *4,* \$360.50. *7,* $\frac{24}{35}$. *8,* $2\frac{2}{3}$ da.

Page **197.**—*9,* 140. *10,* 200 bu. *11,* $\frac{7}{8}$. *13,* $\frac{2}{3}$. *15,* 340. *16,* 180. *17,* \$900. *18,* \$75. *19,* $10\frac{20}{23}$ cts. *20,* \$40. *21,* \$9,500. *22,* $6\frac{3}{4}$ cts. *23,* $8\frac{4}{5}$ cts. *24,* $7\frac{1}{2}\%$. *25,* 25%. *26,* \$4,280. *27,* 2d. \$114, 3d. \$120.

Page **199.**—*2,* \$13.12. *4,* \$520. *6,* 20%. *8,* $6\frac{2}{3}\%$, $6\frac{1}{4}\%$. *10,* \$6250. *12,* \$120. *14,* \$90.

Page **200.**—*16,* 2635 bu. *18,* 800 bu. *20,* 12%. *22,* \$2.40 gain. *24,* \$62.50.

Page **201.**—*26,* \$164.80 loss. *28,* \$64. *30,* \$362.25. *32,* \$2.50. *34,* .\$0. *36,* \$352. *38,* $69\frac{1}{3}$ cts. *39,* \$48. *40,* $53\frac{1}{3}\%$.

Page **203.**—*2,* \$89.10. *4,* \$2.10 agent. *6,* $1\frac{1}{4}\%$. *8,* 3%. *10,* \$5840. *12,* 50. *13,* 5750 lb.

Page **204.**—*15,* \$96.25. *16,* 3200 yd. *17,* 156.8 bar.

Page **205.**—*2,* \$46.67. *4,* \$16.73. *6,* \$271.33. *8,* \$10,173.125. *10,* $3\frac{1}{2}\%$. *12,* $1\frac{3}{4}\%$. *14,* \$361.20. *16,* \$1200. *18,* \$9.375.

Page **206.**—*20,* \$23,764. *21,* \$40,000.

Page **207.**—*2,* \$12,358.97. *3,* 2%, A \$12, B \$15, C \$19.50. *4,* $1\frac{1}{2}\%$; A \$78.60. *5,* $1\frac{1}{8}\%$, A \$65,975.

Page **209.**—*2,* \$1123.20. *4,* \$338.33. *6,* \$920. *8,* \$110.31. *10,* \$822.21. *12,* \$7.30.

Page **210.**—*14,* $250.92. *16,* $339.19. *18,* $27. *19,* $227.42.

Page **211.**—*2,* $128.24. *4,* $28.71. *6,* $4.55. *8,* $19.85. *10,* $1.55. *12,* $122.28.

Page **212.**—*19,* $637.42. *21,* $56.32. *25,* $53.95. *26,* $169.93. *27,* $442.55.

Page **213.**—*2,* $65.625. *4,* $1.26. *6,* $1.50. *9,* $32.01. *11,* $2.72.

Page **214.**—*1,* $1.04. *2,* $881.57. *3,* $7.68. *4,* $9.65. *5,* $253.89. *6,* $16.92. *7,* $1676. *8,* $70.40. *9,* $3531.67. *10,* $6942.19.

Page **215.**—*2,* 5%. *4,* 7% *6,* 7½% nearly. *7,* 7%. *8,* 7%. *11,* 5½%. *12,* 8%.

Page **216.**—*2,* 1 yr. 3 mo. *4,* 5 yr. 7 mo. 25 da. *6,* 1 yr. 1 mo. 10 da. *7,* 3 yr. 4 mo. *8,* 6 yr. 10 mo. 18 da. *11,* $14\frac{2}{7}$ yr., $28\frac{4}{7}$ yr.

Page **217.**—*12,* 2 yr. 2 mo. 25 da. *13,* 6 yr. 16 da.—*2,* $64. *3,* $84.

Page **218.**—*6,* $6568.40. *7,* $864.18. *8,* $1020.96.

Page **220.**—*1,* $130.73. *2,* $224.63. *3,* $46.54. *4,* $149.21.

Page **222.**—*2,* $67.24. *3,* 231.21. *5,* $197.56.

Page **223.**—*2,* $2.64. *3,* $2.85. *5,* $122.66.

Page **224.**—*1,* $80.34. *2,* $61.50. *3,* $155.82. *5,* $484. *6,* $442.16. *7,* $51.13. *9,* $35.11. *11,* $140.72. *12,* $185.21. *13,* $542.23.

Page **225.**—*18,* $61.95. *20,* $520.86. *22,* $895.52. *24,* $3000. *25,* $400+.

Page **228.**—*1,* $6078.90. *2,* $75.82. *4,* $664.525.

Page **229.**—*1,* $322. *2,* $334.40. *4,* $450. *6,* $654.08. *7,* $1013.472. *9,* $159.83. *11,* $1. *13,* 50%.

Page **231.**—*2,* $200. *4,* $400. *6,* $180. *7,* $25. *8,* $60.75. *10,* $1455.93. *11,* $1. *12,* $6.64. *14,* $663.81.

Page **233.**—*1,* $147.64. *3,* $377.16. *5,* $35.78. *7,* $1469.25. *8,* $147.38.

Page **234.**—*9,* $998.73.

Page **235.**—*1,* Date of maturity Aug. 21, term of dis. 36 da., dis. $1.93, proceeds $383.07. *2,* Date of maturity Jan. 5, term of dis. 111 da., dis. $25.90, proceeds $1374.10. *3,* Date of maturity Nov. 23, term of dis. 77 da., dis. $174, proceeds, $133.76. *4,* Date of maturity March 19, term of dis. 156 da., dis. $20.39, proceeds $729.61. *5,* Date of maturity March 3, term of dis. 23 da., dis. $2.69.

Page **236.**—*6,* Term of dis. 36 da., dis. $5.48. *7,* Term of dis. 64 da., dis. $5.33.—*1,* $2000. *2,* $835.70. *3,* $885.25.

Page **237.**—*4,* $1273.10. *5,* $961.87.

Page **238.**—*2,* $4,561.88. *3,* $10,080. *4,* $2163. *5,* $17,980. *7,* 120. *8,* 40. *10,* 250. *12,* $19,300. *13,* $17,388. *14,* $7,750.

Page **239.**—*16,* $400. *17,* $210. *19,* $6\frac{1}{4}$ %. *20,* $6\frac{2}{3}$ %. *21,* $3\frac{1}{3}$ %. *22,* 8 % bonds. *23,* $3\frac{1}{3}$ %. *25,* 70 or 30 % dis. *26,* 60 % prem. *27,* 96. *28,* $422.50.

Page **240.**—*29,* $12,195. *30,* $1,090. *31,* 112. *32,* 150. *33,* $75\frac{1}{4}$. *34,* $2,450.

Page **243.**—*2,* $510. *3,* $364.05. *5,* $838.10. *7,* $270.38. *9,* $1435.68. *10,* $644.15. *11,* $584.55. *12,* $1,234.02. *13,* $2493.40. *15,* $400. *16,* $40. *18,* $33. *20,* $110.

Page **244.**—*22,* $2000. *23,* $1200. *24,* $9704.43. *25,* $1008.11.

Page **245.**—*1,* 2, 5, $\frac{3}{5}$. *2,* $1\frac{1}{3}$, $\frac{3}{7}$, $3\frac{1}{3}$. *4,* $2\frac{2}{3}$, $1\frac{1}{2}$, 4. *5,* $4\frac{2}{7}$, $10\frac{2}{3}$, $\frac{2}{19}$. *6,* $2\frac{1}{5}$, $\frac{1}{30}$, $22\frac{6}{7}$. *7,* 9. *8,* 6. *9,* $3\frac{1}{16}$. *11,* they are equal. *12,* they are equal. *13,* 15 : 20. *14,* they are equal.

Page **246.**—*15,* 0. *16,* $\frac{128}{315}$. *17,* $\frac{11}{200}$. *18,* $\frac{29}{200}$.

Page **248.**—*1,* $x = 8$. *2,* $x = 11\frac{1}{3}$. *5,* $x = 12$. *6,* $x = 150$. *9,* $x = \frac{7}{8}$. *10,* $x = \frac{80}{119}$. *13,* $x = \frac{1}{9}$. *14,* $x = \frac{1}{4}$. *17,* $x = \frac{2}{3}$. *18,* $x = 36$.

Page **249.**—*1,* $40. *3,* 90 ft. *4,* 126. *6,* $151.41. *8,* $256\frac{1}{2}$ mi. *10,* $253.75. *12,* 96.

Page **250.**—*14,* $70.215. *16,* $39.60. *17,* $3\frac{1}{3}$ lb. *18,* $343. *20,* $3.60. *21,* 14 rd. *22,* $25\frac{1}{2}$.

Page **251.**—*24,* \$567. *26,* 14. *29,* 12 mo.

Page **252.**—*30,* 20 da. *32,* 336. *33,* \$179.20. *34,* 10530 lb. *35,* \$$13\frac{1}{9}$. *37,* 15. *38,* 2 ft. *39,* $68\frac{4}{7}$ rd. *40,* 3 in. *41,* 35.

Page **253.**—*2,* \$108, \$270, \$45. *4,* 210 da., 490 da., etc. *6,* 1155 lb., 1848 b., 3850 bb. *8,* 234, 273, 481. *10,* 2 oz. 2 pwt.

Page **254.**—*12,* 33.6 cu. yd. Ox., 126.4 cu. yd. Ni. *16,* 17, 34, 221. *17,* 14, 70, 224. 420.

Page **255.**—*1,* \$420, \$490. *3,* $222\frac{2}{9}$, \$$177\frac{7}{9}$, \$136. *5,* \$187.92, \$221.48, \$281.88, \$308.72. *6,* $19\frac{1}{3}$ ct., $20\frac{2}{3}$ ct.

Page **256.**—*7,* A \$$2828\frac{4}{7}$, B \$$3651\frac{3}{7}$. *9,* \$135.20, \$241.80. *10,* \$429.25, \$681.75. *11,* \$1828.88, \$1516.04, \$1155.08.

Page **257.**—*12,* \$530.96, \$1216.37, \$622.67.— *1,* $21\frac{1}{2}$ ct. *3,* \$5.99.

Page **258.**—*4,* \$2.15. *5,* $74\frac{7}{12}$°.

Page **259.**—*1,* 5, 1, 2, 3. *2,* 3, 3, 2, 1, 2, 2. *4,* 5, 5, 12. *5,* 1, 1, 2, 4. *6,* 6, 6, 3, 2.

Page **260.**—*8,* 18, 9, 18, 27. *10,* 5, 5.

Page **261.**—*12,* 24, 15, 3, 9. *13,* 45, 9, 9. *14.* 5, 1, 94.

Page **262.**—*1,* 4 mo. 18 da. *3,* 12 mo. 17 da. *5,* Feb. 8.

Page **263.**—*7,* Aug. 19. *8,* Feb. 11. *9,* June 27. *10,* Nov. 8.

Page **264.**—*2,* 1764. *3,* 132651. *4,* .0025. *5,* .000015625. *8,* $\frac{9}{16}$. *9,* $410\frac{1}{16}$. *13,* $37\frac{1}{27}$. *17,* $85\frac{9}{16}$.

Page **269.**—*1,* 34. *2,* 125. *3,* 723. *4,* 2401. *7,* 545. *8,* .027. *9,* 49. *12,* 12.96. *14,* 14.2. *16,* 2780. *19,* .733. *20,* .0145. *21,* 37.97 +. *22,* $291\frac{2}{3}$. *23,* .425.

Page **270.**—*24,* 1.414. *25,* 4.242. *26,* .816. *27,* 1.212. *28,* 1.732. *30,* .612. *31,* 3.561. *33,* 7.071. *35,* 5.007.—*2,* 40 yd. *3,* 220. *4,* \$15.12. *5,* 69.57 +. *7,* 85.

Page **271.**—*10,* 52 yd. *12,* 70 yd. *13,* 51.6 ft. *14,* 130 mi. *16,* 117 ft. *17,* 224 ft.

Page **276.**—*1,* 24. *2,* 75. *3,* 36.4. *5,* 27. *6,* 6.25. *9,* 572. *2,* .74. *12,* 3245. *13,* 12. *15,* 243. *16,* 1.2599. *17,* 1.4422. *18,* 1.7099. *19,* 1.8171. *20,* .8735. *21,* .9085. *22,* .8735. *23,* .7368.—*1,* 13 ft. *2,* 78 ft.

Page **277.**—*3,* 1350 sq. yd. *4,* 4.6 ft. +. *5,* 2025. *6,* 12.9 in. +. *7,* 6 ft. *8,* \$74.11$\frac{1}{4}$.

Page **278.**—*1,* 41. *2,* 59. *4,* 8$\frac{1}{2}$. *5,* 46. *7,* 31.

Page **279.**—*8,* \$5. *9,* 46 yr.—*1,* 160. *2,* 220. *3,* 46. *4,* 1580. *6,* 112. *7,* 85. *8,* 525 mi. *10,* \$16.48.

Page **280.**—*11,* 1610 ft. *12,* \$121. *13,* 11$\frac{1}{4}$ mi.

Page **281.**—*1,* 3,584. *2,* 5,120. *3,* 6$\frac{26}{81}$. *5,* $\frac{2187}{8388608}$. *6,* 1215. *7,* \$256,000. *9,* \$638.33.—*1,* 1,275. *2,* 5465. *3,* 127$\frac{15}{16}$.

Page **282.**—*4,* 128$\frac{1}{16}$. *5,* 63$\frac{15}{16}$. *6,* 7$\frac{343}{432}$. *7,* 43$\frac{58592}{78125}$. *8,* \$8857.30. *10,* \$20,971.51.

Page **283.**—*1,* 13 in. *3,* 17 ft. *6,* 102 ft. *7,* 56 in. *8,* 37.6992 ft.

Page **285.**—*10,* 11.45 ft. +. *11,* 134 yd. +. *12,* 14,1372 ft.—*1,* 32$\frac{1}{2}$ sq. ft. *2,* 2 A. 121 sq. rd. *3,* 30. *4,* 1$\frac{3}{4}$ A *5,* 27.

Page **286.**—*7,* 6 sq. in. *8,* $\frac{3}{4}$ A. *9,* 43.3A +. *11,* 9.7 A +. *12,* 140 sq. ft. *14,* 16$\frac{4}{5}$. *15,* \$1031.25.

Page **287.**—*16,* 113,0976 sq. ft. *17,* 38.4846 sq. ft. *18,* 11.45 sq. ft. +. *19,* 4$\frac{1}{2}$ yd. +. *20,* \$220.89. *21,* 166$\frac{1}{2}$ ft. +.

Page **288.**—*22,* 270 sq. ft. *23,* 210 sq. ft. *25,* 47.124 sq. ft. *26,* 150 sq. ft. *27,* 113.0976 sq. ft. *28,* 84 sq. ft.

Page **290.**—*31,* 1485 sq. ft. *32,* 216.77 sq. yd. +. *33,* 310.8 sq. ft. +. *34,* 3.413 sq. yd. +. *35,* 28.2744 sq. in. *36,* 706.86 sq. in. *37,* 196,663,355$\frac{3}{4}$.

Page **291.**—*3,* 294$\frac{2}{3}$ cu. in. *4,* 14.1372 cu. ft. *5,* \$.30. *6,* 4136.14 gal. +.

Page **292.**—*9,* 866 cu. in. +. *10,* 3490$\frac{2}{3}$ cu. yd. *11,* 33$\frac{3}{4}$ cts. *12,* 20.196. *14,* 4188.8 cu. in. *15,* 113.0976 cu. ft. *16,* 14.1372 cu. in. *17,* 69.34 lb. +.

Page **294.**—*1,* 400 A. *2,* 240 A. *3,* 120 A. *6,* 120 A. *8,* 200 A.—*2,* \$112.50. *4,* 21.6. *6,* 20 ft.

Page **295.**—*8,* 29 ft. *9,* 1.414 +. *10,* 1.732 +. *12,* \$149.-

24. *14,* 150 ft. 234 ft., 410 ft. *16,* 6635. *18,* 6.426 A, *20,* 4 ft. $3\frac{1}{4}$ in., nearly. *22,* 345.576 sq. in.

Page **296.**—*24,* 282.744 sq. in. *26,* \$2.45. *28,* 2,632 gal. *30,* 714.08 lb +. *31,* 113. *32,* 235.8818 lb. *33,* 360 A.

Page **300.**—*14,* 229. *15,* 31,320. *16,* 1,475.

Page **301.**—*17,* 39, 9768. *18,* $15\frac{3}{4}$. *19,* $2\frac{1}{5}$. *20,* \$$12\frac{29}{40}$. *21,* 9. *22,* \$88. *23,* $\frac{129}{140}$. *24,* $27\frac{29}{36}$. *25,* $1\frac{1}{5}$. *27,* $\frac{2}{25}$. *28,* 24. *29,* \$35.85. *30,* \$35. *31,* 40 cts. *32,* 40 gal. *33,* 72. *34,* 78, 52, 39. *35,* 26.750375.

Page **302.**—*36,* 110.4155. *38,* 6.73 +. *39,* $98\frac{1}{5}$ ft. *40.* 25 mi. *41,* $39\frac{3}{23}$ mi. *42,* 64 lb. *43,* 23 mi. *44,* \$236.67. *45,* .75. *46,* $\frac{5}{32}$, *47,* \$34,275. *48,* 2500. *49,* 7 oz. 10 pwt. *50,* 7,040. *51,* \$77.76. *52,* $2,681\frac{1}{4}$ *53,* \$24.75.

Page **303.**—*54,* \$1500. *55,* \$40. *56,* 15 %. *57,* $77\frac{1}{7}$ %. *58,* 90 %. *59,* $82\frac{2}{7}$ %. *60,* \$90. *61,* 12 %. *62,* \$13,000. *63,* $32\frac{24}{43}$ %. *64,* $18\frac{2}{11}$ %. *65,* \$12. *66,* $4\frac{1}{2}$ cts. *67,* 68 eggs, cost \$1.98. *68,* $31\frac{59}{81}$ %.

Page **304.**—*69,* 24. *70,* 63.2 +. *71,* 372.02 +. *72,* 49.93 mi. +. *73,* 15 ft. by 9 ft. *74,* 60 rd. *75,* 15 ft. *76,* 7 in., 21 in., 42 in. *77,* 2.18 sq. ft. +. *78,* 720. *79,* 316. *80,* \$2500. *81,* 21.

Page **305.**—*82,* 64. *84,* \$1360.73. *85,* \$13.80, \$19.32, \$22.08. *86,* 6, 2, 4. *87,* 18. *88,* 12 mo. *89,* $163\frac{1}{3}$ sq. in. *90,* $8\frac{4}{5}$ mo. *91,* 180 ft. *92,* $6\frac{2}{9}$ mo. *93,* \$2,400.

Page **306.**—*94,* 336 sq. ft. *95,* 3789. *96,* 124. *97,* 64 sq. ft. *98,* $7\frac{1}{7}$ hr. *99,* $1\frac{7}{17}$ da. *100,* $3\frac{1}{2}$ da. *101,* 3 : 36. *102,* \$60 gain. *103,* $113\frac{1}{14}$. *104,* \$4900. *105,* $41\frac{1}{9}$ %. *106,* $2\frac{3}{5}$ +.

Page **307.**—*108,* 1621.0656 cu. in. *109,* 12 ft. *110,* \$1194.26. *111,* \$518.74. *112,* \$6059.19. *113,* 60. *114,* Saturday, 2 A.M. *115,* \$8.40, \$5.60, \$4.20. *116,* \$120. .*117,* 1,331,000. *118,* $\frac{13}{33}$.

Page **308.**—*119,* \$147.96. *120,* \$1072. *121,* $5\frac{5}{27}$ d. *122,* 625, *123,* 4700.16. *124,* As part \$6648.48. *125,* 18° East. *126,* 11 : $35\frac{5}{17}$. *127,* $12\frac{1}{6}$ mi. *128,* 15 in.

Page **309.**—*129,* 3 m. 5 w. *130,* \$18, \$8, \$2.40. *131,* 10 da.

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